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Calibration and Model Reconstruction in Analytical Close-Range Stereophotogrammetry

Part II: Special Evaluation Procedures for Rasterstereography and Moiré Topography*

The conversion of image data resulting from monoscopic techniques for use with stereophotogrammetric procedures is discussed.

IN THE PRECEDING Part I (Frobin and Hierholzer, 1982) of this paper, the mathematical fundamentals of calibration and model reconstruction according to the bundle method were presented. In many cases of close-range stereophotogrammetry the calculation methods outlined may be applied with minute modifications with respect to image data acquisition. This holds for conventional stereophotography (with arbitrary convergence angles) as well as for stereoradiography and 90° biplane radiography. However, in the case of rasterstereography or Moiré topography special provisions have to be made for the application of standard photogrammetric procedures.

ABSTRACT: *In the preceding Part I of this paper the mathematical fundamentals of calibration and reconstruction in close-range stereophotogrammetry using the bundle method were discussed. The present Part II deals with the application of this general procedure to special close-range techniques. The conversion of the image data resulting from the "monoscopic" close-range techniques of cross rasterstereography, line rasterstereography, and Moiré topography into a standard form which is appropriate for the stereophotogrammetric evaluation of Part I is discussed in detail.*

In the following sections the processing of image data for the "monoscopic" photogrammetric techniques of rasterstereography and Moiré topography is described. As a result we arrive at an image data structure which is similar to that obtained with the conventional biplane techniques. In the subsequent evaluation the converted image data may be processed using the calibration and reconstruction methods in their standard form as outlined in Part I of this study.

In the different versions of rasterstereography, which use a cross raster or a line raster, respectively, different evaluation methods are necessary. The line raster method resembles Moiré topography to some extent. Thus, a Moiré apparatus may be calibrated in much the same way as a line raster device. However, as the production of Moiré fringes represents a special form of reconstruction by "optical computing," an analytical reconstruction is generally not employed in Moiré topography.

CROSS RASTERSTEREOGRAPHY

Rasterstereography is a modification of conventional stereophotography, where one of the two cameras has been replaced by a projector with a regular (cross) raster diapositive (Frobin and Hierholzer, 1978a, 1978b; Hierholzer and Frobin, 1980; Kováts, 1974; Irish and Renner, 1977; Renner, 1977). The raster lines are projected onto the

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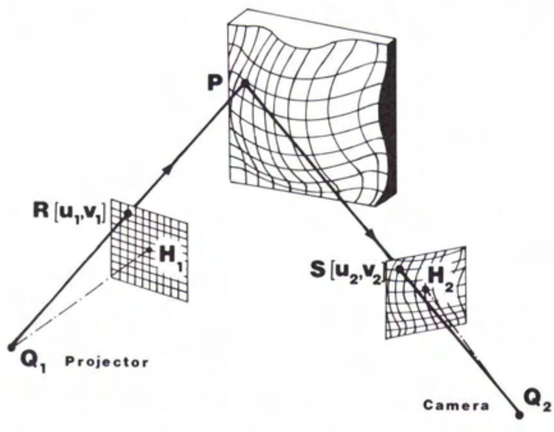

FIG. 1. Reconstruction of a surface point P according to the rasterstereographic method.

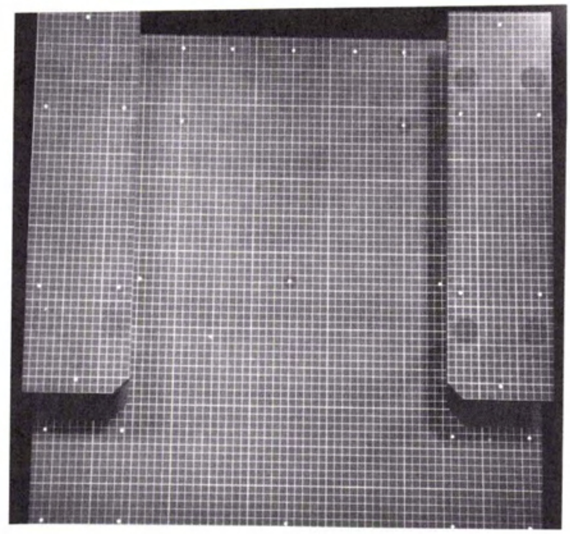

FIG. 2. Control point system for the calibration of raster-stereographic devices consisting of parallel planes (with illuminated control points and projected raster lines).

surface to be measured and are photographed by the camera. Thus, the three-dimensional information is contained in a single image in the form of distorted raster lines. A pair of stereoscopically correlated image points then consists of an intersection point R of the raster lines in the diapositive together with the corresponding intersection S in the camera image. The accompanying object point P is the locus of the projected raster intersection on the object surface (Figure 1).

By this procedure the surface is sampled only at discrete points arranged in rows and columns as given by the projected raster intersections. The reconstruction algorithm is exactly the same as in conventional stereophotography taking as image coordinates of one half-image the coordinates u_2,v_2 of the raster intersection in the camera image, whereas the image coordinates of the other half-image are a priori known from the coordinates u_1,v_1 of the corresponding raster intersection on the raster diapositive. For this purpose, the raster intersections in the camera image have to be identified by their row and column numbers.

It should be mentioned that, in general, the accuracy of the raster intersection coordinates in the diapositive is not equal to that of those measured in the camera image. Consequently, the weight factors (matrices) $\mathbf{G}_l$ and $\mathbf{G}_r$ (see Equations 13 and 19 in Part I of this paper) are of different magnitude. However, when using a cross raster and measuring the intersection points only, the weight factors of the x and y coordinates in each half-image are equal.

The calibration of a rasterstereographic apparatus is a little more difficult. A suitable control point system consists of at least two planes bearing a number of points with (approximately) known coordinates (Figure 2). In the camera image these control points can be measured together with the raster lines projected on the planes. However, unless a control point is accidentally hit by a projected raster intersection, an exact correlation between a control point image on the film and a raster intersection on the diapositive cannot be established. Nevertheless, we can calculate a corresponding point in the diapositive plane by interpolation between the raster lines surrounding the control point image on the film.

The procedure is as follows. As the parallel raster lines are, in general, obliquely projected on the planes, they all intersect at two vanishing points V_1 and V_2 in the camera image (see Figure 3). We may then draw supposed intermediate raster lines through the control point image CP and V_1 or V_2, respectively. The position of these intermediate raster lines in the raster diapositive plane is calculated from the row and column numbers of, say, the lower left-hand corner of the quadrangle $ABCD$ enclosing the control point plus an offset given approximately by the ratios a_i/b_i of the segments of the quadrangle sides (Figure 3). The corresponding control point image is then defined by the intersection of the intermediate raster lines in the diapositive plane.

In this way we may convert the data measured solely in the camera image, namely the coordinates of the control point image CP together with the enclosing quadrangle $ABCD$, into an image pair suitable for the treatment by conventional stereophotogrammetry.

As a result we obtain two sets of orientation elements for the camera and the projector, respectively, the projector being considered as an inverted camera. In addition, the calibration algo-

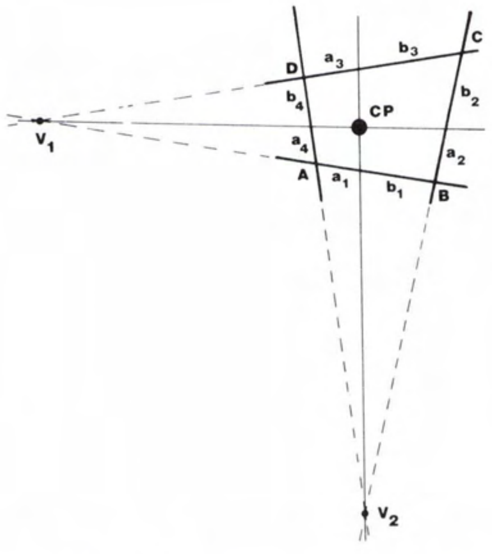

FIG. 3. Control point measurement in a rasterstereographic image.

rithm yields corrections to the approximate a priori control point coordinates. The latter is particularly important in rasterstereography, as it is difficult to form a sufficiently large and exact control point system of the type mentioned above.

LINE RASTERSTEREOGRAPHY

As can be seen from Figure 1, there is an overdeterminancy in the reconstruction of the surface point P. Either the vertical or the horizontal raster lines in the raster diapositive are redundant and may thus be omitted. A very simple structure of the rasterstereographic camera image, which is favorable for automatic image processing, is then obtained (see Figure 4) (Hierholzer and Frobin, 1980).

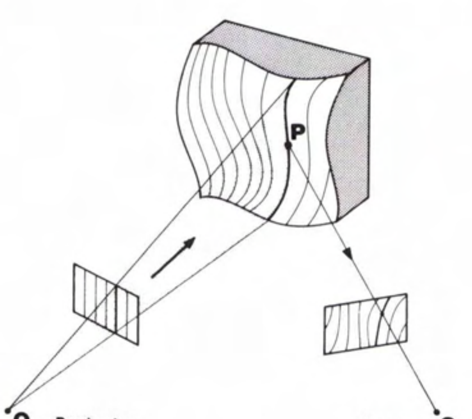

FIG. 4. Principle of line rasterstereography.

In line rasterstereography, uniquely identifiable points like the raster intersections in cross rasterstereography cannot be measured along the raster lines. Therefore, the corresponding coordinate in the raster diapositive plane (e.g., the y coordinate in the case of vertical raster lines, Figure 4) remains indeterminate. Nevertheless, the calibration and reconstruction procedures developed in Part I of this paper may be employed in exactly the same way as in cross rasterstereography.

This is possible, in principle, by introducing a weight factor of zero for the indeterminate coordinate in the half image corresponding to the line raster diapositive. For example, if the projector is assumed to be the left imaging system with the raster lines oriented in the y direction, we obtain for the submatrices in the collected weight matrix **G** (Equation 19 in Part I)

$$\mathbf{G}_{l_i} = \begin{pmatrix} G_{lx} & 0 \\ 0 & 0 \end{pmatrix}$$

However, the system of normal equations (Equations 23 and 24 in Part I) then obviously becomes singular. A possible solution of this problem would be to eliminate the equations belonging to the indeterminate coordinate (Y_{bl} in the above example) from the complete set of normal equations. This means of course that the appropriate computer programs have to be rewritten. A more elegant way, which supersedes any modification of the standard evaluation programs, consists in introducing a small but finite weight for the indeterminate coordinate. The weight factor must be chosen such that it is, on the one hand, small as compared to the weight factor of the other coordinate, but, on the other hand, as large as necessary with respect to the computational precision of the computer. In practice, a relative magnitude between 10^{-2} and 10^{-4} might be adequate (for single precision calculation), i.e., for example

$$\mathbf{G}_{l_i} = \begin{pmatrix} 1 & 0 \\ 0 & 0.001 \end{pmatrix}$$

In effect, the y coordinate in the raster diapositive plane, Y_{bl}, is actually not completely indeterminate but more inaccurate by a factor of $\sqrt{1000}$ than the corresponding x coordinate X_{bl}.

Apart from the fact that the introduction of a small nonzero weight for the indeterminate coordinate has only a negligible effect on the solution of the normal equations, the above mentioned procedure is justified by the consideration that this coordinate is indeed not completely undefined. At least a rough estimation can be obtained by linear interpolation along the raster lines in the rasterstereographic camera image. That is, the degree of indeterminacy is correspondingly reduced and the weight factor may then be increased appropriately. This is favorable with respect to the convergence of the iterative solution of the normal equations, especially in the case of relatively poor starting values.

In order to alleviate the estimation of the indeterminate raster coordinate, it is advantageous to mark an origin on the raster lines (e.g., by an additional line perpendicular to all others, perhaps the lower edge of the raster diapositive in Figure 4). The estimation of the indeterminate coordinate is best if the stereo base of the rasterstereographic device (i.e., the distance between the nodal points of the camera and the projector) is perpendicular to the raster lines and parallel to both the diapositive plane and the film plane.

As a result of the preceding considerations, we state that also in the case of line rasterstereography the data measured solely in the camera image (including of course the determination of the line numbers) can be converted into image pair coordinates suitable for the standard photogrammetric procedure. The only peculiarity consists in a certain degree of indeterminacy of one image coordinate which can be taken into account by an appropriate weight factor in the normal equations.

The calibration of a line rasterstereographic apparatus is carried out in the same way as discussed in the preceding section. The raster lines are projected on a system of planes bearing the control points. However, from the camera image of the raster lines only one vanishing point, say V_2 (Figure 3), and the corresponding intermediate raster line can be constructed. Consequently, only one coordinate of the control point image in the diapositive plane can be calculated with good accuracy, and hence with high weight, whereas the other one can only be estimated (low weight). Anyway, we obtain an image pair of the control point suitable for the standard calibration procedure described in Part I.

Moiré Topography

Although the methods of image preparation and evaluation in Moiré topography are, in general, very different from those used in conventional stereophotogrammetry, the geometry of a Moiré apparatus can be reduced to the basic geometry used in any photogrammetric measurement.

There are essentially two techniques of Moiré topography: the projection method and the shadow method. In the projection method a system of parallel lines is projected onto the surface to be measured, and photographed by a camera. The arrangement is quite similar to that of line rasterstereography (Figure 4) except that the density of the raster lines is generally higher. The Moiré fringes are produced by a superimposition of the camera image with a second raster (reconstruction by "optical computing").

In the case of the shadow Moiré method the projection and superimposition of the raster lines is accomplished by one and the same grid. The surface is illuminated through a large grid by a point source (a line-shaped lamp adjusted exactly parallel to the raster lines is equivalent to a point source), and the photograph is taken through the same grid. The Moiré fringes are produced immediately in the camera image. Even in this case we may consider the Moiré apparatus as a stereophotogrammetric arrangement consisting of a conventional camera and a large "projector" which in turn is composed of the light source as a nodal point (center of perspective) and the grid as a diapositive plane.

Thus, a shadow Moiré apparatus as well as a projection device may be geometrically described by two sets of orientation elements belonging to a "left" and a "right" imaging system, respectively.

As mentioned above, an analytical reconstruction of the surface points is generally not performed in Moiré topography. Instead, the system of Moiré fringes produced by "optical computing" is investigated in the subsequent image analysis. However, an exact calibration of the Moiré apparatus is very desirable, as the Moiré fringes are generally neither planar nor parallel with respect to depth, except for special geometries. In addition, in consequence of the central projection, they are not equidistant. From the orientation elements appropriate corrections can be calculated for arbitrary geometries.

From the design of the projection Moiré method it is evident that the calibration procedure may be performed in exactly the same way as in line rasterstereography (see preceding section). Provisions must be made to associate a line number with each raster line, for example, by marking one line as an origin for the line count. Furthermore, the performance of the calibration procedure may be improved, if a coordinate origin is indicated on each raster line as was discussed in connection with line rasterstereography.

The calibration of a shadow Moiré apparatus is more complicated. The arrangement is shown in Figure 5. For the sake of simplicity only one control point CP on a single control plane is shown.

As already mentioned this setup may be regarded as a stereo-photogrammetric system with the left imaging system consisting of the lamp as a nodal point and the grid as an image plane. The right imaging system is a conventional camera. Consequently, a stereo image pair of the control point CP consists of the camera image (CP on the film) and the intersection between the grid plane and a light ray going from the lamp to the control point. Just as in line rasterstereography, of this intersection only the coordinate perpendicular to the grid lines, say x, is needed, the other one (y) being irrelevant. The x coordinate is measured from some specially marked origin line $\mathcal{O}$ on the grid and is determined by the number of grid lines

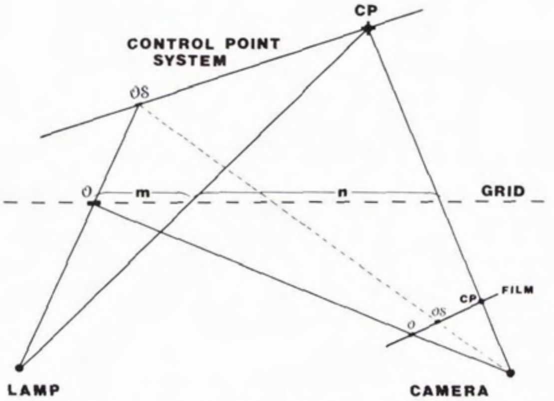

FIG. 5. Calibration of a shadow Moiré apparatus using a control point system.

m between the origin line and the aforesaid intersection point (m may be non-integral by interpolation between the grid lines). The crucial consequence is that the number m and therewith the x coordinate of the left image can be assessed from data extracted solely from the camera image (Moiré topogram Figure 6).

The procedure is as follows. From the Moiré topogram, Figure 6, we determine the number $(m + n)$ of grid lines between the origin line O and the control point image CP. This can be done at best by following up the Moiré fringe which coincides with the control point until some point O on the origin line is reached. Of course, the grid lines must be resolved in the camera image. Thus, a moving grid must not be employed in the calibration procedure. It may be advantageous to mark several grid lines for an easy determination of the line number $m + n$.

The number n (Figure 5) is equal to the ordinal number of the Moiré fringe coinciding with the control point by definition of the Moiré fringe ordinal number (n may be nonintegral by interpolation between the Moiré fringes). It can be determined from the Moiré topogram using the shadow OS of the origin line O (Takasaki, 1973). Thus, by counting the grid lines between CP and O and by a determination of the ordinal number of the Moiré fringe through CP, we may calculate the x coordinate of the projection of CP on the grid plane. Hence, we again arrive at an image pair suitable for the standard photogrammetric procedures.

The considerations of the preceding paragraph concerning the indeterminate coordinate along the grid lines and the corresponding weight factor apply without modification.

From the above considerations it is evident that no assumptions about the relative orientation of the optical components in Figure 5 are necessary

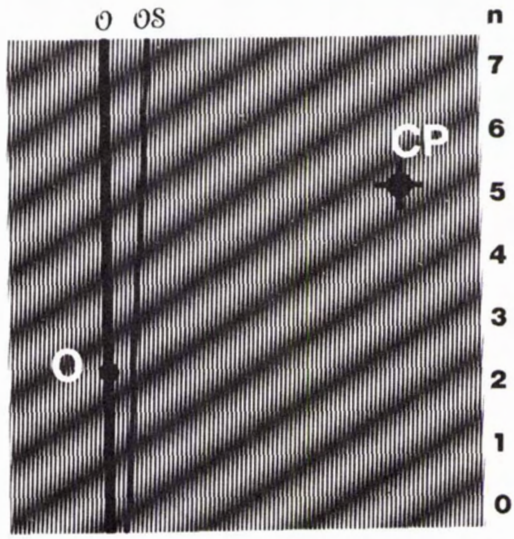

FIG. 6. Moiré topogram produced by the setup of Figure 5.

for the calibration procedure described. However, in most applications of the shadow Moiré technique, the film plane is adjusted parallel to the grid plane, and the distances between the grid and the camera or the lamp, respectively, are equal. These conditions can be checked on the basis of the results obtained from the calibration procedure.

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