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Grid Cell Size in Relation to Errors in Maps and Inventories Produced by Computerized Map Processing

As cell size was allowed to increase, the accuracies of maps and inventories produced by computer processing decreased.

Abstract: *Studies are reported which improve the understanding of the process of converting map data from a graphical representation to a computer compatible format. A uniformly shaped and spaced network of cells, a grid, may be used to determine the spatial characteristics of the map. Investigations were made into (1) a technique for characterizing the spatial nature of a map, (2) the effect of cell size and grid position on computer processing to produce inventory tables and new maps, and (3) the potential for modeling spatial cellularization.*

The frequency distribution of distances between boundary lines enclosing homogeneous map units was employed to characterize spatial characteristics of a map. The accuracies of maps and inventory tables produced by computer processing of a single map with different cell sizes and grid positions were determined. Grid position significantly affects accuracy when one isolated homogeneous map unit is processed; it is not significant in processing maps containing many such units. The importance decreases as the randomness of shape and size of the mapping units increases.

As cell size was allowed to increase, the accuracies of maps and inventories produced by computer processing decreased. Likewise, sample statistics (mean, mode, variance) of the interboundary distance distributions at each cell size were found to decrease systematically with the increases in cell size.

A mathematical modeling process was formulated to allow (1) estimation of the interboundary distance distribution of a map before cellularization and (2) prediction of mapping and inventory accuracies which might be achieved with different cell sizes. Two models were derived on the assumption that the quantities involved were one dimensional and were tested in comparison to experimentally observed accuracies. Although both models overestimated the errors at any particular cell size, the predictions were not erratic and the behavior of the models encourages further research into refinement of the models.

Introduction

Information obtainable through remote sensing techniques is becoming more useful to natural resource management and planning as it becomes integrated with other sources of information.[1] Computer processing is indespensible in storing, manipulating, retrieving, and displaying large quantities of diverse data.

Geographic information systems are used to produce inventories of resources in spatial units (acres, hectares, etc.) and to produce maps specifying the location of the resources. These products must be of known and reasonable accuracy to be acceptable to any decision process. Many articles in the field of remote sensing discuss accuracy of products resulting from process-

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ing to (1) identify mapping units by interpretation or machine assisted classification or (2) remove geometric distortions caused by the sensor. These articles are concerned with the correctness with which map units are identified and with the map geometry itself. This paper is concerned with computer processing of map data assuming the map is error free. The studies reported are concerned with understanding the effect of dividing maps into a grid of cells to allow computer processing.

Background

In order for an information system to be termed geographic and have the capability to generate maps, the data base must be designed to include spatial location information.[2] Location identifiers can be included in the data base by one of four techniques: external index, coordinate reference, arbitrary grid, and explicit boundary.[3] The latter two techniques maintain map boundary information in a form suitable for mapping. They are used in the two most common forms of geographic information systems, known as grid (cell) or line (polygon) systems, respectively.

A systematic comparison of the operating costs of cell and polygon system and of product accuracies was made by Smith.[5] He found that conversion of map data and typical analyses were eight to ten times more expensive with the polygon system although that system exhibits higher spatial accuracy. The faster, simpler cell systems are generally less expensive.[4,6]

A common criticism of cellular systems is that the gridding of the map for computer compatibility forces some selected grid cell size to be the lower limit on the spatial resolution. This makes cell size selection extremely important in creating a data base. Guidelines in the literature include utilizing the resolution of the source data,[7] selecting the smallest cell affordable in the operation of the system,[2] adjusting the size and shape of the cell to match the capabilities of the output device, e.g., rectangular to offset line printer aspect ratios,[8] and selecting a cell size small enough that the smallest mapping unit will be greater than 50 percent of any cell.[4,9]

A better understanding of the effect of cellularization on a map would be useful in (a) selecting the cell size for map conversion to computer format, (b) assigning cell size when converting from a polygon format to the cellular format, and (c) deciding on cell size changes during the course of map processing.

Cell size has significant effect on map accuracy. Nichols[10] reported a brief study of several cell sizes and soil map complexities and concluded that cellularization was too inaccurate. Hord[11] proposed a statistical model for evaluation of map accuracy, and Van Genderen[12] extended the application of the model to the problem of guarding against overconfidence. Note that the Hord-Van Genderen analyses require that the product map be in hand, and Tomlinson[13] reports that data preparation costs for a cellular system run four to five times the analysis costs. Hence, procedures for iterative digitization-evaluation-digitization are not appropriate to the problem.

Switzer[9] developed a map accuracy evaluation technique as a Boolean overlay of an "estimated" map and the "true" map with a two-level resultant map of matching and non-matching categories. Mathematical arguments and approximations allowed him to estimate the map accuracy from the "estimated" map alone. In the course of his analysis, he also justified square cells. His procedure, however, requires that the computer data be created before accuracy can be evaluated.

The Hord, Van Genderen, and Switzer procedures are useful in evaluation of product accuracies only in retrospect. The problem of cell size selection requires predictive capability. An estimation of product accuracy before data entry begins is necessary.

Mapping and Inventory Accuracies

The performance of a geographic information system may be measured by the accuracy of the products it produces, i.e., maps and tables (assuming that the map data in the data base are error-free). An experiment was conducted to seek a relationship between input map data characteristics and output mapping and tabulation accuracies with various cell sizes (for a detailed discussion consult Wehde[14]).

It should be emphasized that it is the cellularization of maps that is being studied, not a particular cellular information system. The information system employed for the study is described only to document the procedures by which the evaluation of cellularization took place.

The Area Resource Analysis System, AREAS, an information system developed at the Remote Sensing Institute, South Dakota State University.[15] AREAS provides the capability to change resolution (cell size), overlay maps, interpret maps, tabulate data sets, plot or record results and analyze data characteristics.[16]

A portion of a detailed soil survey map representing a two mile square area was selected as representative of a map with moderate polygon density yet diverse shapes and sizes of map units. To create a data base which could be employed as the "accurate" or "true" map standard, a very small cell size was selected. The cell size was also constrained to be a small subdivision of approximately 1, 4, and 16, ha (2.5, 10, and 40 acre) cells such that the study of increasing cell sizes would include these historically common sizes. A 0.007 ha (0.017 acre) cell met these requirements and resulted in a map data base of 384 cells per row in 384 rows. The original map and the base data set are shown in Figure 1.

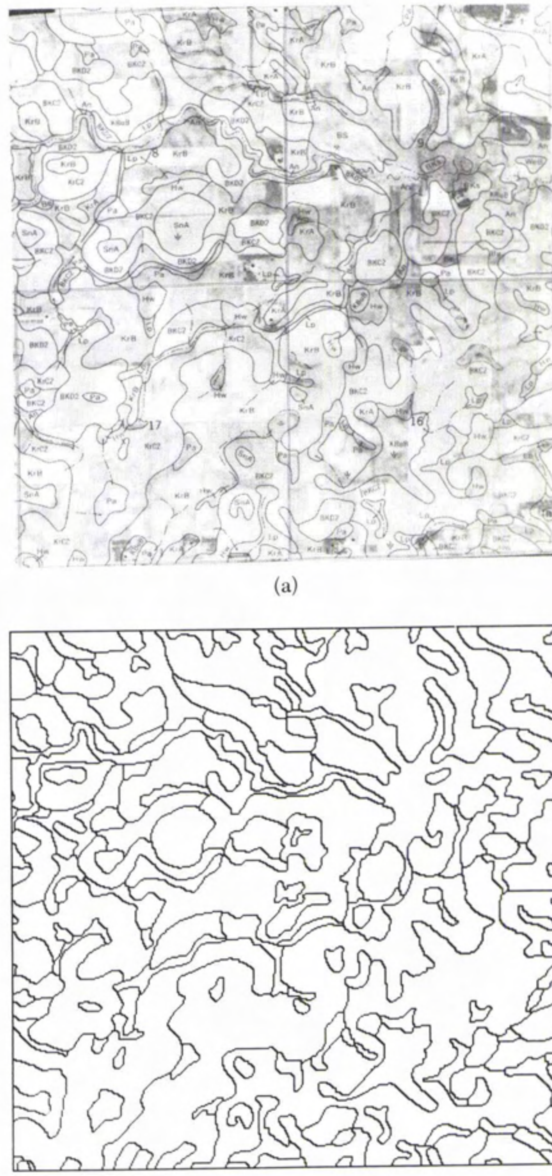
Eleven additional data sets were created by adjusting the cell size. A grouping or aggregation of cells by integral multiples was employed, that is, pairs of cells in pairs of rows combined, groups of three cells in three rows combined, etc. In each succeeding case fewer cells of larger individual area represented the contents of the original map. Only those integral factors which evenly divide the 384 cell by 384 row map were utilized. This eliminated the situations of partial cells being created at the ends of rows or in the last row of the new map data set.

The integral factors employed to group cells into new map data sets were 2, 3, 4, 6, 8, 12, 16, 24, 32, 48, and 64. In the remaining figures and text these factors are termed "resolution numbers" or "resolutions" to maintain a context of spatial extent of the cell on the Earth's surface. The resolution numbers cited correspond to 0.028, 0.063, 0.112, 0.252, 0.448, 1.008, 1.792, 4.032, 7.168, 16.128, and 28.672 ha (0.069, 0.156, 0.278, 0.625, 1.111, 2.500, 4.444, 10.000, 17.778, 40.000, and 71.111 acres). The original, reference map data set and the eleven new map data sets created by the cell aggregation technique are mapped in Figure 2 by a film recording process.

The twelve data sets in the data base represent the same map cellularized at twelve different cell sizes. The AREAS information system was utilized to evaluate the accuracy of maps and inventories produced from each of the twelve data sets by the process shown in flow chart form in Figure 3. The process is shown for one resolution number and was repeated a total of eleven times. With the exception of the COMPARE step, all ovals in the flow chart signify an AREAS processing function, i.e., TABULATE, COMPOSITE, AGGREGATE, and INTERPRET.

The AGGREGATE function was written to relocate map boundarys among cells in a manner simulating larger cell sizes without actually reducing the number of cells or rows. This was necessary to allow the COMPOSITE function to overlay maps with a like number of cells for an analysis of combinations. Also, this kept all maps consistent in size (147,456 cells) to allow calculation of mapping error percentage based on the number of incorrectly assigned cells (INTERPRET as mismatch).

The percentage inventory error could not be obtained from the total cells tallied at each resolution because every inventory counted 147,456 cells. The inventory error had to be obtained from individual map theme or data categories present and then mathematically combined. Categories were inventoried as overabundant (+error) or too sparse (−error) on an individual basis. In total, these errors of commission and omission over



(a)

(b)

FIG. 1. A portion of the soil survey used for the experimental study. (a) the original data sheet and (b) the AREAS product at the 0.007 ha (0.0174 acre) cell size.

categories cancelled out. A root-mean-square error was calculated in order to avoid cancellation of errors. This, however, generated a value representing the average inventory error per map category rather than over the entire map. A root-sum-square calculation avoided the averaging over map categories to produce an inventory error for the map data set. The mapping and inventory errors determined for the twelve data sets are plotted in Figure 4. The relationship to cell size appears well

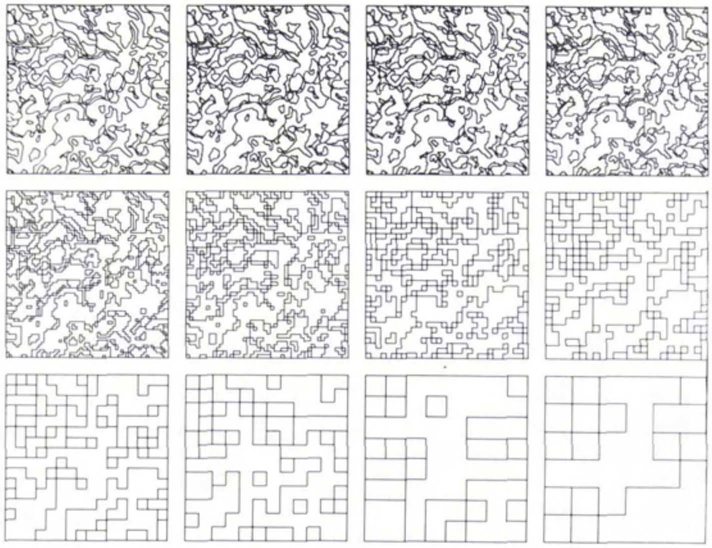


FIG. 2. The twelve maps analyzed for mapping accuracy with changing cell size. Cell sizes from top left to lower right are 0.007, 0.028, 0.063, 0.112, 0.252, 0.448, 1.008, 1.792, 4.032, 7.168, 16.128, and 28.672 hectares.

enough behaved to be modeled by curve fitting techniques, but no applicability beyond the present data would be achieved.

Since the cellularization process introduces error at the borders between homogeneous map regions, one might expect mapping error to depend on region size and perimeter (border length). The INTERPRET function was used to separate each of three map categories of differing abundance and complexity into map data sets. The mapping error evaluation diagrammed in Figure 3 was applied to these single-category data sets. Figure 5 shows the three data sets and the mapping error behavior with changing resolution number.

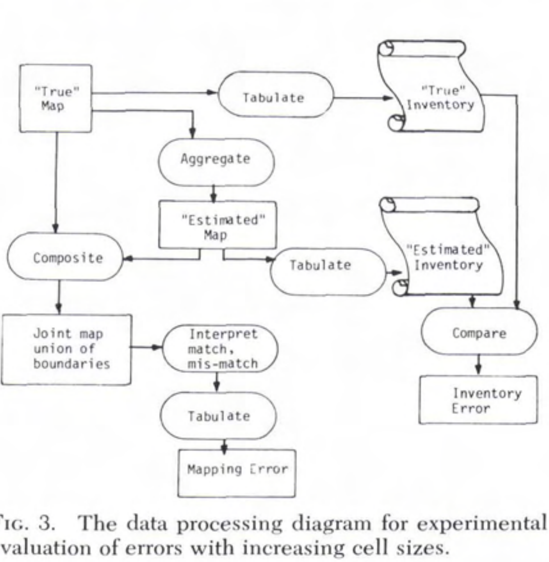


FIG. 3. The data processing diagram for experimental evaluation of errors with increasing cell sizes.

The BNE category with only one mapping unit demonstrates mapping error increasingly erratically with increasing resolution number until the cell size being created is too large for the unit to ever dominate the cell. From that cell size upward BNE is no longer represented at all in the map data set, hence mapping error is 100 percent. Smaller mapping units (polygons) within the category KRA or BKC2 would by themselves also exhibit a similar mapping error behavior with increasing resolution. The increasing abundance and size of polygons in these two categories allows increasing opportunity for more than one polygon to contribute to each of the larger cells being created and thereby provides opportunity for these categories to survive the larger cellular representation.

The mapping error graphed in Figure 4 for all map categories in the study, in comparison to the

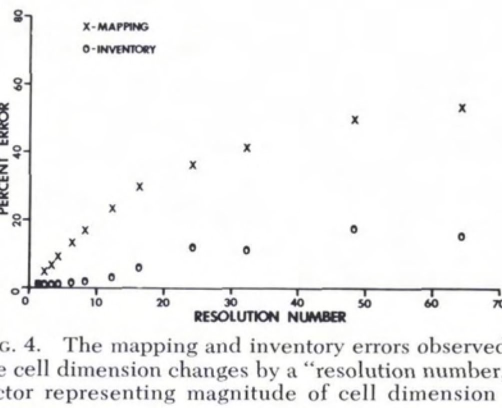


FIG. 4. The mapping and inventory errors observed as the cell dimension changes by a "resolution number," a factor representing magnitude of cell dimension increase.

mapping error graphs for the isolated map categories in Figure 5, demonstrates the averaging effect across map categories.

Importance of Grid Position

When a map unit is of a size approximately equal to that of the cells being used to represent the map, a wide variation in mapping error is possible depending on the position of the grid cells with respect to the map unit (Figure 6). The mapping errors graphed in Figure 4 were those resulting from positioning the sequence of increasingly larger grid cells in alignment with the row-one, cell-one position of the highest resolution data set.

When a cell dimension is doubled, the area increases by a factor of four. The resulting larger cells might be placed in alignment with either of two cell positions and either of two row positions in the smaller cell grid. In general, for a change of cell dimension by a factor of k there would be k^2 ways of positioning the new cell network over the old. The behavior of mapping error for individual mapping units, as shown in Figure 5, might be accounted for by the forced selection of a particular grid placement from the k^2 possibilities at each resolution k.

In order to evaluate this possibility, a mapping error analysis was conducted using a single circular polygon and a single rectangular polygon. At each new resolution, the mapping error for each possible orientation of the new grid cells in alignment with the old grid cells was recorded. The mapping error for the grid position aligned with row one and cell one was noted. Mapping error was averaged over all grid positions at each resolution.

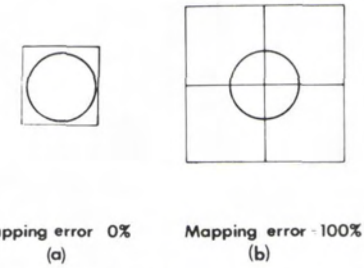


Fig. 6. The effect of grid position on mapping error for a single circular mapping unit. (a) the circle dominates the cell. (b) the circle does not dominate any of the cells.

A conceptually simple linear prediction of mapping errors for various resolutions was also defined for reference. The "linear model" simply predicts zero error at the reference or "true" map resolution of one, and 100 percent mapping error at whatever resolution exceeds twice the area of the mapping unit. Mapping error is linearly interpolated between these points. This model correctly represents the results of the dominant-theme cell encoding rule for simple closed mapping units or polygons of approximately square shape.

Figure 7 shows the mapping error for common point grid alignment, the mapping error averaged over all grid positions at each resolution, and the "linear model" of mapping error.

The results in Figure 7 do demonstrate that mapping error averaged over all possible grid positions is well behaved compared to the erratic behavior of mapping error observed in a sequence of aligned positions. An averaging principle is intuitively accepted: In the limit as k approaches infinity, the average mapping error for k positions of a grid over a single mapping unit is equivalent to the mapping error for k identical mapping units distributed throughout k positions on one grid. The requirement that $k \to \infty$ can be removed and equivalence can still be accurately maintained if the particular k positions of grids over mapping unit correspond one-to-one with the k positions of mapping units within one grid. Although many randomly placed identical mapping units are not often (if ever) found in practice, the averaging may be adequately approximated by a multiplicity of shapes, sizes, and placements of mapping units. In comparing the mapping errors for mapping units in Figure 5 to the overall map results in Figure 4, the averaging effect is apparent. Specifically, the conclusion is that grid positioning is not an important factor in dealing with a map (if the map has

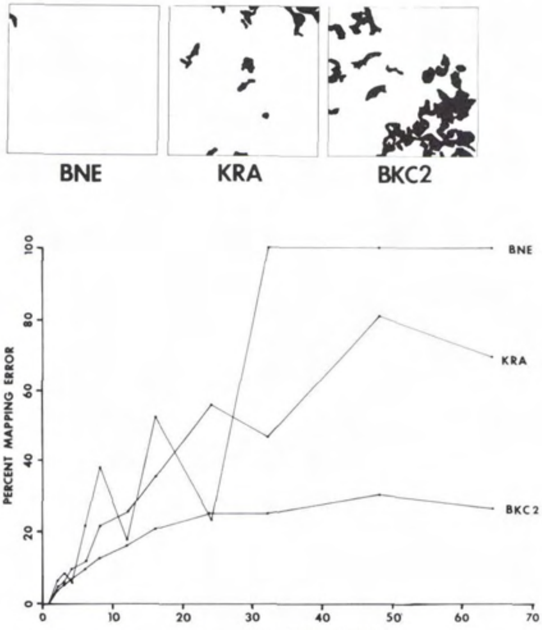


Fig. 5. Three selected soil mapping units and the mapping errors with changing cell dimension.

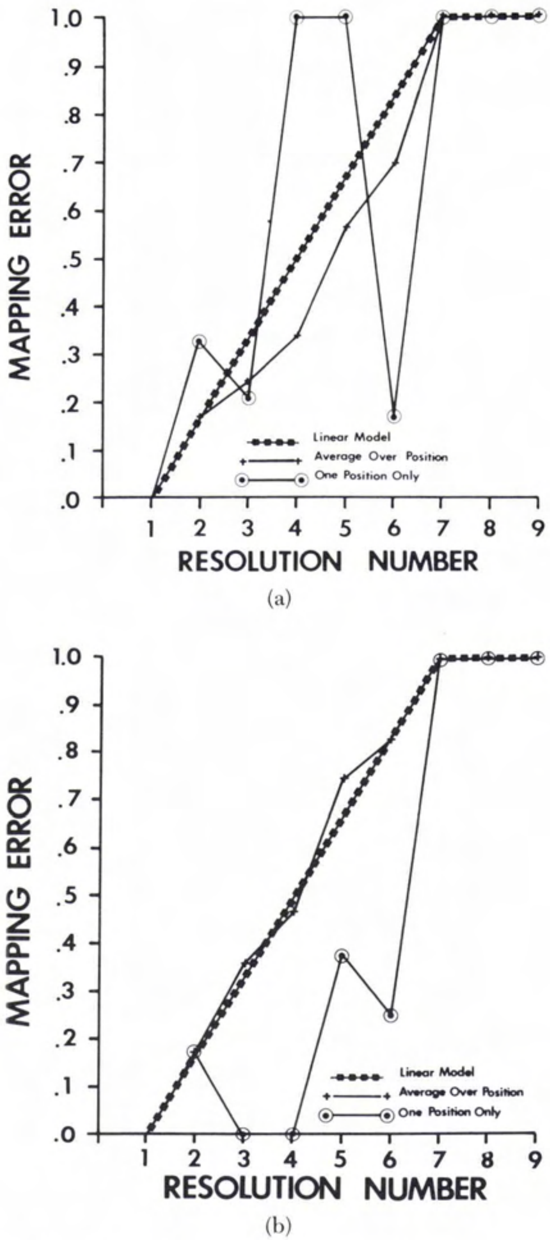


FIG. 7. Mapping errors for selected resolution numbers when the mapping unit is a simple closed figure. (a) a circle and (b) a rectangle.

a sufficient number of diversely shaped and/or diversely oriented mapping units to approximately satisfy the averaging principle).

EVALUATING MAP STRUCTURE

Map structure is the particular arrangement of various sizes and shapes of homogeneous polygons which uniquely defines the map itself. These same factors—size, shape, and arrangement of polygons—also determine the adequacy of any particular grid cell size for representing the map. A characterizing feature which simultaneously represents size, shape, and arrangement must be measurable or estimatible and offer a means for predicting cellular mapping error before maps are actually cellularized at any limiting resolution.

Since a map is a collection of spatially distributed boundary lines, the frequency distribution of distances between these lines must also uniquely represent the map. The distribution is continuous by nature since distance is a real numerical value subject to unit and scale influence. This continuous distribution is termed the inter-boundary distance distribution (IBD). Estimation of the distribution would be possible by randomly placing points and measuring boundary-to-boundary distance along random directions in order to enable construction of a relative frequency table. Since the intent of this paper is to use this IBD as a basis for modeling errors arising from finite cellularization of a map, the estimation of IBD and IBD statistics is restricted to the two orthogonal dimensions which would correspond to the rows of cells.

In a cellular information system, the interboundary distances are forced to take on some value which is an integer multiple of the cell dimension. In this sense the observation of interboundary spacing by analysis of a cellular information base becomes discrete. This discrete interboundary distance (in numbers of cells or multiples of the cell dimension) is called the span distribution.

A boundary analysis program of AREAS simultaneously scans along cells and down rows. The span distribution is generated for the "cells" dimension and the "rows" dimension and is combined for the map data set. Although a number of distribution forms are tabled, graphed, and statistically compared, the primary interest is the behavior of the map span distribution for each of the twelve data sets analyzed. These are shown in Figure 8.

The axis labeled "distance" indicates the number of cells between boundarys. Resolution one is represented in Figure 8a and is the best estimate of the map structure for the "true" map. At resolution six (Figure 8d) the most frequent boundary separation is one cell and certainly in the vicinity of resolutions 12 and 16 (Figures 8g and 8h) the data set becomes dominated by boundary separations of one cell. At approximately this point in the sequence of resolutions, the map structure is being overridden by the grid structure. The pattern of decreasing mean and variance corresponds to decreasing numbers of larger cells required to represent any particular interboundary distance.

The Span Distribution, as a two dimensional

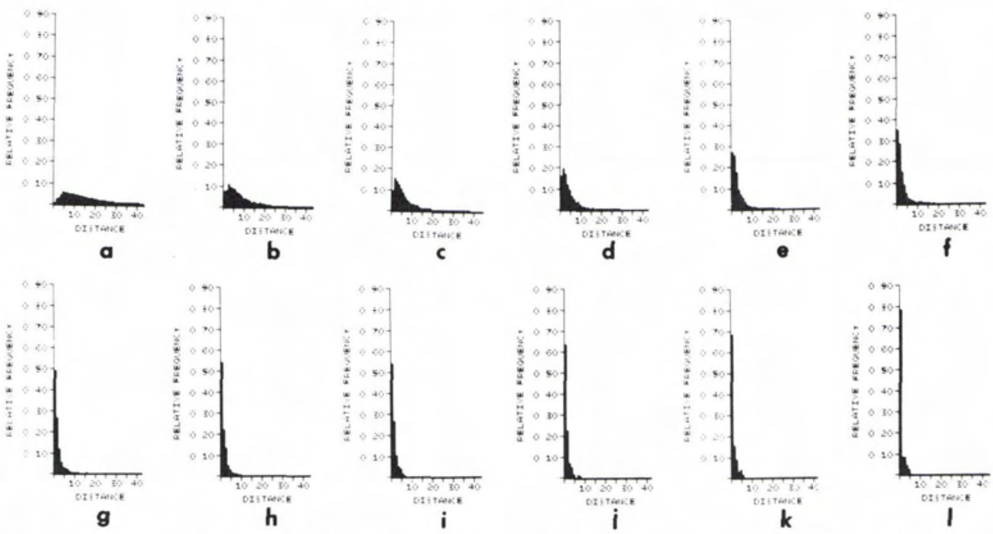


FIG. 8. Span distributions for the combined horizontal and vertical scans of the twelve study data sets in Figure 2. Resolution sequence is from upper left to lower right.

discrete estimate of the map IBD, behaves in a reasonable manner with changing resolution. Recall, however, that the distance in each graph of Figure 8 is measured in number of cells. The cell size for each graph is different. Taking the mean of each distribution times the actual cell size for that distribution allows the mean distance between boundaries to be plotted for each resolution number, as in Figure 9. The high degree of linearity may be accidental. If the placement of map boundaries were not affected by changing resolution, the increase to resolution k would decrease the mean of the distribution by $1/k$ and the graph would be of a constant, i.e., a horizontal line. The departure of the relationship from a horizontal line is an indication of the effect of cellularization on map structure. The slope and linearity of this relationship probably holds only for this particular map, and regression modeling is, therefore, not appropriate.

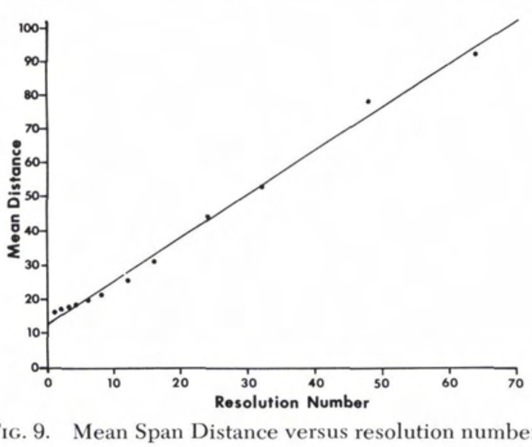


FIG. 9. Mean Span Distance versus resolution number.

In establishing the existence of some relationship between mapping error and cell size (Figure 4) and between Mean Span Distance and cell size (Figure 9), the potential usefulness of the IBD concept for predicting mapping error is demonstrated. One should note that a statistic such as the mean is not unique to a map since more than one IBD could have the same mean. The IBD is unique to a map or set of maps. If more than one map has the same IBD, the application proposed in this paper would merely predict that all maps in the set would exhibit the same error behavior under various cellularizations.

PREDICTING MAPPING ERROR

For a particular map which is to be cellularized, the IBD can represent the important characteristics (polygon shape, size, and arrangement). A model of the errors arising from quantizing distances into discrete units of various sizes would then allow for mapping error prediction. The studies reported hereafter pursue derivation of the proper quantification process model. The span distribution of the "true" map data set was used as the best estimate of the IBD components in the cell and row dimensions.

A two-dimensional array was proposed as the form for the process model. Entries in the array would be some form of error estimate for the span and cell size combination corresponding to the row and column of the array. The process model array would operate on the span distribution to yield map error predictions for each cell size. A diagram of the array modeling technique proposed is shown in Figure 10.

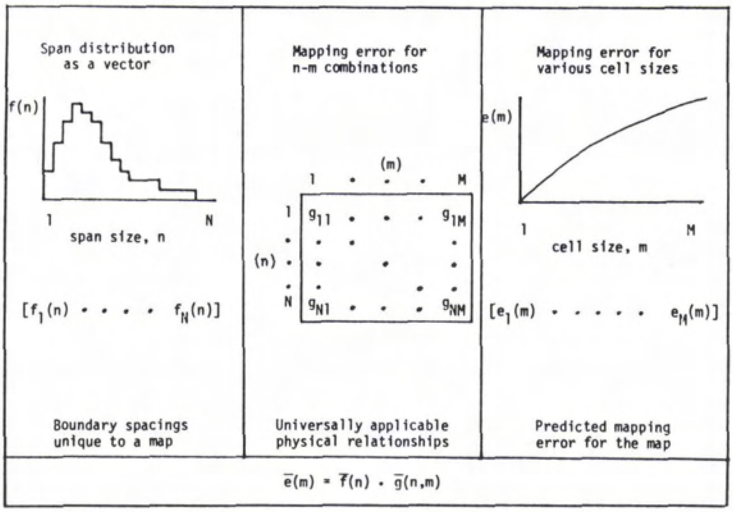


FIG. 10. Components of a simplified mathematical model between span distribution and mapping error vectors.

The span distribution, as an estimate of the interboundary distribution, is the input vector $f(n)$. The array model $g(n,m)$ represents the various choices of quantization sizes (cell sizes), expressed in m discrete-units, in relation to the span sizes, expressed in n discrete-units. The entries are estimates of mapping error for each combination. The vector-array product yields mapping error for various cell sizes, a vector $e(m)$.

THE POSITIONAL AVERAGE MODEL

The array $g(n,m)$ must contain entries representing error for the n-m combination of span size and cell size. Each entry represents a particular span, interboundary map distance. The averaging principle suggests averaging over possible grid positions (instead of averaging over mapping units).

This positional average array of mapping errors was the first experimental attempt at modeling. The derivation of the entries was based on observing enough combinations of spans and cell sizes to establish a predictive pattern in the entries. The simplifying assumptions made were that (1) the observations were of one-dimensional quantities, i.e., line segments (spans) quantized into fixed increments (a cell dimension) with a dominant length coding rule, and (2) each line segment under study was considered isolated on a large homogeneous background.

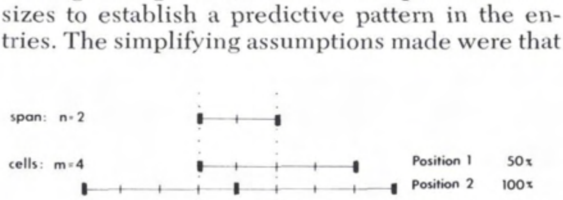


FIG. 11. An example of observing and calculating grid positional average mapping error in a one-dimensional sense.

The way the elements of the array were obtained is illustrated by Figure 11. Two map boundary lines are separated by a distance of 2 units (units are totally arbitrary—feet, inches, miles). If a cell size of 4 units is used to cellularize the map, there are four possible alignments on unit boundaries. Position 1 of Figure 11 shows a cell which lies 50 percent over a map category and 50 percent over a background category. In this tie situation the dominant length is randomly assigned. Hence 50 percent of the time the span is 100 percent incorrectly mapped (50 percent mapping error). In Position 2 the span is split between two 4-unit cells, with neither cell ever being categorized into the map unit. The span is never mapped correctly, hence the 100 percent mapping error. Positions 3 and 4 are equally likely variations of Position 1, and the overall average over possible grid positions is 62.5 percent.

Enough model array elements were calculated to observe a pattern, mathematically expressable for computer implementation. A computer program was written to generate the $g(n,m)$ to the dimensions required. The resulting $g(n,m)$ was applied in the manner outlined in Figure 10. The estimated mapping error from the model is compared to the experimental results in Figure 12.

Although the model strongly overestimates the experimentally observed mapping error, the trend and behavior of the model are encouraging. The

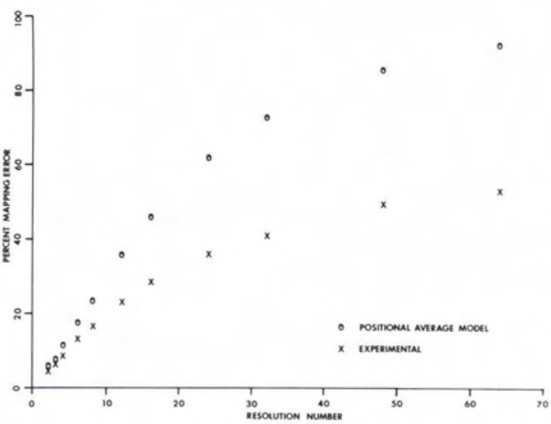


FIG. 12. The experimental mapping error and the mapping error predicted using the positional average model.

comparison is suggestive of a correct approach with perhaps oversimplifying assumptions.

THE SPAN ADJACENCY CORRECTED MODEL

The assumptions of the positional average model were oversimplifications of the two-dimensional cellularization process. Correction of the overestimation in Figure 12 was considered possible by (1) removing the span isolation assumption and (2) changing to a two-dimensional model (which would alter the array concept of Figure 10). The former approach was considered a more convenient alternative.

Removing the span isolation assumption allows spans to occur in sequence adjacent to one another. The occurrence of small spans adjacent to a span under study can alter the result of the dominant length rule used to decide what to do about fractional increments (Figure 13). The corrective effects are observed by comparing Figures 11 and 13.

The map background is homogeneous and of a category different from any of those represented by the adjacent spans. Only Positions 3 and 4 are repeated to demonstrate the corrective influence of the 1-unit span on mapping the 2-unit span with a 4-unit cell. Position 3 is now dominated by the 2-unit span and the category is correctly assigned to the cell, hence mapping error of 0 percent with respect to the 2-unit span.

The correction for span adjacencies requires consideration of the joint event, a span of n_1 units followed by a span of n_2 units. The occurrence of either span alone would be estimated by $f(n_1)$ or $f(n_2)$ from the span distribution. The estimation of relative frequencies for joint events can be simplified by assuming that spans occur independently of one another. Then the occurrence of a span of n units adjacent to any particular span of

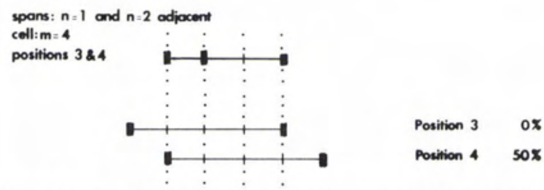


FIG. 13. An example of the positional average mapping error calculation when a one-unit span is known to be adjacent to the span of concern (compare to Figure 11).

interest can also be represented by $f(n)$, and joint event relative frequencies are simply the product of the relative frequencies of the components.

The procedure used in obtaining the Positional Average Array was repeated. The patterns of single and multiple adjacent spans which would reduce the mapping error by altering the dominance encoding step were recorded. The resulting correction array was of the same size $g(n,m)$. The entries were functions of $f(n)$, powers of $f(n)$ and multiple terms $f(n)$, $f(m)$, etc., corresponding to a single span, several equal spans, and a mixture of spans, respectively. The number of such terms in each entry mushroomed as n and m were increased. The span distribution of the reference map data set (highest resolution, smallest cell) had no span more frequent than 0.06; therefore, all higher power and cross product terms were dropped. The first-order, span-adjacency-correction array was generated by computer from the relationships of the first-order coefficients of $f(n)$.

The result of multiplying the array with $f(n)$ as in Figure 10 was a prediction of the correction to be applied to the positional average mapping error predictions. Figure 14 displays the experimental mapping error and the two array predictions for comparision. The span adjacent correction re-

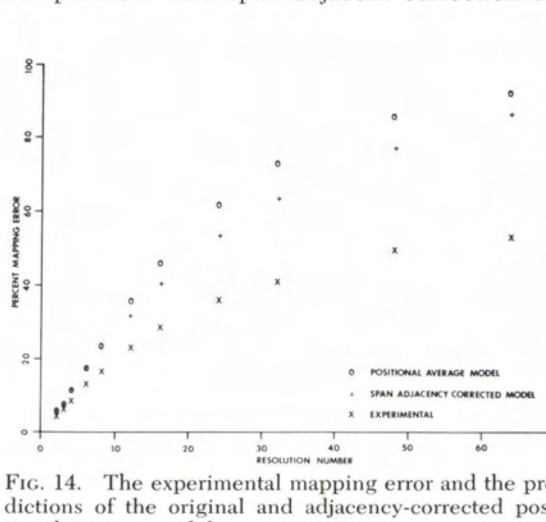


FIG. 14. The experimental mapping error and the predictions of the original and adjacency-corrected positional average models.

moved a part of the deviation of the prediction from the experimentally observed error.

Continued investigations appear warranted. The validity of span independence and first-order assumptions in the span-adjacency correction should be investigated. Also, the more difficult array model for two-dimensional cellularization by positional averaging and span-adjacency corrected positional averaging should be evaluated.

Conclusions

The studies reported have led to the following general conclusions:

- Grid positioning is not an important map accuracy consideration for maps but is a significant source of mapping error variation for individual map polygons.
- The interboundary distance distribution characterizes the size, shape, and arrangement of polygons in a resource map and is an appropriate input to a cellularization-process model.
- Interboundary distance distributions and mapping errors each relate to changing cell size in a well behaved fashion, making it likely that the process can be modeled.
- The grid positional average mapping error array is a significant and important component of a universal cellularization process model.
- The physical process of quantizing distances (cellularizing maps) can potentially be modeled accurately enough to allow prediction of mapping error for various cell size options and a particular map.

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