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Map Projections for Satellite Tracking*

A new series of map projections, on which satellite groundtracks are shown as straight lines, are derived.

Abstract: *New map projections to be used for plotting successive satellite groundtracks show these tracks as straight lines. The map may be made conformal along any two parallels of latitude between the limits of latitude reached by the groundtrack, or the "tracking limits." If these parallels are equidistant from the Equator, they may both be made true to scale, and a cylindrical projection results. If these parallels are not equidistant from the Equator, only one may be made true to scale, and a conic projection results. The groundtracks generally have sharp breaks at either tracking limit. If the tracking limit is one of the parallels at which the map is conformal, there is no break in the groundtrack, and the conic projection may approach (but cannot become) an azimuthal projection.*

Introduction

During the past 15 years, images of the Earth have been obtained from satellites for many purposes such as meteorology and detection of resources. Although satellites circling the Earth near the equatorial plane in a 24-hour orbit (geosynchronous orbits) have been used for other purposes, most image-taking satellites follow orbits with periods and inclinations such that sunlight and satellite position are optimum over regions of interest (sun-synchronous orbits). The groundtrack of a geosynchronous-orbiting satellite is usually a figure 8 with one lobe above and the other below the Equator. The projections utilized for mapping the imagery generated from such a satellite may be of several different well-known types, all of which are based on the concept of a non-rotating Earth.

If the orbit is not geosynchronous nor in the Equatorial plane, the groundtrack, due to Earth rotation, is a curved line from any viewpoint, except for inflection points commonly at the Equator. The groundtrack plotted for this orbit remains a curved line, using any of the conventional map projections.

Colvocoresses first proposed a special projection, the Space Oblique Mercator (SOM), that is especially suitable for mapping of satellite images, especially those of Landsat.[1] This map projection, now mathematically developed, is nearly conformal.[2] The groundtrack plotted on it, however, remains a curved line, so that the problem of plotting the tracks is not thereby simplified.

This paper describes a new series of map projections on which groundtracks are shown as straight lines. The advantage of such a series lies in the simplicity with which groundtracks and the regions viewed from satellites can be shown on the maps.

One approach to the problem of showing groundtracks as straight lines has been the use of B-charts, or the Breckman map projection.[3] This pseudocylindrical projection shows the groundtracks as vertical straight lines and the parallels of latitude as horizontal straight lines. The curved meridians and the coastlines are distorted considerably throughout the map.

The following formulas for preparing graticules on cylindrical and conic map projections allow the plotting of groundtracks as straight lines without the distortions present in B-charts. The map may include two parallels of latitude along which there is no angular distortion, although in the conic

* Publication authorized by the Director, U.S. Geological Survey.

Photogrammetric Engineering and Remote Sensing,
Vol. 47, No. 2, February 1981, pp. 205-213.

form only one of them can be true to scale. The portion of the map within several degrees of these two parallels is relatively free of distortion. These projections, called "satellite-tracking" since they can facilitate the locating of satellite groundtracks, are based on a circular orbit and the Earth taken as a sphere, sufficient for the usual scale of the map and for Landsat orbits. More complicated formulas may be derived for non-circular orbits and the non-spherical Earth if deemed necessary. Several of the formulas derived just below for the cylindrical projection also apply to the conic, discussed subsequently.

Cylindrical Satellite-Tracking Projection

To obtain the basic formulas for the groundtrack on a cylindrical satellite-tracking projection, it will be temporarily assumed that the satellite is orbiting a non-rotating Earth. In Figure 1, let A be the intersection of the groundtrack with the Equator as it crosses from north to south (images from Landsat normally are taken as the satellite moves south). Assigning a longitude of zero to point A, let i be the inclination of the orbit (nominally 99.092° for Landsat), B the pole, and C another point along the groundtrack with geodetic latitude, ϕ, and longitude, L. Longitude, L, where ϕ intersects the groundtrack, is to be distinguished from a general longitude, λ, elsewhere, both L and λ being positive toward the east.

From the elementary Laws of Sines and Cosines, it may be rather readily established that

$$\tan L = \tan \lambda' \cos i \tag{1}$$

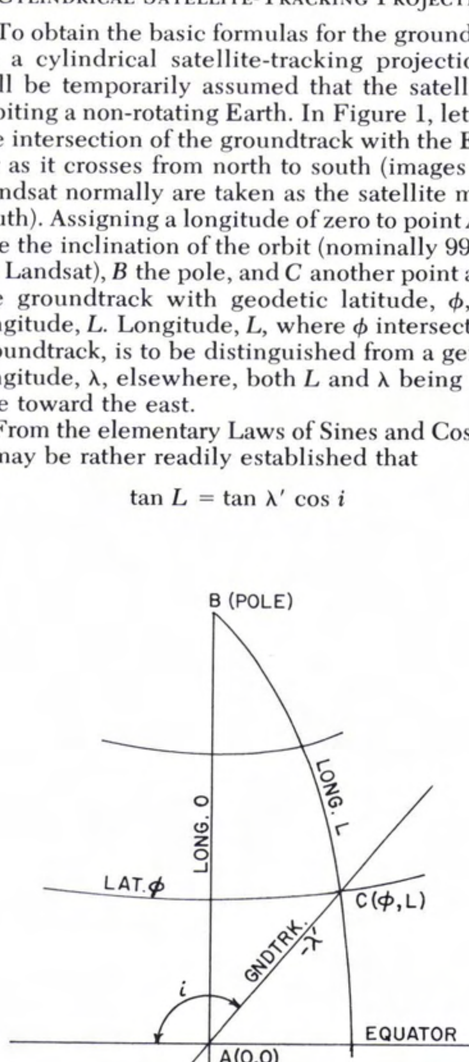

Fig. 1. Satellite groundtrack as projected onto the globe.

and

$$\sin \lambda' = -\sin \phi / \sin i, \tag{2}$$

where λ' is the "transformed" longitude proportional to time along the orbit, with λ' at A called 0.

As discussed in the development of the SOM[4], a "satellite-apparent" longitude, λ_t, should be substituted in place of L in Equation 1 to take into account the Earth's rotation, where

$$\lambda_t = L + (P_2/P_1)\lambda', \tag{3}$$

P_2 is the time of the revolution of the satellite (103.267 min. for Landsat), and P_1 is the time for the Earth's rotation with respect to the ascending node of the orbit. For Landsat, the satellite orbit is sun-synchronous, equating P_1 to the length of the solar day (1440 min.). If the orbit were sidereally fixed, P_1 would equal the approximately 1436 min. of the sidereal day, etc.

To change Equation 1, for a non-rotating Earth, into the corresponding equation for the rotating Earth,

$$\tan \lambda_t = \tan \lambda' \cos i. \tag{4}$$

Rearranging Equation 3,

$$L = \lambda_t - (P_2/P_1)\lambda', \tag{5}$$

where λ_t and λ' are found from Equations 4 and 2, respectively.

If a cylindrical projection is to be devised showing, as is common, parallels of latitude as unequally spaced, horizontal lines, and meridians as equally spaced, vertical lines, the successive groundtracks may be shown as straight lines, provided the parallels of latitude, ϕ, are spaced at distances, L, from the Equator. Such groundtracks would be inclined 45° to the Equator. It is desirable, however, to stretch or compress the projection vertically to produce conformality along a chosen pair of latitudes, $\pm\ \phi_1$, equidistant from the Equator. For pairs of latitudes not equidistant, the conic form, below, must be used.

For conformality, it is necessary that the scale factor, h, along the meridian equals the scale factor, k, along the parallel. For a regular cylindrical or conic projection, this is also sufficient.

For a cylindrical projection,

$$h = dy/(R\ d\phi) \tag{6}$$
$$k = dx/(R \cos \phi\ d\lambda), \tag{7}$$

where the x- and y-axes are taken in the plane of the map projection, usually horizontal and vertical, respectively, and R is the radius of the globe at the scale of the map.

For conformality at ϕ_1, $h = k$, and therefore

$$dy = dx/\left[\cos \phi_1 (d\lambda/d\phi)_{\phi_1}\right]. \tag{8}$$

For true scale at ϕ_1, from Equation 7,

$$k = 1.0 = dx/(R \cos \phi_1 d\lambda). \tag{9}$$

Integrating,

$$x = R\lambda \cos \phi_1. \tag{10}$$

This is the general equation for x, with vertical meridians. From Equation 8, partly integrating, and inserting Equation 10,

$$\begin{aligned} y &= \left\{1/\left[\cos \phi_1 (d\lambda/d\phi)_{\phi_1}\right]\right\} R\lambda \cos \phi_1 \\ &= R\lambda/(d\lambda/d\phi)_{\phi_1}. \end{aligned} \tag{11}$$

For groundtracks plotted as straight lines, it is necessary to make y a linear function of λ along the groundtrack. This is done by substituting the above longitude, L, for λ in Equation 11:

$$y = R\, L/(dL/d\phi)_{\phi_1} \tag{12}$$

Differentiating Equations 5, 4, and 2 in order,

$$dL/d\phi = d\lambda_t/d\phi - (P_2/P_1) d\lambda'/d\phi \tag{13}$$

$$\sec^2\lambda_t (d\lambda_t/d\phi) = \sec^2\lambda' \cos i\, (d\lambda'/d\phi) \tag{14}$$

$$\cos \lambda' (d\lambda'/d\phi) = -\cos \phi/\sin i. \tag{15}$$

Combining Equations 14 and 15,

$$(d\lambda_t/d\phi) = -\cos i \cos \phi/(\sec^2\lambda_t \sin i \cos^3\lambda'). \tag{16}$$

Substituting from Equations 15 and 16 into 13, rearranging, and then substituting from Equation 4,

$$dL/d\phi = (\cos \phi/\sin i \cos \lambda')\left[P_2/P_1 - \cos i/(1 - \sin^2 \lambda' \sin^2 i)\right]. \tag{17}$$

Substituting from Equation 2 into Equation 17 to eliminate λ',

$$\begin{aligned} dL/d\phi &= \left[\cos \phi/\sin i\, (1 - \sin^2 \phi/\sin^2 i)^{1/2}\right] \cdot \left[P_2/P_1 - \cos i/(1 - \sin^2 \phi)\right] && (18) \\ &= \left[(P_2/P_1) \cos^2 \phi - \cos i\right] / \left[(\cos^2 \phi - \cos^2 i)^{1/2} \cos \phi\right]. && (19) \end{aligned}$$

Introducing a new symbol, F, the angle on the globe between groundtrack and meridian, let

$$\tan F = \left[(P_2/P_1) \cos^2 \phi - \cos i\right]/(\cos^2 \phi - \cos^2 i)^{1/2}. \tag{20}$$

Then

$$dL/d\phi = \tan F/\cos \phi. \tag{21}$$

If ϕ equals ϕ_1, from Equation 12,

$$y = R\, L \cos \phi_1/\tan F_1, \tag{22}$$

where L is found from Equations 2, 4, and 5, and

$$\tan F_1 = \left[(P_2/P_1) \cos^2 \phi_1 - \cos i\right]/(\cos^2 \phi_1 - \cos^2 i)^{1/2}. \tag{23}$$

The graticule may then be drawn according to Equations 10 and 22 (see Figure 2). The groundtracks are shown as a series of parallel lines, inclined at angle F_1 to the meridians, since the tangent, dx/dy, of this angle, from Equations 8 and 21, is

$$\begin{aligned} dx/dy &= \cos \phi_1 (dL/d\phi)_{\phi_1} \\ &= \tan F_1. \end{aligned} \tag{24}$$

If the full orbits are shown, there is a sharp break at the northern and southern limits of latitude reached by the groundtrack, or the "tracking limits", so the tracks appear to be a sequence of zig-zag lines.

For the scale factors at any given latitude, ϕ, from Equations 6, 12, and 21,

$$\begin{aligned} h &= (dy/dL)(dL/d\phi)/R \\ &= \cos \phi_1 \tan F/(\cos \phi \tan F_1), \end{aligned} \tag{25}$$

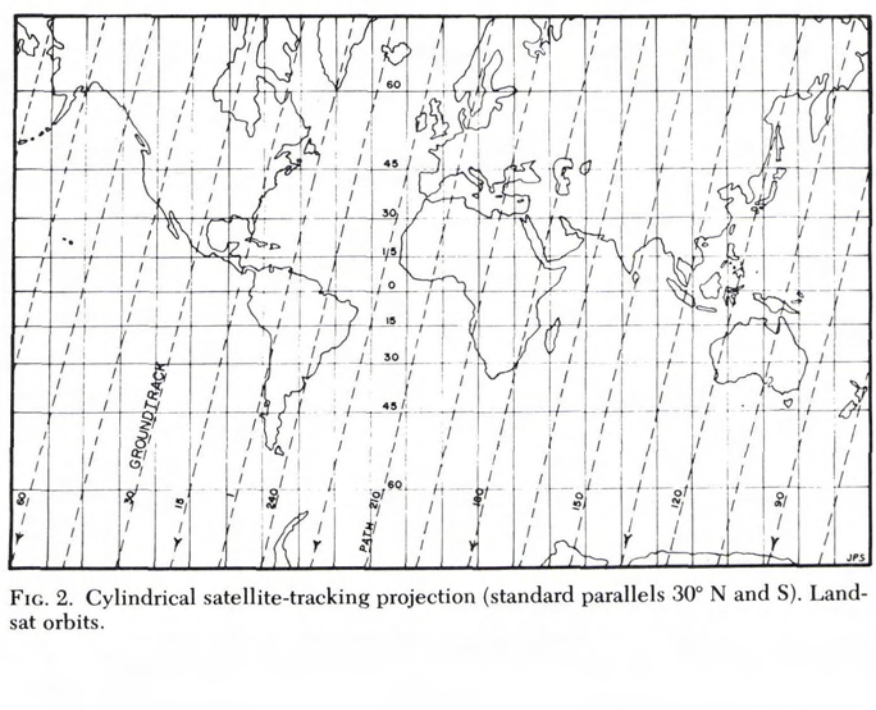

FIG. 2. Cylindrical satellite-tracking projection (standard parallels 30° N and S). Landsat orbits.

TABLE 1. RECTANGULAR COORDINATES FOR CYLINDRICAL SATELLITE-TRACKING PROJECTION
Landsat orbits: $i = 99.092°$
$P_2 = 103.267$ min.
$P_1 = 1440.0$ min.
Globe radius: $R = 1.0$

ϕ_1 / F_1 / x	0° / 13.09724° / 0.017453λ°			±30° / 13.96868° / 0.015115λ°			±45° / 15.71115° / 0.012341λ°		
$\pm\phi$	$\pm y$	h	k	$\pm y$	h	k	$\pm y$	h	k
TL*	7.23571	∞	6.32830	5.86095	∞	5.48047	4.23171	∞	4.47479
80°	5.35080	55.0714	5.75877	4.33417	44.6081	4.98724	3.12934	32.2078	4.07207
70	2.34465	6.89443	2.92380	1.89918	5.58452	2.53209	1.37124	4.03212	2.06744
60	1.53690	3.18846	2.00000	1.24489	2.58266	1.73205	0.89883	1.86473	1.41421
50	1.09849	2.01389	1.55572	0.88979	1.63126	1.34730	0.64244	1.17780	1.10006
40	0.79741	1.49787	1.30541	0.64591	1.21328	1.13052	0.46636	0.87601	0.92306
30	0.56135	1.23456	1.15470	0.45470	1.00000	1.00000	0.32830	0.72202	0.81650
20	0.35952	1.09298	1.06418	0.29121	0.88532	0.92160	0.21026	0.63921	0.75249
10	0.17579	1.02179	1.01543	0.14239	0.82766	0.87939	0.10281	0.59758	0.71802
0°	0.00000	1.00000	1.00000	0.00000	0.81000	0.86603	0.00000	0.58484	0.70711

* Tracking limit, 80.908° = (180° − i)
See Appendix for other symbols.

and from Equations 7 and 10,

$$k = (dx/d\lambda)/(R \cos \phi) \\ = \cos \phi_1/\cos \phi. \qquad (26)$$

Coordinates and scale factors for the graticule of Figure 2, as well as alternates with $\phi_1 = 0$ and ± 45°, are given for representative purposes in Table 1. If ϕ_1 is zero, there is only one standard parallel. No coordinates of latitudes nearer to the poles than the tracking limits may be calculated with the above formulas, since there is no groundtrack in those regions (the denominator of Equation 20 becomes imaginary). Such latitudes may be plotted arbitrarily for esthetic reasons, or omitted altogether.

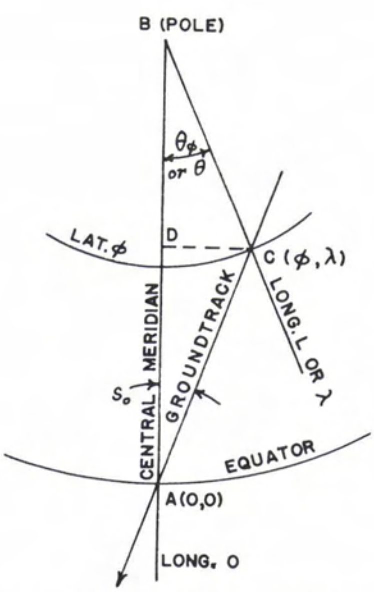

FIG. 3. Elements of the conic satellite-tracking projection.

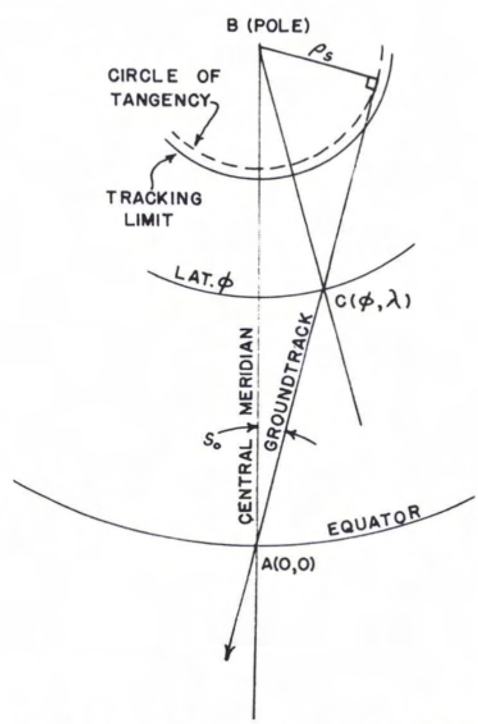

FIG. 4. Circle of tangency for the conic satellite-tracking projection.

Conic Satellite-Tracking Projection with Conformality at Two Parallels

While the cylindrical form of the satellite-tracking projection is of more interest if much of the world is to be shown, the conic form applies to most continents and countries, just as do the usual cylindrical and conic projections. In Figure 3, showing elements of a conic satellite-tracking projection, AB is the radius, ρ_0, of the circular arc representing the Equator, BC is the radius, ρ, of the circular arc for latitude, ϕ, and θ is the angle between the central meridian, AB, for which λ is zero, and line BC representing longitude, λ. When applied to the longitude of the groundtrack at latitude ϕ, θ is called θ_ϕ and λ is again called L. The groundtrack is to be a straight line passing through C and A, the equatorial crossing.

The cone constant, n, is defined, as usual, as the ratio of θ to λ, and therefore of θ_ϕ to L, or

$$\theta_\phi = n\,L \tag{27}$$

and

$$\theta = n\,\lambda, \tag{28}$$

where longitude is measured east of the central meridian.

If s_0 is the angle of intersection at the Equator between the groundtrack and the meridian on the map, the Law of Sines leads to

$$\rho = \rho_0 \sin s_0/\sin(\theta_\phi + s_0). \tag{29}$$

For a conic projection, scale factors may be calculated as follows:

$$h = -\,d\rho/(R\,d\phi) \tag{30}$$

$$k = \rho\,n/(R\cos\phi). \tag{31}$$

Combining Equations 29 and 31,

$$k = n\,\rho_0 \sin s_0/\left[R\cos\phi\sin(\theta_\phi + s_0)\right]. \tag{32}$$

Combining Equations 29 and 30,

$$h = \left[\rho_0 \sin s_0 \cos(\theta_\phi + s_0)/R\sin^2(\theta_\phi + s_0)\right] d\theta_\phi/d\phi. \tag{33}$$

Differentiating Equation 27, and substituting from 21,

$$d\,\theta_\phi/d\phi = n\,dL/d\phi = n\tan F/\cos\phi. \tag{34}$$

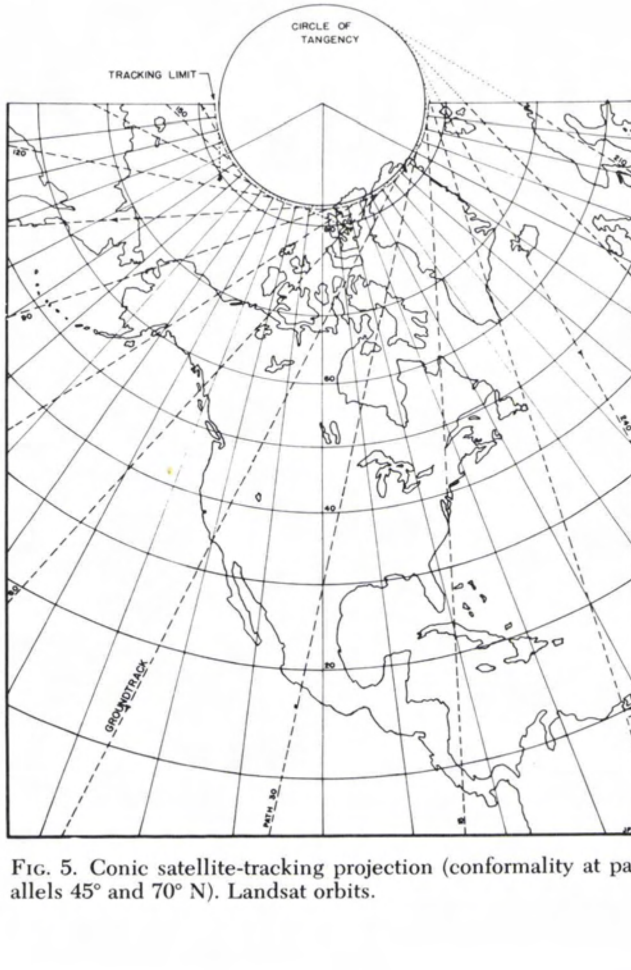

Fig. 5. Conic satellite-tracking projection (conformality at parallels 45° and 70° N). Landsat orbits.

Substituting into Equation 33 from Equation 34 and then from Equation 32,

$$h = k \tan F / \tan (\theta_\phi + s_o). \quad (35)$$

If conformality is to exist along latitudes ϕ_1 and ϕ_2, $h = k$ at each of these parallels, or, from Equation 35,

$$\tan (\theta_1 + s_o) = \tan F_1$$

and

$$\tan (\theta_2 + s_o) = \tan F_2,$$

or

$$\theta_1 + s_o = F_1$$

and

$$\theta_2 + s_o = F_2. \quad (36)$$

Then

$$s_o = F_1 - \theta_1 = F_2 - \theta_2, \quad (37)$$

and

$$\theta_2 - \theta_1 = F_2 - F_1.$$

Substituting from Equation 27,

$$n = (F_2 - F_1)/(L_2 - L_1), \quad (38)$$

where F_n is calculated from Equation 20 and L_n is calculated from Equations 5, 4, and 2, applying the same subscripts to F and ϕ, and to L and ϕ.

Although conformality occurs along ϕ_1 and ϕ_2, these parallels are not of equal scale. If ϕ_1 is chosen to be the latitude which is true to scale, k in Equation 32 equals 1.0 when ϕ equals ϕ_1 and θ_ϕ equals θ_1. Combining Equations 32 and 36 for this condition,

$$\rho_o = R \cos \phi_1 \sin F_1 / (n \sin s_o). \quad (39)$$

Substituting Equation 39 into Equation 29,

$$\rho = R \cos \phi_1 \sin F_1 / [n \sin (\theta_\phi + s_o)]. \quad (40)$$

Note that F cannot be substituted for $(\theta_\phi + s_o)$.

We now have the polar coordinates of the conic satellite-tracking projection, from Equations 40, 38, and 28, as well as Equations 20, 5, 4, and 2. For rectangular coordinates, the usual conversions are employed:

$$x = \rho \sin \theta \quad (41)$$
$$y = \rho_{\phi_0} - \rho \cos \theta, \quad (42)$$

where ϕ_0 is the arbitrary latitude which intersects the central meridian at the origin of coordinates ($x, y = 0$). Equation 35 is simplest to use for computing h, and Equation 31 for k.

As on the cylindrical projection, the straight groundtracks will break at the tracking limit, except as noted later, but the groundtracks are no longer parallel to each other. They are similarly placed with respect to the radiating meridians, however.

After drawing the basic graticule, the plotting of the straight groundtracks can be facilitated if the map extends near enough to the northern or southern tracking limit to permit including a "circle of tangency" to which every projected groundtrack is tangent. Referring to Figure 4, the dashed inner circle, to which groundtrack AC is tangent, has a radius ρ_s, where

$$\rho_s = AB \sin s_o = \rho_o \sin s_o$$
$$= R \cos \phi_1 \sin F_1 / n. \quad (43)$$

after substitution from Equation 39.

Figure 5 shows a graticule with coastlines for Landsat groundtracks with the circle of tangency. Conformality occurs at latitudes 45° and 70° N. The polar coordinates for this map, and for a map with conformality at a different pair of latitudes (30° and 60°), as well as for Figure 6, are given in Table 2. These coordinates can serve as a check for those using the formulas.

If the tracking limit is beyond the limits of the map, it is necessary to make sure each ground-

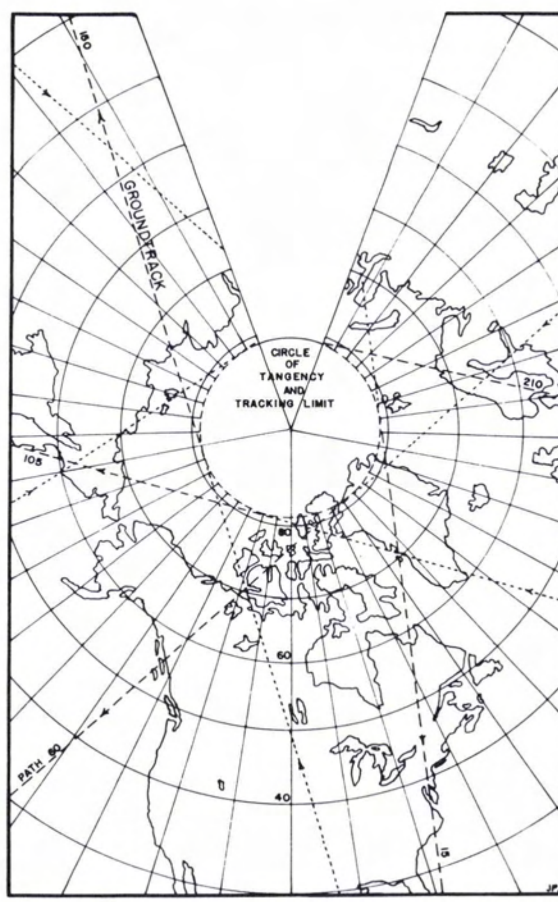

FIG. 6. Conic satellite-tracking projection (conformality at parallels 45° and 80.9° N). Landsat orbits.

TABLE 2. POLAR COORDINATES FOR CONIC SATELLITE-TRACKING PROJECTIONS WITH TWO CONFORMAL PARALLELS
Landsat orbits (i, P_2, P_1 same as Table 1)
Globe radius: $R = 1.0$

ϕ_1		30°			45°			45°	
ϕ_2		60°			70°			80.908°	
n		0.49073			0.69478			0.88475	
F_1		13.96868°			15.71115°			15.71115°	
ρ_s		0.42600			0.27559			0.21642	
ϕ	ρ	h	k	ρ	h	k	ρ	h	k
TL*	0.50439	∞	1.56635	0.28663	∞	1.26024	0.21642	1.21172	1.21172
80°	0.59934	3.72928	1.69373	0.33014	1.93850	1.32093	0.23380	1.08325	1.19121
70	0.98470	1.61528	1.41283	0.57297	1.16394	1.16394	0.40484	0.90832	1.04727
60	1.22500	1.20228	1.20228	0.75975	1.00596	1.05572	0.55875	0.87290	0.98871
50	1.41806	1.03521	1.08260	0.93154	0.97914	1.00689	0.71504	0.93344	0.98421
45	1.50659	0.99771	1.04556	1.01774	1.00000	1.00000	0.79921	1.00000	1.00000
40	1.59281	0.98135	1.02035	1.10669	1.04212	1.00374	0.89042	1.09569	1.02840
30	1.76478	1.00000	1.00000	1.30060	1.19708	1.04342	1.10616	1.40901	1.13008
20	1.94551	1.08181	1.01599	1.53188	1.47984	1.13263	1.39852	2.00877	1.31675
10	2.14662	1.23677	1.06965	1.82978	1.98371	1.29091	1.84527	3.28641	1.65780
0	2.38332	1.49781	1.16956	2.25035	2.94795	1.56351	2.66270	6.72124	2.35583
−10	2.67991	1.94172	1.33539	2.92503	5.10490	2.06361	4.79153	22.2902	4.30472
−20°	3.08210	2.75586	1.60953	4.26519	11.6380	3.15356	29.3945	898.207	27.6759
ML**		−60.65° ($\rho = \infty$)			−38.52° ($\rho = \infty$)			−21.86° ($\rho = \infty$)	

* Tracking limit, 80.908° = (180° − i)
** Minimum latitude, at infinite radius
See Appendix for other symbols.

track is inclined with respect to the intersecting meridian at an angle F_1 at parallel ϕ_1, or F_2 at parallel ϕ_2. At other parallels the angle on the map is not F, but $(\theta_\phi + s_0)$.

CONIC SATELLITE-TRACKING PROJECTION WITH CONFORMALITY AT TWO PARALLELS, ONE BEING A TRACKING LIMIT

If a tracking limit coincides with ϕ_1, one of the two parallels at which conformality is specified,

TABLE 3. POLAR COORDINATES FOR NEAR-AZIMUTHAL CONIC SATELLITE-TRACKING PROJECTION
Landsat orbits (i, P_2, P_1 same as Table 1)
Globe radius: $R = 1.0$
$\phi_1 = 80.908°$
$n = 0.96543$
$F_1 = \pm 90°$
$\rho_s = 0.16368$

ϕ	ρ	h	k
TL*	0.16368	1.00000	1.00000
80°	0.17953	1.00076	0.99813
70	0.35986	1.09115	1.01579
60	0.57095	1.36647	1.10243
50	0.85650	1.99000	1.28641
40	1.31643	3.53452	1.65907
30	2.28682	8.83705	2.54931
20°	6.22402	58.0828	6.39449
ML**		13.70° ($\rho = \infty$)	

* Tracking limit, 80.908° = (180° − i)
** Minimum latitude, of infinite radius
See Appendix for other symbols.

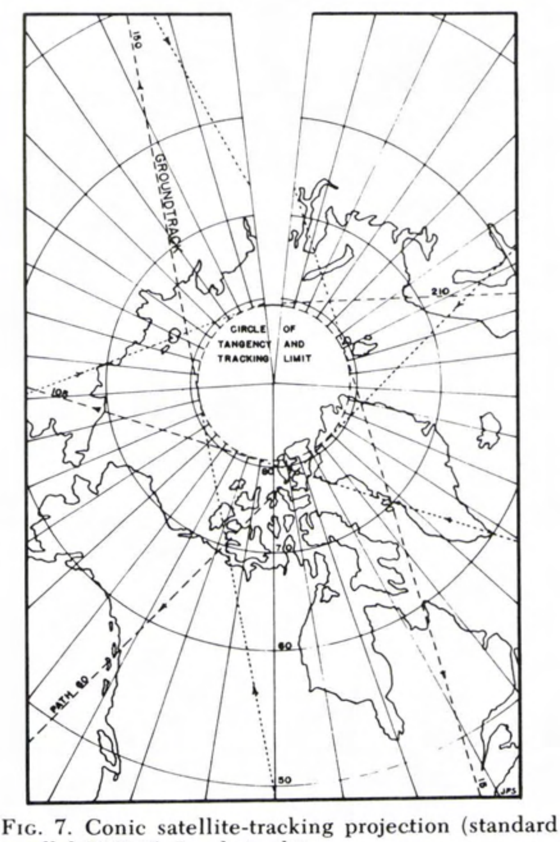

FIG. 7. Conic satellite-tracking projection (standard parallel 80.9° N). Landsat orbits.

the circle of tangency coincides with the tracking limit, and the straight groundtracks have no break at this point, as they do in the foregoing cases. In this case, from Equation 23, since the denominator is 0 and the numerator is not, $F_1 = \pm 90°$. From Equation 2, $\lambda' = -90°$; from Equation 4, $\lambda_t = 90°$, if $i > 90°$; from Equation 5, $L_1 = 90° \cdot (1 + P_2/P_1)$, if $i > 90°$. For other constants, $\theta_1 = n L_1$ (from Equation 27), and $s_0 = F_1 - \theta_1$ (from Equation 37). If the tracking limit is ϕ_2 (with conformality, but not true scale), the same values apply with 2 instead of 1 as the subscript.

Figure 6 illustrates this form of the projection for Landsat groundtracks with conformality at latitudes at 45° N. and the upper tracking limit, 80.908° N. Polar coordinates are given in Table 2.

It may be desirable to show equal intervals of time along the orbit on any of these projection forms (or, for Landsat groundtracks, various row numbers, which are spaced at equal time intervals). Equation 2 may be used to determine the value of ϕ corresponding to a given λ', which in turn is directly proportional to time for the circular orbit, and various meridians or the edge of the map can be marked with the equivalent time (or Landsat row number).

Conic Satellite-Tracking Projection with Conformality at a Single Parallel

If, in Equations 20, 5, 4, and 2, ϕ_1 is made equal to ϕ_2, for a conic projection with one standard parallel, Equation 38 becomes indeterminate, although Equation 40 is usable, if n is known. Since this form is of some interest, especially in a near-azimuthal case to be described, the actual cone constant is derived: As ϕ_2 approaches ϕ_1, the numerator and denominator of Equation 38 approach dF_{ϕ_1} and dL_{ϕ_1}, respectively, where ϕ_1 is the one parallel at which conformality is to occur. It is also made true to scale and is, thus, a standard parallel in the same sense as that on common conic projections. Then

$$n = \left[dF/dL\right]_{\phi_1}, \qquad (44)$$

which is equivalent to

$$n = \left[(dF/d\phi)/(dL/d\phi)\right]_{\phi_1}. \qquad (45)$$

To find $[dF/d\phi]_{\phi_1}$, Equation 20 is differentiated:

$$\sec^2 F(dF/d\phi) = \frac{\sin\phi\cos\phi\left[(P_2/P_1)(2\cos^2 i - \cos^2\phi) - \cos i\right]}{(\cos^2\phi - \cos^2 i)^{3/2}} \qquad (46)$$

Substituting from Equation 23 into 46 and from Equations 46 and 19 into 45, inserting subscripts,

$$n = \frac{\sin\phi_1\left[(P_2/P_1)(2\cos^2 i - \cos^2\phi_1) - \cos i\right]}{\left[(P_2/P_1)\cos^2\phi_1 - \cos i\right]\left\{(P_2/P_1)\left[(P_2/P_1)\cos^2\phi_1 - 2\cos i\right] + 1\right\}}. \qquad (47)$$

Conic Satellite-Tracking Projection Nearest to an Azimuthal Projection

Probably the most useful application of Equation 47 is to make ϕ_1 equal to the upper tracking limit i, if $i < 90°$, or $(180° - i)$, if $i > 90°$. In either case, $\cos^2\phi_1 = \cos^2 i$. Considerable simplification is possible:

$$n = \sin i/\left[(P_2/P_1)\cos i - 1\right]^2. \qquad (48)$$

This form of the satellite-tracking projection is the closest approximation to an azimuthal form, but it remains a conic projection, with $n < 1$. As in the conic projection with conformality at two parallels, of which one is a tracking limit, the groundtracks extend straight across the map through the polar approach without a break, the tracking limit is the circle of tangency, and constants F_1, λ', etc., are the same for both variations.

Polar coordinates for Landsat groundtracks in the near-azimuthal form are given in Table 3, and the graticule is shown in Figure 7. Because of the near-polar orbit, this cone as developed is less than 4 percent (n is 0.96543) from a full circle. With orbits of lower inclination, the approach to azimuthal becomes less.

Acknowledgments

The author wishes to thank Dr. Alden P. Colvocoresses for his constructive comments concerning this paper.

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(Received 7 March 1979; revised and accepted 30 August 1980)

Appendix
Summary of Formulas for Calculation

1. Cylindrical Satellite-Tracking Projection:

$$x = R\ \lambda \cos \phi_1 \qquad (10)$$
$$y = R\ L \cos \phi_1/\tan F_1 \qquad (22)$$
$$h = k \tan F/\tan F_1 \qquad (25)$$
$$k = \cos \phi_1/\cos \phi \qquad (26)$$

(see below for additional formulas)

2. Conic Satellite-Tracking Projection:

$$x = \rho \sin \theta \qquad (41)$$
$$y = \rho_{\phi_0} - \rho \cos \theta \qquad (42)$$
$$\rho = R \cos \phi_1 \sin F_1/n \sin (\theta_\phi + s_o) \qquad (40)$$
$$s_o = F_1 - \theta_1 \qquad (37)$$
$$\theta = n\lambda \qquad (28)$$
$$\theta_\phi = n\ L \qquad (27)$$
$$\theta_1 = n\ L_1 \qquad (27)$$
$$n = (F_2 - F_1)/(L_2 - L_1), \text{ for} \qquad (38)$$

conformality at two parallels, one of which is also standard

$$n = \frac{\sin \phi_1[(P_2/P_1)(2 \cos^2 i - \cos^2\phi_1) - \cos i\,]}{[(P_2/P_1)\cos^2\phi_1 - \cos i]\{(P_2/P_1)[(P_2/P_1)\cos^2\phi_1 - 2 \cos i] + 1\}} \qquad (47)$$

for one standard parallel

$$n = \sin i/[(P_2/P_1) \cos i - 1]^2 \qquad (48)$$

for standard parallel only at tracking limit

$$\rho_s = R \cos \phi_1 \sin F_1/n \qquad (43)$$
$$h = k \tan F/\tan (\theta_\phi + s_o) \qquad (35)$$
$$k = \rho n/(R \cos \phi) \qquad (31)$$

Applicable to Cylindrical or Conic forms:

$$L_n = \lambda_{tn} - (P_2/P_1)\lambda'_n \qquad (5)$$
$$\tan \lambda_{tn} = \tan \lambda'_n \cos i \qquad (4)$$
$$\sin \lambda'_n = - \sin \phi_n/\sin i \qquad (2)$$
$$\tan F_n = [(P_2/P_1)\cos^2 \phi_n - \cos i]/(\cos^2 \phi_n - \cos^2 i)^{1/2}. \qquad (20, 23)$$

Subscript n is equal to 1 or 2 or is omitted, as required by preceding formulas.

If the maximum angular deformation ω is desired, the following formula applies to all of the above cases:

$$\sin \tfrac{1}{2}\, \omega = |(h - k)/(h + k)| \qquad (49)$$

Symbols:

- ϕ,λ geodetic latitude and longitude, respectively.
- ρ,θ polar coordinates (radius and polar angle, respectively).
- x,y rectangular coordinates.
- h,k scale factors along meridian and parallel, respectively.
- n cone constant.
- ϕ_1 standard parallel (true to scale and with conformality).
- ϕ_2 second parallel at which conformality is specified.
- R radius of globe at scale of map.
- F_1 inclination of groundtrack to meridian at latitude ϕ_1.
- i inclination of satellite orbit to Earth equatorial plane.
- P_2 period of revolution of satellite.
- P_1 period of rotation of Earth relative to satellite orbit.
- ρ_s radius of circle to which groundtracks are tangent on map.
- ρ_{ϕ_0} radius of circle for latitude crossing central meridian at $x,y = 0$.