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Algorithms for Dense Digital Terrain Models

The algorithms were developed to derive quantitative data and topographic and thematic maps from a dense DTM.

Abstract: *A family of computer algorithms has been developed at the University of Guelph to derive quantitative data and topographic and thematic maps from dense digital terrain models. The sources of such models are discussed briefly, and the rules that have been adopted for generating new programs are described. Some of the programs derive completely new quantitative and qualitative descriptors of the terrain, and all of them are intended for operations on 10^5 to 10^7 points or more. Finally, a new geocoding algorithm is described for deriving thematic maps from arbitrary combinations of topographic and thematic data obtained from a wide variety of sources.*

Introduction

A digital terrain model is an array of numbers that represents the spatial distribution of a set of properties of the terrain. When there is only one property, elevation, the term digital elevation model (DEM) is coming into use and will be used in this paper. DTM will be used to indicate a more general model, which may contain a DEM.

Some years ago, attempts were made to derive terrain descriptors from the DEM by using various methods of algebraic surface-fitting, but the inherently non-analytical nature of real terrain has hindered such investigations. The uses of both DTM and DEM are now entering a new phase. All terrain data from whatever source can now be digitized, and every possible computation or graphic display can now be handled by the computer. It is thus important to think of the computer data base at all stages of terrain data digitization and representation. In this paper, algorithms will be suggested for extending the uses of the DEM far beyond the stage of contour production and volume computation, and the groundwork will be laid for a complete system of data management and for producing new types of topographic and thematic maps from DTM.

This paper starts with the assumption that a dense DEM exists, with its points attached to an accurate coordinate system. Admittedly, this avoids or assumes as available all of the excellent techniques that have been evolved for producing such models; these will only be mentioned briefly below. But it is a logical starting point, and it avoids the trap of developing a mapping method in which the acquisition, processing, and presentation of data are inextricably wound together so that new inputs and outputs are very difficult to introduce. For a complete mapping system, these three phases must be well separated and each must be independently flexible.

Definition of a Dense DTM

To be called 'dense' for the purposes below, the spacing of the points must meet the following criteria:

- Every characteristic point is approximated within the map tolerance by one or more DTM points (For

Photogrammetric Engineering and Remote Sensing,
Vol. 47, No. 1, January 1981, pp. 71-76.

a DEM, a characteristic point is a singular point or one where there is a significant change of slope. This notion may be generalized for the DTM);
- The state (numerical specification) of a general point may be taken as the state of the nearest DTM point;
- Adjacent DTM points have states that do not differ by more than the tolerance on the state for the intended application (except for a limited number of discontinuities such as cliffs and sharp boundaries between terrain types); and
- A smooth line can be approximated closely enough by a continuous series of straight lines joining adjacent DTM points.

The methods that are proposed below for the analysis of DTM are somewhat similar to the methods of calculus applied to physical systems. These methods imply continuity, but they can be applied to granular systems as long as the granularity is small compared to the dimensions of the mass. Methods have been developed to analyze many types of physical systems by representing finite particles of the system by elements in computer storage, and the DTM methods below have just the same dependence upon the integrity of the data with respect to large-scale states and phenomena.

Another way of stating the assumption above is to say that, if the DTM is not fine enough to meet these conditions, either it must be densified by the addition of new real data or it must be abandoned. This is not to say that, if the terrain is in fact represented adequately, an arbitrarily fine grid spacing may not be used for convenience. For example, soil-type data with a precision of location of 10m might be represented on a 2-m grid, as long as no unjustified inferences are drawn about the accuracy of the final result.

Sources of Dense DTM

Sparse irregular data, such as the elevations derived by digitizing characteristic points of the surface, may be used in a wide variety of interpolation and smoothing processes to produce a grid DEM (Schut, 1976). If the irregular data are chosen well and are dense enough to represent all significant changes in slope, then linear interpolation will provide accurate grid DEM values. Direct application of some of the algorithms below to map production may reveal the nature of the interpolation, but the actual accuracy of the map cannot be questioned on that basis. Smoothing after derivation of map elements can be carried out at much less cost than smoothing of the whole model.

Densification of contour data and profile data, with the addition of significant points and lines such as drainage and break-lines, is a method that is now widely practiced and is available to many private and public mapping agencies. Ackermann and his associates in Stuttgart have carried this type of DEM productions to a high level. They have solved special problems that occur in profiling by cross-correlation techniques. They also apply auto-correlation and other methods of smoothing in order to obtain a DEM that will immediately yield a smooth contour map. (The Stuttgart Contour Program SCOP—Institute for Photogrammetry. Stuttgart University).

Automatic image correlation (AIC) offers a means of producing very dense grid DEM, and several of the programs described in this paper were written because computer tapes from the Gestalt GPM II were available. The DEM produced by the GPM II from each photogrammetric model contains more than 700,000 points, giving a grid spacing of 0.182 mm at contact scale (about 9m at ground scale from 1:50,000 scale photos).

The grid elevations of the Gestalt DEM are not independent because some smoothing is inherent in the electronic and optical systems. There are difficulties in applying automatic image correlation in some types of terrain (Allam, 1978), and the original DEM may be supplemented by other data in order to provide a more attractive representation of drainage and terrrain break-lines. However, the accuracy and density of the data are excellent for some mapping and computing tasks.

The Nature of the New Algorithms

All of the programs that have been written or that are now being developed by the authors have been designed to satisfy a few elementary but important requirements. Search techniques are avoided wherever possible, and are never applied to large DTM's (of the order of 10^6 points). The basic assumptions above about the density and validity of the DTM are constantly observed, so that smoothing, interpolation, or surface fitting are also completely avoided in dealing with the large models. What smoothing is necessary is applied only to the reduced amounts of data that constitute the map elements after these elements have been defined. The fundamental operations that are adopted are as follows:

(1) The DTM is scanned row by row, each point being dealt with once and once only and always in order of its column number.
(2) Each point is assigned a state, or a change of state, according to the states of its four rectangular neighbors.
(3) The four neighbors of a point may have states or changes of state assigned to them, according to the state of the central point.

It may be noted that the second and third operations are analogous to the operations of setting up a differential equation, as mentioned above.

(4) Each point may be assigned to many different types of states at the same time.
(5) For time-dependent phenomena, one pass through the whole or part of a model may be taken as one instant (a second, or microsecond, or year) of elapsed time.

These operations do not appear to offer much scope for data manipulation, but the computer handles them very easily; and setting the rules of the game in advance helps to avoid unnecessary complications. It places burdens on the ingenuity of the programmer and reduces the requirements on time and capacity of the computer.

The States of DTM Points

From the DEM, the following states of the grid points may be defined:

- Elevation itself
- Extreme value (hilltop or valley bottom)
- Elevation class
- Slope class
- Watershed membership (a point lies within a given watershed or depression)
- Divide membership (a point lies on a given divide)
- Singularity (a point lies on a break-line or is a saddle-point).

Some of these states are derived directly from a DEM by means of a previous algorithm (Collins, 1975). When the DTM is defined more broadly, any number of states may be ascribed to a point. A few examples follow:

Soil type
Geomorphological status
Vegetal cover
Land use
Precipitation rate
Permeability

Specific Algorithms

The following algorithms are now in various stages of development. Some have been programmed in efficient languages. Others have been tested in high-level languages, and still others are only in the logic-statement stage.

CONTOUR OUTLINE PROGRAM SYSTEM: COPS

This system has been fully developed in a combination of Fortran and Assembler languages. It may be used to draw an outline around any assembly of points that carry a specific coding. For contours, the coding is the elevation class. The program has been used to draw contours from an 800,000-point model in 154 CPU seconds on an IBM 370-155 computer, using a maximum of 130K bytes of main memory. Of the core time, 25 seconds was spent in reordering the Gestalt patch data into row form, 43 seconds was used to identify and sort the contour points, and 86 seconds was used to string the 36,000 contour points for plotting. In mountainous terrain the last operation requires more time, in linear proportion to the number of contour points to be plotted. An algorithm for smoothing the outlines under given constraints has also been developed.

WATERSHED ANALYSIS

This programming system was developed several years ago (Collins, 1975) and has been extended to new uses since that time. It consists of two parts. In the first part the watershed parameters are computed and the watershed characteristics are plotted. In the second a precipitation event is simulated and the surface flow patterns and quantities are determined. The output from the system at present consists of the following:

- Watershed boundaries (divides)
- Depression storage volumes and area
- Runoff contributing areas for any depression or watershed
- Low points and high points
- Number of depressions and hills
- Volume and area of hills down to saddle points
- Contributing areas to any stream point
- Flow of water over any point
- Map of drainage determined directly from the DEM

The watershed analysis is of obvious importance, and provision is being made for the entry of many hydrologic factors to provide an accurate quantitative analysis of watershed behaviour in a runoff event.

One highly practical program was developed as a special case of watershed analysis. This program computes the volume and area of the water stored by a dam for many different elevations.

A hypothetical dam is created by inserting false elevations into the DEM, and this is done automatically by digitizing a line and assigning new elevations to the grid point defined by the line. This line can be carried all the way around the area that may be flooded at the highest water level. In this case it delimits the area of interest, and points outside of it do not have to be processed.

In one example of the use of this program, a DEM of 400,000 points was reduced to 250,000 points by means of a delimiter. The volume and area of water for each of 11 water levels was calculated in 60 seconds of CPU time, on an Amdahl V5 computer. It is interesting that the computing time is increased negligibly by an increase in the number of levels, and is approximately linear with the number of points inside the delimiter.

Table 1 gives the output of this program for the example quoted.

Thematic Mapping and Terrain Analysis

The purpose of these algorithms is to accept topographic and thematic digitized data from a variety of sources, to permit the selection and amalgamation of the data in any arbitrary manner, and to map or quantify the desired thematic information. Figures 1 and 2 show how this process is carried out, using combinations of individual themes in deriving a new map. The output map should be

TABLE 1. EXAMPLE OF OUTPUT OF THE PROGRAM SYSTEM STAN
(Reservoir Stage Analysis)

RESERVOIR STAGE ANALYSIS
THE ANALYSIS PROCEEDS FROM LEVEL 650
TO LEVEL 850 WITH INCREMENTS OF 20 M
THE DAM HEIGHT IS 850 M
THE TAPE IDENTIFICATION LABEL IS
LARGE DELIMITER—DAM TEST RUN 00001730
60224 DELIMITED POINTS FELL ABOVE THE DAM
AREA OF A GRID SPACE— 29.681 SQM
TOTAL NUMBER OF POINTS WITHIN DELIMITED AREA 240179
NUMBER OF POINTS IN MODEL IS 432000
55.6% OF THE MODEL IS DELIMITED

RESERVOIR STAGE STORAGE TABLE

Level Number	Stage Elevation	Number of Points	Total Volume (×100,000 cu m)	Surface Area (×100,000 sq m)	Surface Area as % of Delimited Area
1	650	29925	508.96265	8.88204	12.5%
2	670	38372	708.82837	11.38919	16.0%
3	690	48545	965.05420	14.40864	20.2%
4	710	59249	1282.91650	17.58568	24.7%
5	730	71871	1668.73926	21.33202	29.9%
6	750	87820	2137.05249	26.06584	36.6%
7	770	107333	2712.92139	31.85750	44.7%
8	790	125927	3404.12158	37.37639	52.4%
9	810	143642	4201.92578	42.63437	59.8%
10	830	161311	5104.19141	47.87871	67.2%
11	850	179955	6111.91016	53.41243	74.9%

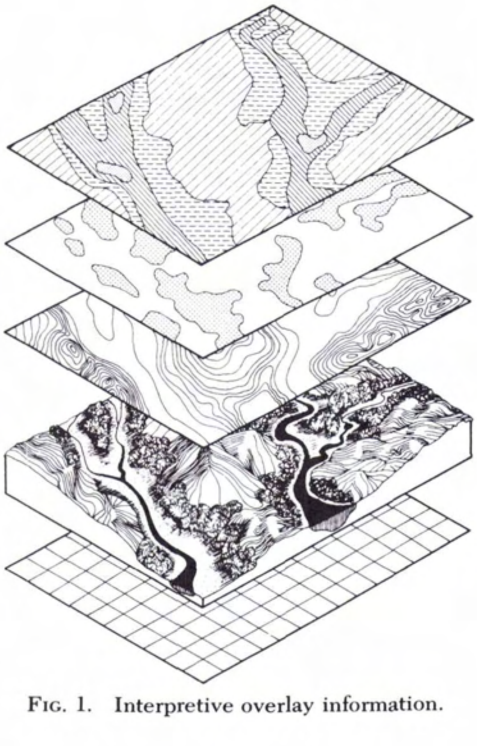

FIG. 1. Interpretive overlay information.

accurately attached to a geographic coordinate system.

The first step is to associate the input data with a square grid coordinate system. The DEM itself is already in that form or can be converted into it. Thematic data digitized in outline form are used to delineate continuous lines with points that coincide with adjacent grid points. These outlines are then scanned and the grid points interior to them are identified. This step is the key one in the whole process of thematic map derivation. With several or many codes associated with the grid points (including elevation), decision functions are now applied to select points according to arbitrary rules. The original contour-outline program can then be used to draw the outlines of the derived thematic material.

The thematic mapping and analysis system (THEMAPS), described above, consists of the following individual programs:

CONNECT:

Interpolates grid points between the points of any digitized line. These grid points are separated by no more than one row or one column, or one row and one column.

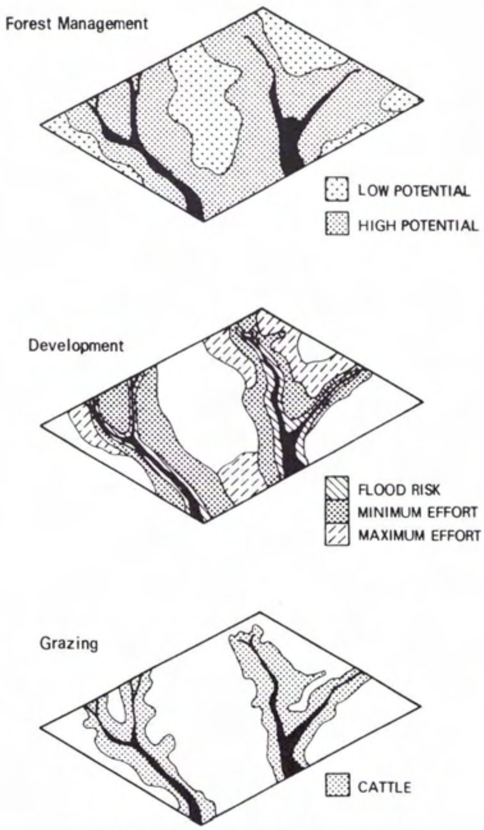

FIG. 2. Derived thematic results.

FILL:

The interior of each grid-connected line is identified, thus creating a grid data base. The process does not require that the direction of digitization, or the inside of the line, be specified. "Donuts" are also accounted for.

SELECT:

A selection program is written by the user, assisted by an interactive program that uses logic algebra. Alternatively, a selection program using arithmetic algebra can be supplied.

Sorting and merging programs are also part of the complete system, which now includes virtual grid storage for large data bases. Virtual grid storage retains the data in polygon form throughout the analysis and plotting stage, but retains the economy of data analysis associated with a grid approach.

As an example of the use of THEMAPS, one thematic mapping project illustrated in Plate 1 involved outline data which allowed the use of the Universal Soil Loss Equation to indicate potential soil loss. The numbers of digitized points from the overlays varied from 9,000 to 64,000, and the data were connected to a grid base containing 220,000 points. Creation of the grid data base required a total of 162 seconds of CPU time on an Amdahl V5 computer. Once the data base was created, only 2 to 5 seconds of CPU time was required to query and analyze the data base and create a data file for transmission to the display unit, in this case a Tektronix 4027 color terminal.

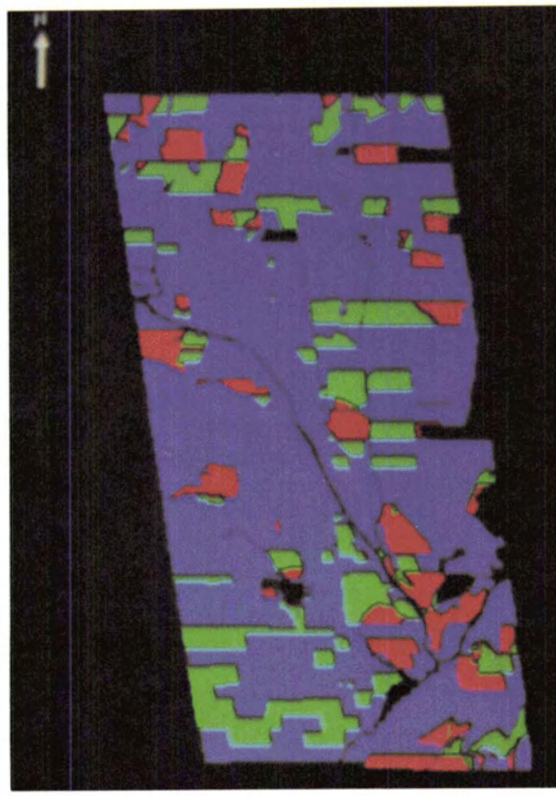

PLATE 1. Potential soil erosion. Erosion Class (t/ac). Blue, 0-1; green, 1-2; red, 2-3. Scale approximately 1:100,000.

As well as thematic mapping and analysis, the basic logic has been extended to a number of interesting techniques for terrain analysis. For example, the ability to handle large amounts of terrain data in a row-by-row manner has led to the implementation of algorithms for determining adjacency and intersections of terrain features. These have not been trivial problems in the past, but the new algorithms permit very rapid determination of such occurrences as the following starting from files of digitized lines, outlines, and points:

- Location of all road (or stream) intersections;
- Order of the intersection (three-way, four-way, etc.);
- Location of intersections of roads with streams (or of any two types of features);
- Simultaneous occurrences (e.g., intersection of road, railroad, and river); and
- Location of all lakes within a given distance of a given road (or other linear feature).

The following is a specific example of a two-way analysis of a particular area, for which complete topographic map data were available in digitized-line form:

Area: 31 km by 36 km

Grid size: 25 m square.

Analyses performed:

- All road intersections and their orders, and
- Number of gravel pits within 200 metres of a railway.

The tasks were carried out on an Amdahl V5 computer and the programs (written in Fortran) required approximately 110 K bytes of main memory for execution. The runs were performed on roughly 100,000 connected feature points (stored on magnetic tape) and required on the order of 25 seconds of CPU time per analysis. The intersection run produced about 300 intersection points, and around 50 railroad points were identified adjacent to gravel pits during the adjacency run.

Conclusions

These algorithms, taken together, form the basis of a complete system for terrain data utilization. All of the new developments in instrumentation and surveying and cartographic procedures seem to be headed toward computerized data handling. The square grid DTM is a format that is most suitable for computer operations. The algorithms that are presented in this paper show how new and valuable information can be extracted directly from the DTM, and how a basic grid of geographic coordinate values can be used as a reference base for unifying data from many sources and for deriving accurate new maps from those data.

Acknowledgments

This work was supported in part by contracts with the Surveys and Mapping Branch, Department of Energy, Mines and Resources, and with the Defense Research Establishment, Department of National Defense. Valuable assistance was given by Miss Lorraine Fetter and Mr. Don Genner of the University of Guelph.

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(Received 26 February 1979; revised and accepted 6 August 1980)

8th Biennial Workshop on Color Aerial Photography in the Plant Sciences

Skyland Lodge, Shenandoah National Park, Virginia
21-23 April 1981

The School of Forestry and Wildlife Resources at Virginia Tech will host the 8th Biennial Workshop on Color Aerial Photography in the Plant Sciences on 21-23 April 1981. The Workshop is co-sponsored by the American Society of Photogrammetry, the Society of American Foresters, and the U.S. Geological Survey. It will be held at Skyland Lodge in Shenandoah National Park near Luray, Virginia.

The Planning Committee is putting together an interesting and stimulating program. In addition to formal papers, three workshop sessions will be held on (a) advanced theory of color science, (b) methods for estimating photo interpretation accuracy, and (c) techniques for estimating timber volume. Several companies will have equipment and representatives present. A field trip to the George Washington National Forest is planned for the third day of the Workshop. This trip will emphasize applications of color-infrared aerial photography for forest management.

Further information is available from

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