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Accuracy of the Normal Case of Close-Range Photogrammetry

Equations are developed, and results are compared with simulation methods.

Abstract: *Normal case photogrammetry is the most commonly used in analog and analytical applications. The present formulas which are used to estimate the accuracy of normal case aerial photogrammetry do not give good results in the case of close-range photogrammetry. Therefore, the method of mathematical models (simulation) is used for estimating the accuracy of the normal case of close-range photogrammetric systems.*

The estimated accuracy obtained from simulation closely matches the accuracy obtained in real applications. The simulation method is costly and complicated. This article gives the mathematical and experimental proofs of newly developed formulas for estimating the accuracy of normal case close-range and aerial photogrammetry.

Introduction

Saudi Arabia has many places of historical importance not only for Saudis but also for many Moslems around the world. Three-dimensional recording is a must for restoration and reservation of these historical monuments. The photogrammetric technique is used for recording those monuments. The problem which faces the researcher in recording these historical monuments is in establishing the optimum layout of the camera-stations relative to the monument in order to obtain the required accuracy.

So far, there have been no satisfactory mathematical formulas, in the field of photogrammetry, which allow the photogrammetrist to determine the optimum positions of the two camera stations. The main objective of this research project is to solve the accuracy problem by developing formulas which will make it possible for the photogrammetrist to determine the object accuracy, for any camera set-up, right in the field. The accuracy of the ground coordinates is a function of the elements of the photogrammetric system. These elements are

- *The Camera Used.* The accuracy achieved by any camera is a function of (1) the principal distance, (2) the format size, and (3) the errors of image points. The photogrammetrist has to estimate these errors (random and systematic) and consider their values in his design.
- *The Plotter Used.* The accuracy achieved by any plotter is a function of (1) the reading error, (2) the pointing error, and (3) the errors of the inner, relative, and absolute orientation parameters. The photogrammetrist has to estimate the values of these errors and consider these values in his design.
- *The Layout of the Two Camera Stations.* The layout of the two camera stations relative to the object is a function of the external orientation parameters. These parameters determine the distribution, transformation, and magnification of the image errors to the corresponding object points.

The mathematical relationship expressing the accuracy of the ground points as a function of the twelve parameters of the external orientation are complicated. In normal case photogrammetry, the twelve parameters of external orientation are reduced to only six parameters. These six parameters are the space coordinates of the two camera stations relative to the object. In the case of a stereo-metric camera, these six parameters are reduced to four (the space coordinates of one camera station and the base distance between the two

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stations). The mathematical relationships which express the accuracy of ground points as a function of these four parameters have not been developed. The existing formulas for normal case photogrammetry express the accuracy of ground points as a function of only two of the external orientation parameters: the base distance (B) and the object distance (D). These formulas, according to Karara (1967), Abdel-Aziz (1974), and Marzan and Karara (1976), have these forms:

$$\sigma X = \frac{D}{c}\,\sigma \tag{1}$$

$$\sigma Y = \frac{D}{c}\,\sigma \tag{2}$$

$$\sigma Z = \frac{D^2}{cB}(2)^{1/2}\,\sigma \tag{3}$$

where σX is the average accuracy of the X-coordinate,
- σY is the average accuracy of the Y-coordinate,
- σZ is the average accuracy of the Z-coordinate,
- B is the distance between the two camera stations,
- c is the camera constant,
- D is the average object distance from the two stations, and
- σ is the standard deviation of the image coordinates.

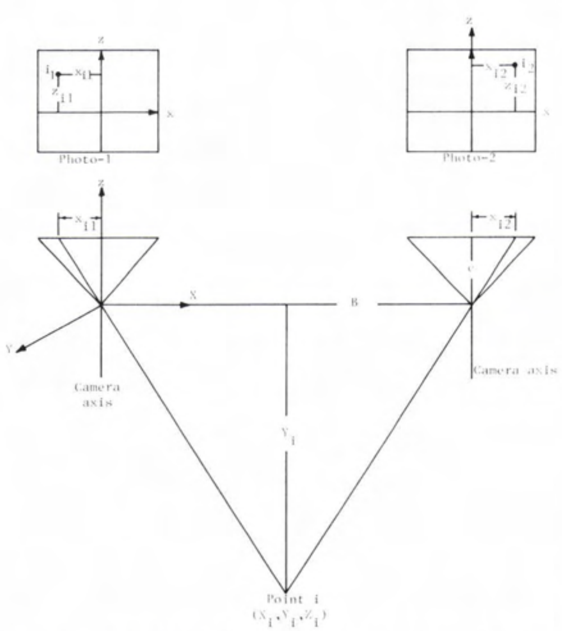

Fig. 1. Data acquisition set-up for the normal case of photogrammetry.

Malhotra and Karara (1971), and Hottier (1976) proved experimentally that these formulas do not give satisfactory results. Accordingly, Malhotra and Karara and Hottier used the simulation technique for estimating the accuracy of the ground points. The estimated accuracy by simulations yields results which closely match the accuracy obtained from close-range applications. The simulation technique cannot be recommended for real applications because it is costly and complicated. Moreover, the photogrammetrist cannot estimate the ground accuracy immediately in the field for any camera stations. Accordingly, new formulas have been developed in this article expressing the accuracy of the ground points as a function of the above mentioned four parameters of external orientation. These formulas may be used in the field for determining the positions of the two camera stations.

This paper discusses two points: (1) the mathematical and the experimental proofs of the developed formulas, (2) a comparison between the estimated accuracies from the developed formulas and the accuracies from a simulation.

The Accuracy of Ground Point Coordinates

The ground coordinates X_i, Y_i, and Z_i of a point i are obtained either analytically or mechanically from the image coordinates on the two photographs.

The equations which are used for calculating the X_i, Y_i, and Z_i coordinates of point i in normal case photogrammetry, according to Figure 1, are

$$X_i = -\frac{Y_i}{c}\,x_{i1} \tag{4}$$

$$Z_i = \frac{Y_i}{2}(z_{i1} + z_{i2}) \tag{5}$$

$$Y_i = -\frac{Bc}{(x_{i1} - x_{i2})} \tag{6}$$

where x_{i1}, z_{i1} are the image coordinates of point i on photo 1,
- x_{i2}, z_{i2} are the image coordinates of point i on photo 2,
- B is the distance between the two camera stations, and
- c is the camera principal distance.

The error in the X_i coordinate, (VX_i), resulting from errors in the image coordinates, Vx_{i1} and Vx_{i2}, can be calculated from Equation 4. The value of Y_i in Equation 4 has to be replaced by its value from Equation 6. Accordingly, Equation 4 takes the following form:

$$X_i = \frac{B}{(x_{i1} - x_{i2})}\,x_{i1}$$

Applying Taylor's expansion to the above equation, one gets the error VX_i as a function of the image errors Vx_{i1} and Vx_{i2}; that is,

$$VX_i = \frac{B}{(x_{i1} - x_{i2})^2}(-x_{i2}\,Vx_{i1} + x_{i1}\,Vx_{i2})$$

$$VX_i = -\frac{Y_i}{c}\left(-\frac{x_{i2}}{b_i}Vx_{i1} + \frac{x_{i2}}{b_i}Vx_{i2}\right)$$

where Vx_{i1} is the error of the image coordinate x_{i1},
Vx_{i2} is the error of the image coordinate x_{i2}, and
$b_i = x_{i1} - x_{i2}$.

The standard deviation σX_i, calculated from the above equation, takes the form of

$$\sigma X_i = \frac{Y_i}{c}\left[\left(\frac{x_{i2}}{b_i}\right)^2 + \left(\frac{x_{i1}}{b_i}\right)^2\right]^{1/2}\sigma$$

Substituting $x_i = x_{i1}$ and $x_{i2} = x_i - b_i$, the above equation takes the form of

$$\sigma X_i = \frac{Y_i}{c}\left[2\left(\frac{x_i}{b_i}\right)^2 - 2\left(\frac{x_i}{b_i}\right) + 1\right]^{1/2}$$

Substituting into the above equation the value (X_i/B) instead of (x_i/b_i), one gets

$$\sigma X_i = \frac{Y_i}{c}\left[2\left(\frac{X_i}{B}\right)^2 - 2\left(\frac{X_i}{B}\right) + 1\right]^{1/2}\sigma \qquad (7)$$

where σ is the standard deviation of the image coordinates (x_{i1}, x_{i2}, z_{i1}, and z_{i2}),
B is the distance between the two camera stations, and
X_i is the ground X-coordinate of point i.

The error of the Z_i-coordinate resulting from errors in the image coordinates Vx_{i1}, Vz_{i1}, Vx_{i2}, and Vz_{i2} can be calculated from Equation 5. The value of Y_i in Equation 5 has to be replaced by its value from Equation 6. Accordingly, Equation 5 takes the form

$$Z_i = 0.5\frac{B}{(x_{i1} - x_{i2})}(z_{i1} + z_{i2}).$$

Applying Taylor's expansion to the above equation, one gets the errors VZ_i as a function of errors Vx_{i1}, Vz_{i1}, Vx_{i2}, and Vz_{i2}

$$VZ_i = \frac{Y_i}{c}\left|0.5(Vz_{i1} + Vz_{i2}) - \frac{Z_i}{B}(Vx_{i1} - Vx_{i2})\right|$$

The standard deviation σZ_i of Z_i, calculated from the above equations, takes the following form:

$$\sigma Z_i = \frac{Y_i}{c}\left|0.5 + 2\left(\frac{Z_i}{B}\right)^2\right|^{1/2}\sigma \qquad (8)$$

The error of the Y_i coordinates VY_i resulting from errors of the image coordinates Vx_{i1} and Vx_{i2} can be calculated from Equation 6. Applying Taylor's expansion to Equation 6, one gets

$$VY_i = \frac{B}{(x_{i1} - x_{i2})^2}(Vx_{i1} - Vx_{i2})$$

The standard deviation σY_i, calculated from the above equation, takes the form

$$\sigma Y_i = \frac{Y_i^2}{cB}(2)^{1/2}\sigma \qquad (9)$$

Equations 7, 8, and 9 give the standard deviations σX_i, σY_i, and σZ_i of the coordinates X_i, Y_i, and Z_i of point i relative to the 1st camera station.

The Average Accuracies of Points on a Vertical Plane j

The values σX_i, σY_i, and σZ_i, given in Equations 7, 8, and 9, represent the accuracy of the ground coordinates of point i. The value of σY_i is constant for all the points on the plane j while the values of σX_i and σZ_i are different for different points on that plane. The accuracies σX_j, σY_j, and σZ_j represent the average accuracies of all the points on plane j at a distance Y_j from the two camera stations. The values σX_j, σY_j, and σZ_j are obtained by integrating the σX_i^2, σY_i^2, and σZ_i^2 values, Equations 7, 8, and 9, for the values X_i and Z_i of all points on that plane. The integration of each equation is treated separately in the next sections.

The average accuracy of X-coordinates X_j

The accuracy σX of point i on the vertical plane j is a function of constant parameters Y_j, c, B, and σ, and variable parameter X_i. The value $\sigma^2 X$ at a point i on plane j, as given in Equation 7, has the following form

$$\sigma X_i^2 = \left(\frac{Y_i}{c}\right)^2\left|2\left(\frac{X_i}{B}\right)^2 - 2\left(\frac{X_i}{B}\right) + 1\right|\sigma^2.$$

$$\text{Let } S_i^2 = 2\left(\frac{X_i}{B}\right)^2 - 2\left(\frac{X_i}{B}\right) + 1.$$

The function $S^2 - X$ has the following properties:

- S^2 has a unit value at $X = 0$ and $X = B$
- S^2 has the minimum value of 0.5 at $X = 0.5B$
- The area A under the curve from a to X_i is equal to

$$A = \left[\frac{2}{3}\frac{X_i^3}{B^2} - \frac{X_i^2}{B} + X_i\right]$$

- The average ordinate S^2 for $H_1 \leq (X/B) \leq H_2$ is

$$S^2 = \frac{2}{3}(H_1^2 + H_2^2 + H_1H_2) - (H_1 + H_2) + 1 \qquad (10.1)$$

- The average ordinate S^2 for $-H_1 < (X/B) \leq -H_2$ is

$$S^2 = \frac{2}{3}(H_1^2 + H_2^2 - H_1H_2) + (H_2 + H_1) + 1 \tag{10.2}$$

- The average ordinate S^2 for $-H_1 \leq (X/B) \leq H_2$ is

$$S^2 = \frac{2}{3}(H_1^2 + H_2^2 - H_1H_2) + (H_1 - H_2) + 1 \tag{10.3}$$

where $H = H_1 + H_2$

HB represent the total stereo-coverage of the object along the X-direction.

H_1B and H_2B represent the distances of the first camera station from the edges of the stereophoto coverage of the object.

The values of H_1B and H_2B are shown in Figure 2.

In positioning the two camera stations symmetrically with respect to the object, $H_1 = 0.5\ (H - 1)$, $H_2 = 0.5\ (H + 1)$, and the corresponding value of S^2 is

$$S^2 = \frac{H^2}{6} + 0.5 \tag{11}$$

The average accuracy σX_j of all the points on a plane j at a distance Y_j from the two camera stations is

$$\sigma X_j = \frac{Y_j}{c}\left[\frac{2}{3}(H_1^2 + H_2^2 - H_1H_2) + (H_1 - H_2) + 1\right]^{1/2} \sigma \tag{12}$$

where Y is the distance between this plane to the two camera stations, and

c is the camera constant.

In the case of aerial photogrammetry, if the overlaps range from 55 percent to 65 percent, the corresponding values of H would range from 1.22 to 1.86, and the values of S would range from 0.87 to 1.04. In the case of 60 percent overlap, $H = 1.25$ and $S = 0.94$. This explains why in the case of aerial photogrammetry S is taken as unity, and Equation 12 takes the same form as Equation 1.

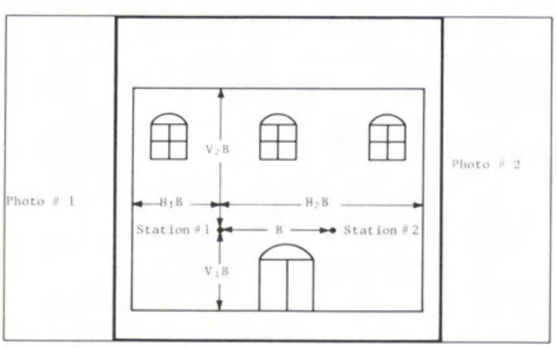

FIG. 2. The stereometric coverage of an object.

THE AVERAGE ACCURACY OF Z-COORDINATES σZ_j

The average accuracy σZ_j of the points on a plane j, at a distance Y_j from the two camera stations, is a function of constant parameters Y_j, c, B, and σ, and the variable parameter Z_i. The equation for σZ_i of a point i on plane j, as was given in Equation 8, has the following form

$$\sigma Z_i^2 = \left(\frac{Y_i}{c}\right)^2 (0.5 + 2\left(\frac{Z_i}{B}\right)^2)\ \sigma^2.$$

$$\text{Let } R^2 = 0.5 + 2\left(\frac{Z_i}{B}\right)^2.$$

The function $R^2 - (Z/B)$ has the following properties:

R^2 has a value 0.5 at $Z = 0$

The area A under the curve $R^2 - (Z/B)$ is

$$A = 0.5Z + \frac{2}{3}\frac{Z^3}{B^2}$$

The area A under the curve for $-V_1B < Z < V_2B$ is

$$A = 0.5B\ (V_2 + V_1) + \frac{2}{3}B\ (V_2^3 + V_1^3)$$

The average value of R^2 for $-V_1 < (Z/B) \leq V_2$ is

$$R^2 = 0.5 + \frac{2}{3}(V_1^2 + V_2^2 - V_1V_2) \tag{13}$$

where

V_1B is the stereophotos' vertical coverage of the two camera stations.

V_2B is the stereophotos' vertical coverage of the object below the level of the two camera stations.

The values of V_1B and V_2B are shown in Figure 2.

In the case of aerial photographs, if the overlap in the X-direction ranges from 55 percent to 65 percent, the corresponding stereo-photo coverage along the Z-direction would range from 2.22 to 2.85 and the corresponding values of R would range from 1.14 to 1.36. In case of 60 percent overlap, $V_1 = 1.25$, $V_2 = 1.25$, and $R = 1.24$. That explains why, in normal case aerial photogrammetry, R has a unit value and Equation 14 takes the same form as Equation 2.

The average accuracy σZ_j of all the points on plane j has the following form:

$$\sigma Z_j = \frac{Y_j}{c}(0.5 + \frac{2}{3}(V_1^2 + V_2^2 - V_1V_2))^{1/2}\ \sigma \tag{14}$$

THE AVERAGE ACCURACY OF Y-COORDINATES σY_j

The accuracy of all the points on plane j at a distance Y_j from the two camera stations is constant for all the points on that plane. Accordingly, the average accuracy has the form

$$\sigma Y_j = \frac{Y_j^2}{cB}(2)^{1/2}\,\sigma \qquad (15)$$

Finally, the average accuracies σX_j, σY_j, and σZ_j of all points on a plane j at a distance Y_j from the two camera stations have the forms given in Equations 12, 14, and 15.

The Average Accuracies σX, σY, and σZ of Three-Dimensional Objects

Equations 12, 14 and 15 give the average accuracies of all points on a plane. The average accuracies of a three-dimensional object can be obtained by integrating the equations for σX_j, σY_j, and σZ_j for all planes of the object. The equations of the average accuracies σX_j, σY_j, and σZ_j as given in Equations 13, 14, and 15 have the following forms:

$$\sigma X_j = \frac{Y_j}{c}\left(\frac{2}{3}(H_1^2 + H_2^2 - H_1H_2) + (H_1 - H_2) + 1\right)^{1/2}\sigma$$

$$\sigma Z_j = \frac{Y_j}{c}(0.5 + \frac{2}{3}(V_1^2 + V_2^2 - V_1V_2))^{1/2}\;\sigma$$

$$\sigma Y_j = \frac{Y_i^2}{cB}(2)^{1/2}\,\sigma$$

The object space has many planes. The exact distribution of ground points in each plane is not known. Accordingly, it is impossible to have general numerical formulas for the integration of Equations 12, 14, and 15. A reasonable estimation of σX, σY, and σZ values is obtained from the average accuracies σX_j, σY_j, and σZ_j of the center of gravity plane. This plane passes through the center of gravity of the object surface on the stereophotos and it contains the perspective projection of all the object's points. The values Y_j, H_1, H_2, V_1, and V_2 in Equations 12, 14, and 15 are estimated for the center of gravity plane. The comparison between the values of σX, σY, and σZ which are obtained from simulation and those obtained from the center of gravity plane are given in Table 1.

The values σX, σY, and σZ can be estimated, in analytical photogrammetry only, from the numerical integration of σX_i^2, σY_i^2, and σZ_i^2 which is given in Equations 7, 8, and 9. The average values of σX, σY, and σZ, estimated from the numerical integration, take the following form:

$$\sigma X = \left[\frac{1}{n}\sum\left(\frac{Y_i}{c}\right)^2 (2\left(\frac{X_i}{B}\right)^2 - 2\left(\frac{X_i}{B}\right) + 1)\right]^{1/2} \qquad (16)$$

$$\sigma Y = \left[\frac{2}{n}\sum\left(\frac{Y_i^4}{c^2B^2}\right)\right]^{1/2}\sigma \qquad (17)$$

$$\sigma Z = \left[\frac{1}{n}\sum\left(\frac{Y_i}{B}\right)^2 (0.5 + 2\left(\frac{Z_i}{B}\right)^2)\right]^{1/2}\sigma \qquad (18)$$

The values of σX, σY, and σZ obtained from numerical integration, Equations 16, 17, and 18, and from simulation (Table 1) are equal for all cases studied.

Table 1. Comparison between σX, σY, and σZ Estimated from Simulation (Sim), Proposed Formulas (Pro) and Present Formulas (Pre)

	σX			σY			σZ		
	Sim	Pro	Pre	Sim	Pro	Pre	Sim	Pro	Pre
1	1.7	1.7	2.2	5.0	5.1	5.1	1.8	1.7	2.2
2	1.6	1.6	1.9	4.6	4.6	4.7	1.6	1.6	1.9
3	1.6	1.5	1.7	4.8	4.9	4.9	1.6	1.5	1.7
4	1.7	1.6	1.4	5.6	5.7	5.7	1.7	1.6	1.4
5	4.5	4.5	1.1	15.6	15.7	15.5	4.6	4.5	1.1
6	2.9	2.8	1.2	9.5	9.7	9.6	2.9	2.6	1.2
7	1.9	2.0	1.2	7.6	7.7	7.7	2.3	2.2	1.2
8	2.0	2.0	1.2	7.7	7.7	7.7	2.4	2.2	1.2
9	2.1	2.1	1.2	7.6	7.7	7.7	2.4	2.2	1.2
10	2.2	2.2	1.2	7.6	7.7	7.7	2.4	2.2	1.2
11	15.2	14.0	1.8	49.5	50.1	50.1	15.2	12.9	1.8
12	10.1	10.0	1.4	30.4	31.4	30.9	10.1	10.0	1.4
13	6.0	5.2	1.0	17.2	16.9	16.4	6.0	4.7	1.0
14	2.4	2.1	0.7	6.9	7.0	6.4	2.4	2.1	0.7
15	1.8	1.4	0.5	5.3	4.6	3.7	1.8	1.2	0.5
16	0.5	0.5	0.3	1.8	2.0	1.0	0.5	0.5	0.3
17	3.4	3.0	0.8	10.2	10.3	9.8	3.4	2.7	0.8
18	3.3	2.8	0.8	11.4	10.3	9.8	3.3	2.7	0.8
19	3.1	3.0	0.8	9.5	10.3	9.8	3.1	2.7	0.8
20	3.7	3.3	0.8	10.4	10.3	9.8	3.7	2.7	0.8

TABLE 2. THE DESCRIPTION OF THE INVESTIGATED CASES OF P-31 CAMERA

Case #	Number of Points	Camera Stations Coordinates Xo_1 (m)	Yo_1 (m)	Zo_1 (m)	Xo_2 (m)	Stereo coverage on Plane 1 H_1B (m)	H_2B (m)	V_1B (m)	V_2B (m)
1	200/185/181	−1	45	2	25	0	25	−2	12
2	200/168/168	1	40	2	23	−1	23	−2	12
3	200/168/168	4	35	2	20	−4	20	−2	12
4	200/184/168	7	30	2	17	−7	17	−2	12
5	184/147/147	11	22.6	2	13	−10	12	−2	12
6	200/184/168	10	25	2	14	−10	14	−2	12
7	120/112/104	0	25	2	5	−0	14	−2	12
8	160/152/144	5	25	2	10	−5	14	−2	12
9	184/176/160	8	25	2	13	−8	14	−2	12
10	200/169/160	10	25	2	15	−9	14	−2	12

$Yo_2 = Yo_1$
$Zo_2 = Zo_1$

EXPERIMENTS AND RESULTS

The experimental investigation for the validity of the developed formulas is performed by applying these formulas to some simulation cases.

The author attempted to apply the developed formulas to the simulation cases of Hottier (1976) and Malhotra and Karara (1971). Unfortunately, these formulas were not applicable to the above cases due to the lack of information about the three-dimensional positions of the two camera stations relative to the object points. Although this information is so important in the proposed formulas, it was neglected in Hottier and Malhotra and Karara's investigations. Accordingly, the simulation technique was used to represent some close-range cases. The three-dimensional ground points, which are used in the simulation, are in three parallel planes. Each plane has 200 points. The points are at equal intervals of one metre along the X-direction ($0 \leq X \leq 24$) and two metres along the Z-direction ($0 \leq Z \leq 14$). The origin of the ground axes is the point (0,0,0) on the first plane. The Y-coordinates of all the points on the first, second, and third planes are 0, 2, and 3 m, respectively.

Two metric cameras were used in the investigation, the stereo-metric camera C-120 and the metric camera P-31. The principle distance of the C-120 is 64 mm and the format size is 80 by 60 mm. The principle distance of the P-31 camera is 100 mm and the format size is 117 by 90 mm.

In all the simulation cases studied, the base between the two camera stations was parallel to the ground X-axis. Random errors of N (0, 5 μm) have been given to the image coordinates. The estimated standard deviations from simulations (σX, σY, and σZ) are calculated from the differences between the calculated and the known values of the ground coordinates.

The description of the investigated cases are given in Tables 2 and 3. The position of the first

TABLE 3. THE DESCRIPTION OF THE INVESTIGATED CASES OF C-120 CAMERA

Case #	Number of Points	Camera Stations Co-ordinates Xo_1 (m)	Yo_1 (m)	Zo_1 (m)	Xo_2 (m)	Stereo coverage on Plane 1 H_1B (m)	H_2B (m)	V_1B (m)	V_2B (m)
11	200/200/200	11.4	25	2	12.6	−11.4	12.6	−2	12
12	184/147/147	11.4	20	2	12.6	−10.4	11.6	−2	12
13	102/90/65	11.4	15	2	12.6	− 7.4	8.6	−2	8
14	55/32/28	11.4	10	2	12.6	− 4.4	5.6	−2	6
15	36/14/10	11.4	8	2	12.6	− 3.4	4.6	−2	4
16	10/3/1	11.4	5	2	12.6	− 1.4	2.6	−0	2
17	40/35/24	0.0	12	2	1.2	− 0.0	7.0	−2	6
18	55/49/37	3.0	12	2	4.2	− 3.4	7.0	−2	6
19	65/60/40	5.0	12	2	6.2	− 5.0	7.0	−2	6
20	70/60/40	8.0	12	2	9.2	− 6.0	7.0	−2	6

$Yo_2 = Yo_1$
$Zo_2 = Zo_1$

camera station Xo_i, Yo_i, and Zo_i is given in columns (3), (4), and (5), respectively, and the position of the second camera station Xo_2, Yo_2, and Zo_2 is given in columns (6), (4), and (5) for the cases in column (1). The number of points of each plane on the stereophotos is given in column (2).

THE MATHEMATICAL CALCULATIONS

The values of σX, σY, and σZ obtained from the proposed formulas, Equations 13, 14, and 15, represent the average accuracy of all the ground points as if they were on the center of gravity plane. The procedures for calculation is carried out as follows:

- Calculate the stereo-coverage parameters (H_1B, H_2B, V_1B, and V_2B) of the object from (a) the position of the two camera-stations, (b) the format size, (c) the principle distance, and (d) the maximum object distance (Y_m). Tables 2 and 3 show the values of these parameters for different cases studied of C-120 and P-31 cameras.
- Locate the position of the center of gravity plane. This plane passes through the center of gravity of the object surface on the stereophotos.
- Measure the distance between this plane and the two camera stations (Y_c). The average object distance is a good estimator of Y_c. The value of Y_c is calculated from the simulation cases by using the following relationship:

$$Y_c = \frac{n_1 Y_1 + n_2 Y_2 + n_3 Y_3}{n_1 + n_2 + n_2}$$

$$Y_c = \frac{1}{3}(Y_1 + Y_2 + Y_3)$$

where n_1, n_2, and n_3 are the number of points of plane-1, plane-2, and plane-3, respectively; and

Y_1, Y_2, and Y_3 are the object plane-2 and plane-3, respectively.

- Calculate the stereo-coverage parameters (H_1B, H_2B, V_1B, and V_2B) of the center of gravity plane. These values can be obtained by multiplying the stereo coverage parameters of the maximum plane by a scale factor (Y_c/Y_m).
- Calculate σX, σY, and σZ values from Equations 13, 14, and 15 using the stereo-coverage parameters of the center of gravity. The values of σX, σY, and σZ calculated from the proposed formulas are given in columns (3), (6), and (9) of Table 1. The values of σX, σY, and σZ calculated from the present formulas are given in columns (4), (7), and (10), respectively, in Table 1, while the values of σX, σY, and σZ estimated from simulation are given in columns (2), (5), and (8) of the same table. The values of σX, σY, and σZ, calculated from the numerical integration formulas, Equations 16, 17, and 18, and from the simulation technique in Table 1, are equal for all the cases studied.

CONCLUSIONS

A brief glance at Table 1 reveals that the proposed formulas give better values of σX, σY, and σZ when compared to the present formulas. It also becomes clear that the values estimated through the use of the proposed formulas are very close to the values produced by the simulation technique. Accordingly, one can suggest using the proposed formulas to replace the simulation technique and the present formulas, particularly because of its many advantages.

The main advantage of the proposed method is that the photogrammetrist can easily choose the appropriate position of the two camera stations from the available positions right on the field. Another main advantage is the simplicity of using these formulas since all the photogrammetrist needs is a small calculator rather than using a computer for simulation. Needless to say, these advantages combined make these proposed formulas a must for most photogrammetric designs.

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