Ismat M. El Hassan
College of Engineering
University of Riyadh
Riyadh, Saudi Arabia

Analytical Techniques for Use with Reconnaissance Frame Photography

When treated as a dynamic imaging system, significantly improved results were obtained.

Introduction

The geometry of the image produced by reconnaissance frame cameras was investigated by El Hassan (1978). Theoretical investigation showed that only when the camera is equipped with a focal plane shutter and no image movement compensation (IMC) is applied will the geometry of the image be significantly distorted. Since conventional analog types of stereoplotting machines are not equipped with devices which can compensate for the type of displacements produced by focal plane shutters, an analytical approach to the problem is therefore obligatory.

Two possible approaches which could allow relative orientation and the formation of corrected model and terrain coordinates are devised and examined. The basic input for all these operations will be the photo or image coordinates as measured in a monocomparator or stereocomparator. These techniques are, of course, equally suitable for use with an analytical plotter.

Tilt Variations During Shutter Transit Time

In conventional photogrammetric work using metric photography, the tilts present during exposure and derived by standard orientation techniques are assumed to be fixed. Such an assumption is justified

Abstract: *Analytical techniques for use with reconnaissance frame photographs are outlined. The first approach is a point-by-point space resection in which the dynamic properties of the camera are taken into account. In the second approach appropriate parameters are added to correct for image distortions, caused by the focal plane shutter, during the space resection phase. Test results showed that the analytical techniques developed will significantly improve the planimetric and height accuracy obtained by conventional methods.*

with metric photographs taken using an intra-lens shutter—the exposure time is commonly 1/300 to 1/500th second and any tilt variation in this short time period will be negligible. With a focal plane shutter, while the exposure time, t_e, for any individual image point will be the same, the transit time t_i, of the shutter across the focal plane will be quite different. It is then much more difficult to assume that the tilt values remain constant during the exposure and one will have a type of *dynamic imaging system*, the term introduced by Case (1967) in his analysis of strip and panoramic photography. In this situation the exposure station position and the camera attitude are both changing while the shutter is in motion exposing the whole of the focal plane. Therefore, the *elements of exterior orientation should be considered as varying with time.*

Space Resection (Point-by-Point)

Because the focal plane shutter causes the exposure station to change in position and attitude while the negative is being exposed, the image points must be treated individually (i.e., for each point on the photo, the corresponding exposure station position and camera attitude must be determined). The

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projective relationship between the object and image in space (see Schmid, 1959), when applied to each individual point on the reconnaissance frame photograph, will be written as

$$\begin{bmatrix} X_i \\ Y_i \\ Z_i \end{bmatrix} = \lambda_i \mathbf{A}_i \begin{bmatrix} x_i \\ y_i \\ -f \end{bmatrix} + \begin{bmatrix} X_{0_i} \\ Y_{0_i} \\ Z_{0_i} \end{bmatrix} \tag{1}$$

where X_i, Y_i, Z_i are the coordinates of the object point in a fixed exterior coordinate system; X_{0_i}, Y_{0_i}, Z_{0_i} are the coordinates of the perspective center in the exterior orientation system at the instant at which the image point i is exposed; and $\mathbf{A}_i$ is the orthogonal transformation matrix, composed of the rotation elements ω_i, ϕ_i, and κ_i present at the moment of exposure of point i and referred to the instantaneous projection center (X_{0_i}, Y_{0_i}, Z_{0_i}), and relating the photo system to the ground system. The photo-coordinate system has the x- and y-axis normal to and along the slit line respectively, with the geometric center, p, of the photo as the origin (Figure 1).

The corresponding inverse form for this relation will be

$$\begin{bmatrix} x_i \\ y_i \\ -f \end{bmatrix} = \frac{1}{\lambda_i} \mathbf{A}_i^T \begin{bmatrix} X_i - X_{0_i} \\ Y_i - Y_{0_i} \\ Z_i - Z_{0_i} \end{bmatrix} \tag{2}$$

The following collinearity equations can be derived from the relations of Equation 2:

$$\begin{aligned} x_i &= -f \frac{a_{11_i}(X_i - X_{0_i}) + a_{12_i}(Y_i - Y_{0_i}) + a_{13_i}(Z_i - Z_{0_i})}{a_{31_i}(X_i - X_{0_i}) + a_{32_i}(Y_i - Y_{0_i}) + a_{33_i}(Z_i - Z_{0_i})} \\ y_i &= -f \frac{a_{21_i}(X_i - X_{0_i}) + a_{22_i}(Y_i - Y_{0_i}) + a_{23_i}(Z_i - Z_{0_i})}{a_{31_i}(X_i - X_{0_i}) + a_{32_i}(Y_i - Y_{0_i}) + a_{33_i}(Z_i - Z_{0_i})} \end{aligned} \tag{3}$$

where a_{11_i}, a_{12_i}, etc., are the elements of the orthogonal matrix, $\mathbf{A}_i$.

These equations can be linearized by Taylor's series and can be written in the form:

$$\begin{aligned} V_{x_i} &= \left(\frac{\partial x_i}{\partial \omega_i}\right) d\omega_i + \left(\frac{\partial x_i}{\partial \phi_i}\right) d\phi_i + \left(\frac{\partial x_i}{\partial \kappa_i}\right) d\kappa_i + \left(\frac{\partial x_i}{\partial X_{0_i}}\right) dX_{0_i} \\ &\quad + \left(\frac{\partial x_i}{\partial Y_{0_i}}\right) dY_{0_i} + \left(\frac{\partial x_i}{\partial Z_{0_i}}\right) dZ_{0_i} - J_i \\ V_{y_i} &= \left(\frac{\partial y_i}{\partial \omega_i}\right) d\omega_i + \left(\frac{\partial y_i}{\partial \phi_i}\right) d\phi_i + \left(\frac{\partial y_i}{\partial \kappa_i}\right) d\kappa_i + \left(\frac{\partial y_i}{\partial X_{0_i}}\right) dX_{0_i} \\ &\quad + \left(\frac{\partial y_i}{\partial Y_{0_i}}\right) dY_{0_i} + \left(\frac{\partial y_i}{\partial Z_{0_i}}\right) dZ_{0_i} - K_i \end{aligned} \tag{4}$$

where V_{x_i} and V_{y_i} are corrections to the measured photo coordinates; J_i and K_i are the discrepancies of the measured photo coordinates from the values computed using approximate exterior orientation ele-

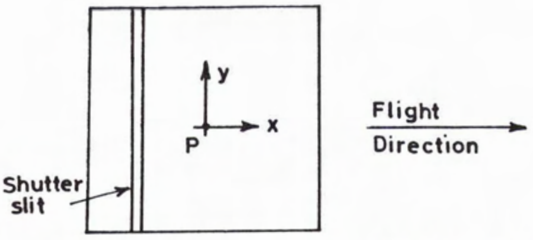

FIG. 1. Photocoordinate system, slit moving parallel to the flight direction.

ments experienced at the moment of exposure of the geometric center; the partial derivatives of x_i and y_i with respect to the unknown orientation elements are also evaluated at the approximate values of the exterior orientation elements.

Now, if the approximate values of the exterior orientation elements corresponding to the exposure of the central point, p, of the photograph are given by ω_p, ϕ_p, κ_p, X_{0_p}, Y_{0_p}, Z_{0_p} (expressed by the vector $\mathbf{X}_p$), then the approximate exterior orientation elements ω_i, ϕ_i, κ_i, X_{0_i}, Y_{0_i}, Z_{0_i} (expressed by the vector $\mathbf{X}_i$) corresponding to the exposure of point i on the photo will be given by

$$\mathbf{X}_i = \mathbf{X}_p + \mathbf{x}_i \tag{5}$$

where $\mathbf{x}_i$ is the vector representing $d\omega_i$, $d\phi_i$, $d\kappa_i$, dX_{0_i}, dY_{0_i}, and dZ_{0_i}, which are the changes in the exterior orientation elements from their position at which point i is exposed. However, these changes are functions of the craft speed, V, (with components $\dot{X}_{0_i}$, $\dot{Y}_{0_i}$, $\dot{Z}_{0_i}$) and the roll, $\dot{\omega}_i$, yaw, $\dot{\phi}_i$, and pitch, $\dot{\kappa}_i$, and can be expressed as follows:

$$\mathbf{x}_i = t_i\, \dot{\mathbf{X}}_i \tag{6}$$

where

$$\dot{\mathbf{X}}_i = [\dot{\omega}_i\ \dot{\phi}_i\ \dot{\kappa}_i\ \dot{X}_{0_i}\ \dot{Y}_{0_i}\ \dot{Z}_{0_i}]^T$$

If the slit speed is given by u_s, assumed to be constant, then the time, t_i, for photo point i, measured from the instant at which the central point p of the photograph is exposed will be

$$t_i = \frac{x_i}{u_s} \tag{7}$$

If point i is exposed before the central point of the photograph, the corresponding value of t_i will be negative.

From Equation 7 it is clear that t_i can be considered as a linear function of x_i. Thus, Equation 6 can be written in the following form:

$$\begin{aligned} d\omega_i &= a_0 + a_1 x_i \\ d\phi_i &= b_0 + b_1 x_i \\ d\kappa_i &= c_0 + c_1 x_i \\ dX_{0_i} &= d_0 + d_1 x_i \\ dY_{0_i} &= e_0 + e_1 x_i \\ dZ_{0_i} &= f_0 + f_1 x_i \end{aligned} \tag{8}$$

The constants $a_0, b_0, \ldots, f_0$ are to correct for the approximate values of the exterior orientation elements corresponding to the exposure of the central point of the photograph. The correction values of Equation 8 are then substituted in Equation 4 to give

$$\begin{aligned} V_{x_i} = {} & \left(\frac{\partial x_i}{\partial \omega_i}\right) a_0 + \left(\frac{\partial x_i}{\partial \omega_i}\right) a_1 x_i + \left(\frac{\partial x_i}{\partial \phi_i}\right) b_0 + \left(\frac{\partial x_i}{\partial \phi_i}\right) b_1 x_i \\ & + \left(\frac{\partial x_i}{\partial \kappa_i}\right) c_0 + \left(\frac{\partial x_i}{\partial \kappa_i}\right) c_1 x_i + \left(\frac{\partial x_i}{\partial X_{0_i}}\right) d_0 + \left(\frac{\partial x_i}{\partial X_{0_i}}\right) d_1 x_i \\ & + \left(\frac{\partial x_i}{\partial Y_{0_i}}\right) e_0 + \left(\frac{\partial x_i}{\partial Y_{0_i}}\right) e_1 x_i + \left(\frac{\partial x_i}{\partial Z_{0_i}}\right) f_0 + \left(\frac{\partial x_i}{\partial Z_{0_i}}\right) f_1 x_i - J_i \\ V_{y_i} = {} & \left(\frac{\partial y_i}{\partial \omega_i}\right) a_0 + \left(\frac{\partial y_i}{\partial \omega_i}\right) a_1 x_i + \left(\frac{\partial y_i}{\partial \phi_i}\right) b_0 + \left(\frac{\partial y_i}{\partial \phi_i}\right) b_1 x_i \\ & + \left(\frac{\partial y_i}{\partial \kappa_i}\right) c_0 + \left(\frac{\partial y_i}{\partial \kappa_i}\right) c_1 x_i + \left(\frac{\partial y_i}{\partial X_{0_i}}\right) d_0 + \left(\frac{\partial y_i}{\partial X_{0_i}}\right) d_1 x_i \\ & + \left(\frac{\partial y_i}{\partial Y_{0_i}}\right) e_0 + \left(\frac{\partial y_i}{\partial Y_{0_i}}\right) e_1 x_i + \left(\frac{\partial y_i}{\partial Z_{0_i}}\right) f_0 + \left(\frac{\partial y_i}{\partial Z_{0_i}}\right) f_1 x_i - K_i \end{aligned} \tag{9}$$

Equation 9 contains 12 unknown orientation parameters (a_0 to f_1), two for each orientation element. Since each observed point gives a set of two equations of the form of Equation 9, then for a single photo, a minimum number of six full control points (known in X, Y, and Z) are needed to determine the unknown parameters.

Values of the computed parameters would be used to determine new correction values from Equation 8. The new values for changes in orientation elements are then used in Equation 4 to determine new approximate values for the orientation elements and the procedure is repeated. Hence, an *iterative solution* will finally lead to the exact exterior orientation elements for each point on the photograph, determined by parameters $a_0, b_0, \ldots, f_1$.

Space Intersection

Given the image coordinates in the photo system and the orientation elements being derived by the space resection solution, the projective relationship given in Equation 1 can be used to solve for the ground coordinates (X, Y, and Z) of the object. This is done by *space intersection* where the measurements made on the two overlapping photographs of the stereomodel and the exterior orientation elements determined by the space resection method are used to solve for the scale factor.

The projective equations relating the object on the ground and its corresponding images on the two photographs would be given by

$$\begin{bmatrix} X_i - X'_{0_i} \\ Y_i - Y'_{0_i} \\ Z_i - Z'_{0_i} \end{bmatrix} = \lambda'_i \, \mathbf{A}'_i \begin{bmatrix} x'_i \\ y'_i \\ -f \end{bmatrix} \tag{10}$$

and

$$\begin{bmatrix} X_i - X''_{0_i} \\ Y_i - Y''_{0_i} \\ Z_i - Z''_{0_i} \end{bmatrix} = \lambda''_i \, \mathbf{A}''_i \begin{bmatrix} x''_i \\ y''_i \\ -f \end{bmatrix} \tag{11}$$

where the terms with a single prime correspond to one photograph (say the left-hand photograph) and the terms with double prime correspond to the other (right-hand photograph). From these equations, the scale factor will be given by

$$\lambda'_i = \frac{(X''_{0_i} - X'_{0_i})\, W''_i - (Z''_{0_i} - Z'_{0_i})\, U''_i}{U'_i W''_i - U''_i W'_i} \tag{12}$$

and

$$\lambda''_i = \frac{(Z'_{0_i} - Z''_{0_i})\, U'_i - (X'_{0_i} - X''_{0_i})\, W'_i}{U'_i W''_i - U''_i W'_i} \tag{13}$$

where

$$\begin{bmatrix} U'_i \\ V'_i \\ W'_i \end{bmatrix} = \mathbf{A}'_i \begin{bmatrix} x'_i \\ y'_i \\ -f \end{bmatrix} \tag{14}$$

and

$$\begin{bmatrix} U''_i \\ V''_i \\ W''_i \end{bmatrix} = \mathbf{A}''_i \begin{bmatrix} x''_i \\ y''_i \\ -f \end{bmatrix} \tag{15}$$

Substituting for the scale factor, and the given values of the image coordinates and the orientation elements in Equation 10 or Equation 11, the ground coordinates (X, Y, Z) of the object in question can be determined.

Space Resection with Additional Parameters

The method of self calibration has been applied particularly to aerial triangulation using the bundle method, especially by Brown (1976) and by Bauer and Muller (1972). The principal of this method is to add corrections to the measured image coordinates so that the projective relations used with conventional photography are adhered to. The correction parameters are selected such that they model and correct for the appropriate distortions in the photography.

Since photography taken with reconnaissance cameras equipped with focal plane shutters is taken with a totally uncalibrated system and one which has dynamic imaging characteristics, the additional parameter method appeared to be an obvious one to experiment with in the context of this photography.

The solution for the additional parameters is carried out during the space resection phase; in other words, they are solved for simultaneously with the camera exterior orientation elements. The space intersection phase is then carried out using the corrected image coordinates.

The following error model has been chosen (Brown, 1976; Ebner, 1976) to be applied as a solution for reconnaissance frame photography:

$$\begin{aligned} \Delta x &= a_1x + a_2y + b_1xy + b_2xy^2 + b_3x^2y + c_1xr^2 + c_2xr^5 + d_1 \\ \Delta y &= -a_1y + a_2x + b_4xy + b_5xy^2 + b_6x^2y + c_1yr^2 + c_2yr^5 + d_2 \end{aligned} \tag{16}$$

where the terms a_1 and a_2 correct for change in image scale and for rotation of the photograph, which are the displacements caused by the focal plane shutter (El Hassan, 1978); the b-terms are to correct for film deformation; the c-terms are to correct for radial distortion; and the d-terms are to correct for the position of the principal point.

Very high correlation between the parameters is a serious problem and would probably lead to an unstable solution. To avoid this correlation, it can be tested by the correlation matrix generated from the covariance matrix. The most ineffective parameters, having high correlations with others, could then be excluded from the error model.

Experimental Tests

In order to evaluate their effects, the above techniques were tested using Skylab S-190B photography. The body of the S-190B camera is a modified Hycon KA-74 reconnaissance camera equipped with a bi-directional focal plane shutter (McLaurin, 1972).

Image motion compensation is achieved by rocking the entire camera in its mount during the exposure. The S-190B camera was equipped with a lens having a focal length of 18 in. (460 mm), a maximum operature of $f/4$, and a maximum radial distortion of $\pm$ 10 μm. The format size of 11.5 by 11.5 cm at the Skylab altitude (H = 435 km) covers a terrain area of 109 by 109 km at the scale of 1/945,600.

The photography used in the test consisted of a strip of three photographs exposed on SO 242 high resolution color film. The three photographs formed two stereomodels with a 60 percent forward lap. Small scale map coverage for the area (1/24,000 and 1/62,500) produced by the U.S. Geological Survey was kindly made available by Professor Welch of the University of Georgia and was used to provide ground coordinates of the selected control and check points.

The photocoordinates were measured on the new Zeiss Jena Stecometer belonging to the Department of Civil Engineering of the City University of London.

Results of the tests are given in Table 1.

Table 1. Root Mean Square Errors of Residuals at Check Points in μm at Negative Scale and in m. at Ground Scale

	Model No. 01334/5 No. of control points 20 No. of check points 40				Model No. 01335/6 No. of control points 20 No. of check points 32			
	Planimetry		Height		Planimetry		Height	
Technique	μm	m	‰H	m	μm	m	‰H	m
1. Conventional space resection/space intersection	28	26.7	0.36	158.0	30	28.4	0.30	130.3
2. Point-by-point space resection/ space intersection	22	21.2	0.15	66.2	28	26.2	0.19	83.4
3. Space resection with additional parameters	25	23.2	0.24	106.2	21	19.8	0.16	69.8

TABLE 2. PERCENTAGE IMPROVEMENT IN ACCURACY

Techniques	Percent Improvement in Planimetric accuracy		Percent Improvement in Height accuracy	
	01334/5	01335/6	01334/5	01335/6
1. Point-by-point resection/intersection	21%	7%	58%	37%
2. Additional parameters	15%	30%	33%	46%

ANALYSIS OF RESULTS

Attempting an overall assessment of the results, one notices immediately that the techniques devised to treat this type of photography improved the accuracy of planimentric and height coordinates obtained by the conventional space resection/space intersection method.

The percentage improvement in accuracy of the two techniques is summarized in Table 2.

From Table 2 it can be seen that the additional parameters technique is more effective on the planimetric accuracy, with an average improvement of 22 percent for the two models. Both techniques gave significant improvement in height accuracy. The average improvements for the two models are 47 percent and 39 percent as obtained by the point-by-point space resection and the additional parameters technique, respectively.

CONCLUSIONS AND RECOMMENDATIONS

The results of the tests show that satisfactory results can be achieved with reconnaissance frame photography using the analytical techniques and that these would provide useful metric and topographic information to a wide spectrum of users.

The results of these tests are limited by the small *B/H* ratio of the S-190B photography (0.10), hence further work should be carried out on reconnaissance frame photography at different focal lengths and *B/H* ratios to define more exactly the limits of the methods used.

Further tests are also needed with cameras equipped with focal plane shutter but without IMC. Such photography taken with older cameras is widely available in the United Kingdom and application of studies of movement, for example, dunes, spits, bars, etc., in a coastal situation would depend on measurements made on such photographs.

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