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Stereographic Prediction of Grazing Solar Illumination

The moment of time when rays of sunlight at any given time of the year and at any given geographic locality will exactly graze a topographic slope having any given orientation and steepness may be predicted.

Introduction

A meridional stereographic net, the general operation of which is described and explained fully in most standard textbooks on structural geology, can be a useful and simple tool for graphic solution of various problems of three-dimensional relationships of lines and planes in many other fields. For example, suppose that one wants to know the time of day, for a given day of the year, when the rays of sunlight would just graze a planar slope such as a fault scarp, so that optimum photographic emphasis of that slope is possible. The advantages of some low-angle sun photography for detection of obscure fault scarps have been described (Cluff and Slemmons, 1972; Glass and Slemmons, 1978), together with various tabular values for sun altitude and azimuth derived by standard astronomical calculations or by reference to some ingenious computer-generated solar position diagrams (Clark, 1971). The solar position diagrams for 2° intervals of latitude are essentially tabular values presented through a family of curves plotted on a polar stereographic net.

Abstract: *Simple graphic constructions on a tracing paper overlay of a meridional stereographic net permit easy visualization and prediction of the moment of time when rays of sunlight at any given time of the year and at any given geographic locality will exactly graze a topographic slope having any given orientation and steepness. Techniques described in most standard textbooks of structural geology permit use of a meridional stereonet as an analog of the familiar geographic grid of meridians and parallels, on which the geographic position of the slope and the plane of the slope can be plotted. The intersection of the trace of the plane of the slope and the apparent path of the sun during the day provides an estimate of the moment of time for exactly grazing illumination. An additional rotation of the overlay sheet also can yield the altitude and azimuth of the sun at that moment of time or any other moment of time.*

The meridional stereographic method described in the present paper has the advantage of providing easy visualization of the interrelationships among an infinite variety of the variables of solar position, date, time, latitude and longitude, and azimuth and angle of topographic slope. This method also uses the readily available meridional stereonet and does not require special ordering of a multitude of the Mylar copies of the specialized solar position diagrams from the U.S. Geological Survey.

The paper by Clark does not address specifically the question of grazing illumination of a given topographic slope having a certain angle of inclination and direction of slope, although such use of the solar position diagrams is implicit. The papers by Slemmons and his associates do not explicitly and fully relate the infinite variety of declivity vectors of a given topographic slope to the altitude of the sun and its azimuth at a specific moment of time. A possible misconception in use of the solar position diagrams is evident in the statements that (Glass and Slemmons, 1978, pp. 39-41)

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"... approximate slope angles can be determined for scarps by noting the degree of shadowing present on the face of the scarp. Grazing incidence angles occur on slopes with angles equal to the sun angle at the time the photograph was taken ... "

What should have been said is that grazing illumination occurs when the "sun angle" or altitude is equal to the slope-component (analogous to "apparent dip" in structural geology) that has the same azimuth as the direction of the sun's rays at that moment. The acute angle, γ, between the azimuth or direction of the slope-component, σ', and the azimuth or direction of the maximum inclination, σ, of the topographic slope is given by the expression

$$\cos \gamma = \frac{\tan \sigma'}{\tan \sigma}$$

and the slope-component can range from 0° (along the contour or trend of the slope) to a maximum directly down the slope, while γ varies from 90° to 0°. Thus, a particular slope could even be grazed at sunrise or sunset, or at a wide variety of times and seasons (or combinations of altitude and azimuth), and not just at a particular moment once or twice a year when the sun's altitude and azimuth just happen to permit rays to shine directly down the slope.

For example, suppose that a certain topographic slope, at a latitude of 40° N and on a standard meridian of a time zone, slopes 30° in a N30°E (030°) direction. According to the solar position diagram illustrated in the paper by Glass and Slemmons, the sun would indeed graze the slope with rays directly down the slope, when the sun simultaneously has an altitude of 30° and an azimuth of S30°W (210°), shortly before 1400 Standard Time (neglecting the equation of time!) in either early November or early-middle February. What may not be clear, however, but what is easily determined and visualized by the use of the meridional stereographic method described in the present paper, is that the sun's rays will also graze the specified topographic slope at noon at the time of the winter solstice, despite the sun's rays having an azimuth 30° away from the azimuth of the slope.

Further, the sun's rays at sunset in very late May and again in early-middle July will also graze the slope specified, with the sun's rays at that time having an azimuth almost 90° away from the north-northeasterly azimuth of the specified slope. In each of the latter cases, however, the sun's altitude is too far below the 10° lower limit of the optimum altitudes to be useful photographically (Glass and Slemmons, 1978, p. 39).

Using the 10° sun altitude value, one can either calculate trigonometrically the appropriate sun azimuth that corresponds to the azimuth of a slope-component of 10° (or any particular value less than the slope maximum) and then consult the solar diagram again, or one also can easily visualize the relationships directly by the graphical construction explained in the present paper. In either case, the prediction is for grazing illumination with a sun altitude of 10° at about 1800 Standard Time in early May or early-middle August. Of course, grazing illumination of the specified slope occurs also at successively higher sun altitudes and successively more southerly sun azimuths each day between early-middle August and the winter solstice, with a reversal of the changes between the winter solstice and early May. From early May, through the summer solstice, on to early-middle August, the sun would illuminate the specified slope all day long, and grazing illumination would not be possible, in this case, during that interval.

The meridional stereographic construction for each specific case may seem somewhat tedious, but it does have the advantage of easy visualization of the variables, perhaps more so than does the solar position diagram. The following pages describe and illustrate the use of the meridional stereographic technique on an actual topographic slope that does not happen to lie on either a zone meridian for standard time or on one of the even-numbered parallels of latitude for which the solar position diagrams were calculated.

Procedure

The complete stereographic solution of a typical problem is shown in Figure 1; the various steps in the graphic construction are explained in the following paragraphs. The slope used as an example is illustrated in Figure 2. The fault scarp below the "Lute" triangulation monument is an essentially planar slope that trends N65°W/S65°E and slopes approximately 32° in a north-northeasterly direction (N25°E, or an azimuth of 025°). The fault scarp has the geographic coordinates of approximately 33°N and 116°W.

If the time of year is specified as mid-April, the sun's declination would be about 10°N and the equation of time essentially zero. Thus, the sun would lie on the 120th meridian at noon PST, and its geographic position (GP) along the 10th parallel would advance 15° for each hour. For purposes of the graphic solution, astronomical niceties such as the slightly changing declination of the sun in a 12-hour period, the horizontal parallax of the sun, and atmospheric refraction are negligible.

The stereographic net should be oriented first with its central meridian upright, and a sheet of tracing paper should be impaled on a thumbtack inserted through the center of the net. The meridians can be numbered with a range of longitudes or Greenwich hour angles appropriate to the geographic location of the problem. In the example given, the answer to the problem is anticipated to

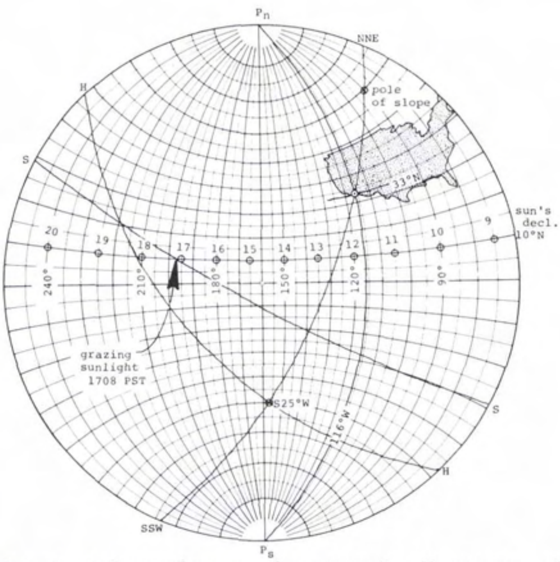

FIG. 1. Solution for time of grazing solar illumination of a fault scarp in southeastern California in mid-April, shown on a 5° meridional stereographic net. Symbols explained in text.

lie in the afternoon hours; therefore, the meridian for the GP of the fault scarp should be made to lie well eastward of the central meridian of the net. An outline map of the United States is shown, incidentally, to emphasize the relationship between the stereographic net and the familiar geographic coordinates.

With the tracing paper held in place, tick marks should be placed at opposite ends of the central meridian, to represent the geographic North Pole (P_n) and South Pole (P_s). The position of the "Equator" is obvious. The GP of the fault scarp should be plotted as a point at the appropriate latitude and longitude, and the meridian of the fault scarp should be interpolated and traced on the overlay sheet. The GP of the fault scarp locality is equivalent also to the zenith of that locality projected onto the celestial sphere.

The GP of the sun at successive hours of the appropriate zone time should be plotted along the parallel of latitude corresponding to the sun's declination for the chosen day, interpolating the small circle or parallel of latitude on the net, if necessary. This apparent path of the sun constitutes a sun/time line on the overlay sheet.

The next step involves rotation of the overlay sheet until the plotted GP of the fault scarp lies on the "Equator" of the underlying stereonet, so that the southerly horizon (*H-H*) of the fault scarp locality can be drawn as a great-circle arc exactly 90° away from the GP of the fault scarp. The intersection of the meridian of the fault scarp locality and the horizon for that locality, of course, is the direction of due south for that locality. Incidentally, the intersection of the horizon and the sun/time line defines the time of local sunset (or sunrise) for the fault scarp locality.

With the tracing paper still held in this second position, 25° should be counted off west of south along the horizon, in exactly the same manner that one would use in counting off the difference in latitude of two points on the same meridian in geography. This latter operation is done to establish a S25°W direction, so that a vertical plane trending N25°E/S25°W can be defined, a vertical plane that trends in a direction at right angles to the trend of the fault scarp. That vertical plane also contains the pole of the fault scarp, a line that is perpendicular to the plane of the fault scarp.

To define and trace the NNE/SSW vertical plane, the tracing paper should be rotated until both the polotted GP of the fault scarp locality and the plotted S25°W point on the horizon lie on the same interpolated great circle of the underlying net. If the fault scarp faces north-northeasterly and slopes 32° in a N25°E direction, its pole will have a *zenith distance* of 32° from the GP of the fault scarp locality, also in a N25°E direction. Thus, the pole of the fault scarp should be plotted 32° northerly from the GP of the fault scarp, along the trace of the NNE/SSW vertical plane.

Now it is a simple matter to rotate the tracing paper once more, until the point representing the pole of the fault scarp lies on the "Equator" of the underlying net, so that the stereographic trace of the plane of the fault scarp (*S-S*) can be drawn exactly 90° away from the pole of the fault scarp, in a manner similar to tracing the horizon for the GP of the locality. The resulting intersection of the trace of the fault scarp plane and the sun/time line gives the moment of time at which the sun exactly grazes the slope—a few minutes after 5 P.M. (1700) PST.

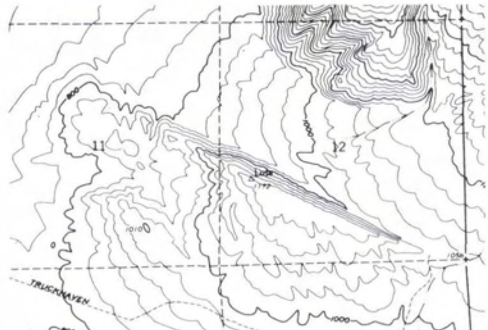

FIG. 2. Central portion of U.S. Geological Survey topographic map (Fonts Point 7.5-minute quadrangle, San Diego County, California) showing expression of fault scarp used as example.

Additional Applications

Further rotation of the overlay sheet until the grazing sun's GP at that moment of time and the fault scarp's GP are on the same great circle allows for easy solution of the grazing sun's altitude as about 13° above the horizon and the grazing sun's azimuth as a few degrees north of due west. Somewhat earlier than the moment of grazing rays from the sun, the sun is "above" the plane of the fault scarp and slightly illuminates the slope. Somewhat later, the sun is "below" the plane of the fault scarp and casts a deepening shadow that may obscure the fault scarp.

A further implication of this plotted solution is that the sun at summer solstice (23.5°N declination) would always illuminate the fault scarp, even right up to the time of sunset, because the intersection of the trace of the plane of the fault scarp and the sun/time line for such a northerly declination would occur only below the horizon. Furthermore, the sun at winter solstice (23.5°S declination) would just graze the fault scarp shortly before 1 P.M. (1300) PST, casting a lengthening shadow all during the rest of the afternoon.

With certain combinations of time of year and attitude of the slope, there might be two times of grazing solar illumination, one in the morning and one in the afternoon. A special combination would give grazing illumination exactly at noon; still another combination would give grazing rays shining exactly down the slope, in a N25°E direction. Experimentation with a variety of conditions is recommended, to achieve thorough understanding of this method. Thorough understanding of these principles can lead to challenging use of the stereonet for solution of similar geometric problems in celestial navigation, astronomy, sundial design, architectural and solar energy design, etc.

For most days of the year, when the equation of time is not negligible, the correction to clock time or zone time can be obtained from an almanac or even a generalized analemma. Alternatively, the hourly positions of the sun on the sun/time line can be shifted by an angular amount corresponding to the equation of time.

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