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Production of Photogrammetric Stereopairs by Beam Division in Single-Lens Cameras

The optical system, image geometry, and calibration procedure are described.

Introduction

In close-range measurements of small objects, owing to the camera dimensions, it is very often not possible to arrange two synchronously operating photogrammetric cameras on a necessarily small base. The sequential taking of the photos of a stereopair, after the single photogrammetric camera has been displaced transverse to the camera axis, is only possible when the taking situation is stationary and the scene does not move. If this condition cannot be satisfied, use of a single-lens camera system is suggested, in which the photographs of a stereoscopic pair are obtained by pupil division.

Abstract: *In close-range photogrammetry, owing to the camera dimensions, it is often not possible to arrange two synchronously operating cameras on a necessarily small base. In this case the application of a single-lens camera is suggested, in which photographs of a stereopair are generated by pupil division.*

A suitable optical system and the image geometry of such a special camera are described. Furthermore, procedures for the calibration of the taking system are offered. Suggested is a mixture between adjustment-predictions and a method of self-calibration with the help of reference points. Presented is the mathematical model with the adjustment of redundant observations.

New possibilities in very close range photogrammetric measurement of spatial objects and processes are opened by this photographic technique. A single-lens imaging system with pupil division can be used as an attractive complement of conventional photogrammetric camera instrumentation for close-range photogrammetry.

A very important application emerges in the medical sector. The fitting of contact lenses, the determination of the anterior chamber, as well as the multitemporal topography of the eye's fundus are representative examples.

Photographic Arrangement

A variety of single-lens stereoscopic systems for the direct visual observation or for taking photographs are known[1-4] which are generally based on the optical functional principle of Figure 1. The path of rays of the camera system O is limited by the pupils I and II. In this way one obtains two image points, P_I and P_{II}, defined by a circle of confusion, of an object point, P, lying outside the object's focal plane, which pertains to the image plane, B. Thus, the two pictures of a stereogram are formed by the totality of all image points designated by the index I, on the one hand, and the index II, on the other. The two stereoscopic pictures being superimposed according to Figure 1 can be separated by appropriately directing the optical path of rays and recording on photographic material or relaying to a binocular stereoscopic viewing device.

One possible configuration of the design is shown in Figure 2. In this arrangement the beams to the image planes, B_I and B_{II}, are reflected at right angles to the optical axis of the imaging system by the front-surface mirrors, Sp. Generally, the optical system, O, consists of a multi-lens complex which, through movable focusing elements, produces absolutely sharp images in the image planes B_I and B_{II}

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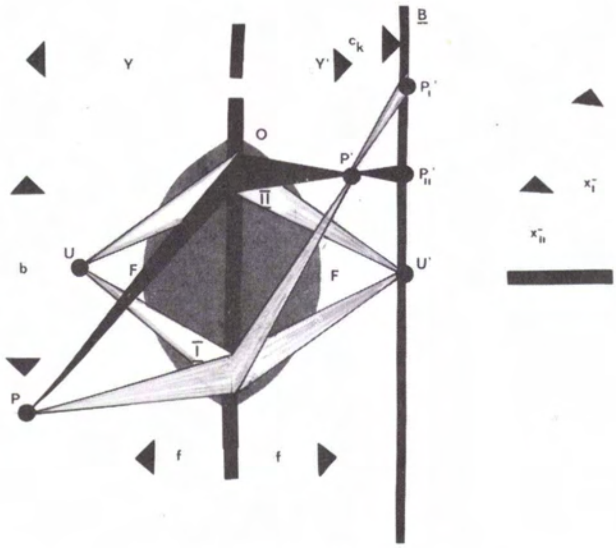

FIG. 1. Optical train.

for any object sections lying at right angles to the optical axis. Arrangements have also become known which, by means of switchable optical components, produce different photo scales of the object in the image planes, the scale being variable in steps.

CAMERA GEOMETRY

The camera geometry may be derived by referring to Figure 1, which shows the general model of production of stereoscopic photographs by means of a pupil-divided single-lens camera system. If the coordinate origin of the spatial rectangular coordinate system is according to Figure 3 placed in the optical axis of the imaging system, the space coordinates of points of the recorded object can be calculated from the image coordinates to be measured, $x_{\bar{I}}$, $z_{\bar{I}}$, and $x_{\bar{II}}$, in the two stereoscopic pictures with the constants of the photographic system defined in Figure 1 by the following mathematical model:

$$
\begin{aligned}
Y &= \frac{f \cdot c_k \cdot b}{c_k \cdot b - f(b + x_{\bar{II}} - x_{\bar{I}})} \\
X &= \frac{f \cdot b/2\,(x_{\bar{II}} + x_{\bar{I}})}{c_k \cdot b - f(b + x_{\bar{II}} - x_{\bar{I}})} \\
Z &= \frac{z_{\bar{I}} \cdot f \cdot b}{c_k \cdot b - f(b + x_{\bar{II}} - x_{\bar{I}})}
\end{aligned}
\tag{1}
$$

When a pocket calculator is used, the following rearrangement of Equation 1 may be advantagous:

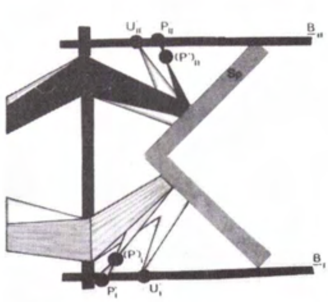

FIG. 2. Separation of stereo pairs.

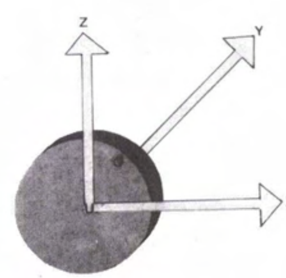

FIG. 3. Coordinate system.

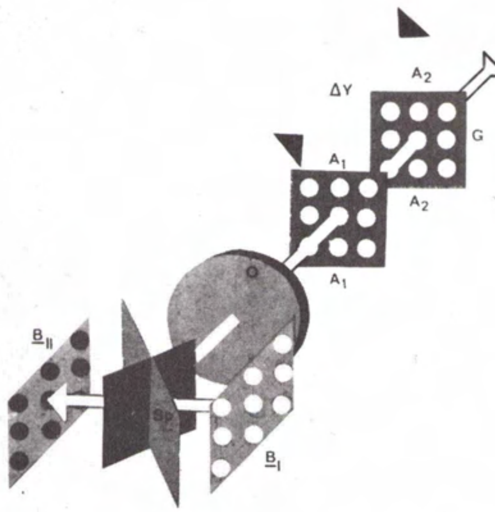

FIG. 4. Adjustment of the optical system.

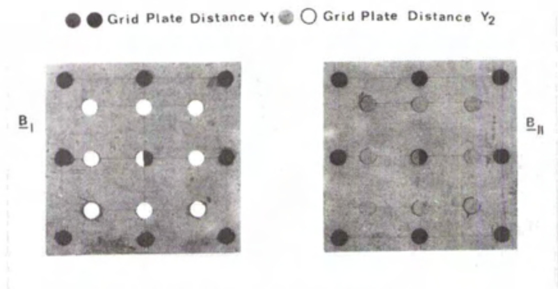

FIG. 5. Correct adjustment.

$$\frac{1}{Y} = \frac{1}{f} - \frac{b + x_{\bar{II}} - x_{\bar{I}}}{c_k \cdot b}$$
$$\frac{1}{X} = \frac{2c_k}{f(x_{\bar{II}} + x_{\bar{I}})} - \frac{b + x_{\bar{II}} - x_{\bar{I}}}{b/2(x_{\bar{II}} - x_{\bar{I}})} \tag{1a}$$
$$\frac{1}{Z} = \frac{c_k}{z_{\bar{I}} \cdot f} - \frac{b + x_{\bar{II}} - x_{\bar{I}}}{z_{\bar{I}} \cdot b}$$

It is evident that the mathematical model described by Equation 1 corresponds in its character to the well-known "normal-case" of stereophotogrammetry.

CONSTANTS OF THE PHOTOGRAPHIC SYSTEM

The available systems for observation and photography have in their present version been designed for a qualitative assesment of stereograms. Their application for obtaining quantitative statements may

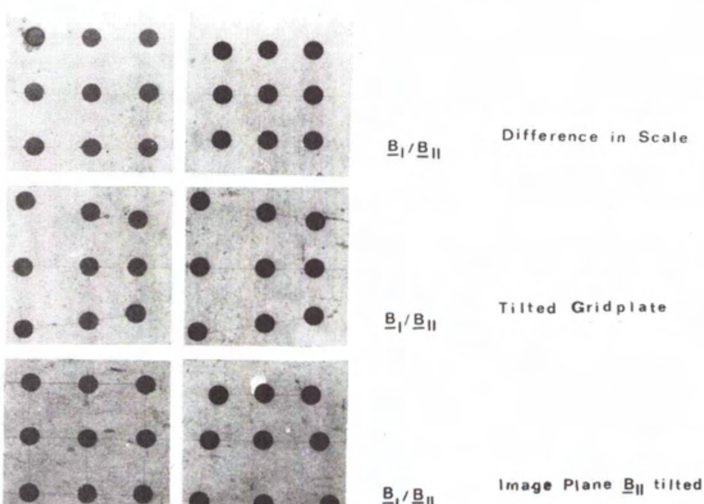

FIG. 6. Incorrect adjustment.

roughly be compared to the occasional use of so-called amateur cameras for tasks of close-range photogrammetry. The inner orientation of the imaging system is neither constant over a longer time nor are appropriate devices available for performing precalibrations as they are common in photogrammetric cameras. Therefore, one will have to determine the camera geometry immediately before taking the photographs with fixed settings (e.g., for focus and magnification) or to provide for a possible self-calibration within the restitution process by taking suitable measures.

For such operations one requires sufficient reference points (with known coordinates) of a test object or on the measuring object itself or in its immediate environment (in the ideal case, completely around the object). The general approach can be described as follows: If the reference point coordinates are known and the corresponding image coordinates are measured in the taken photographs, it is possible to determine the transformation parameters between object and image space in the Equation 1 system. If these transformation parameters are calculated with sufficient rigor (preferably from a least-squares adjustment process by means of redundant determination equations), then the desired space coordinates can be calculated on the basis of the relationship described by Equation 1. Because in the system of Equation 1 the origin of the object coordinate system can be assumed as known only by approximation, the error equations are consequently stated as follows:

$$Y = Y_0 + \frac{f \cdot c_k \cdot b}{c_k \cdot b - f(b + x_{\bar{II}} - x_{\bar{I}})}$$

$$X = X_0 + \frac{f \cdot b/2\,(x_{\bar{II}} + a_{\bar{I}})}{c_k \cdot b - f(b + x_{\bar{II}} - x_{\bar{I}})}$$

$$Z = Z_0 + \frac{z_{\bar{I}} \cdot f \cdot b}{c_k \cdot b - f(b + x_{\bar{II}} - x_{\bar{I}})}$$

one may then obtain the equations

$$\begin{aligned} Y + v_y &= Y_0 + \frac{f \cdot c_k \cdot b}{c_k \cdot b - f(b + x_{\bar{II}} - x_{\bar{I}})} \\ X + v_x &= X_0 + \frac{f \cdot b/2\,(x_{\bar{II}} + x_{\bar{I}})}{c_k \cdot b - f(b + x_{\bar{II}} - x_{\bar{I}})} \\ Z + v_z &= Z_0 + \frac{z_{\bar{I}} \cdot f \cdot b}{c_k \cdot b - f(b + x_{\bar{II}} - x_{\bar{I}})} \end{aligned} \tag{2}$$

From this relation the linearized equations

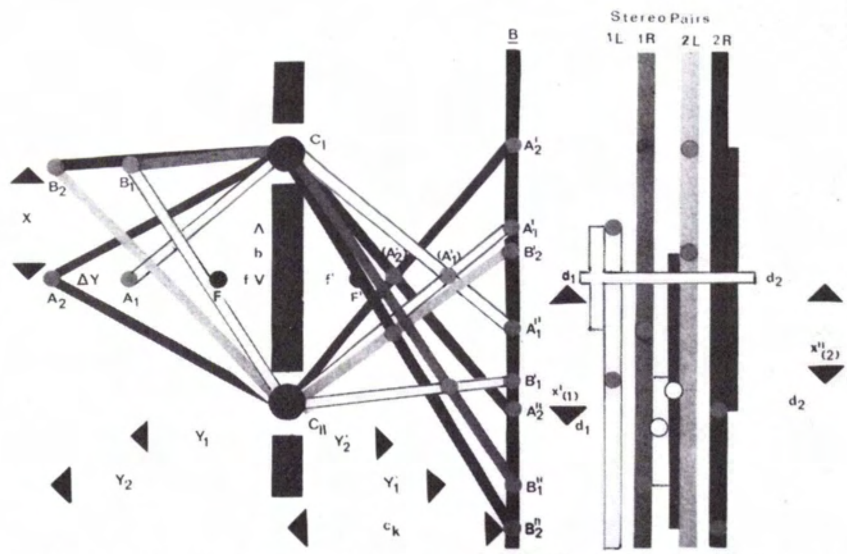

FIG. 7. Approximate values for taking distance, calibrated focal length, focal length, and base.

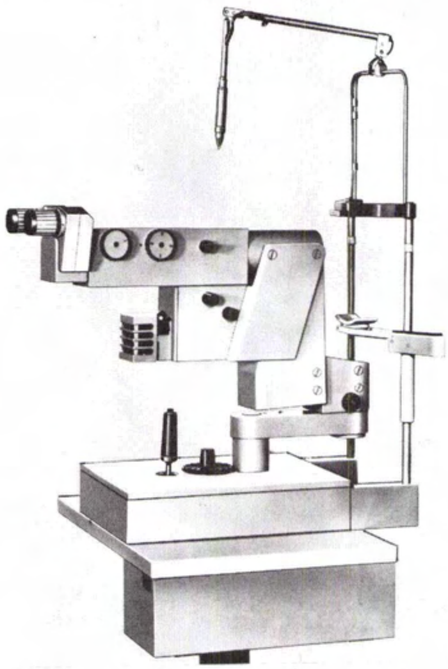

FIG. 8. Stereo-Ophthalmoskop 110 (JENOPTIK JENA GmbH).

$$
\begin{aligned}
v_Y &= \frac{\partial Y}{\partial Y_0} dY_0 + \frac{\partial Y}{\partial f} df + \frac{\partial Y}{\partial c_k} dc_k + \frac{\partial Y}{\partial b} db + Y_0 + \frac{f_0 \cdot c_{k_0} \cdot b_0}{c_{k_0} \cdot b_0 - f_0(b_0 + x_{\overline{II}} - x_{\overline{I}})} - Y \\
v_X &= \frac{\partial X}{\partial X_0} dX_0 + \frac{\partial X}{\partial f} df + \frac{\partial X}{\partial c_k} dc_k + \frac{\partial X}{\partial b} db + X_0 + \frac{f_0 \cdot b_0/2(x_{\overline{II}} - x_{\overline{I}})}{c_{k_0} \cdot b_0 - f_0(b_0 + x_{\overline{II}} - x_{\overline{I}})} - X \\
v_Z &= \frac{\partial Z}{\partial Z_0} dZ_0 + \frac{\partial Z}{\partial f} df + \frac{\partial Z}{\partial c_k} dc_k + \frac{\partial Z}{\partial b} db + Z_0 + \frac{z_{\overline{I}} \cdot f_0 \cdot b_0}{c_{k_0} \cdot b_0 - f_0(b_0 + x_{\overline{II}} - x_{\overline{I}})} - Z
\end{aligned}
\tag{3}
$$

are derived.

This equation system with six unknowns X_0, Y_0, Z_0, df, dc_k, and db is according to Equation 1 determined by at least two reference points (each point yields three determination equations). If more reference points in the object space are available, an adjustment problem exists with the normal equations*

$$
\begin{aligned}
[aa]dX_0 + [ab]dY_0 + [ac]dZ_0 + [ad]df + [ae]dc_k + [af]db - [al] &= 0 \\
[ab]dX_0 + [bb]dY_0 + [bc]dZ_0 + [bd]df + [be]dc_k + [bf]db - [bl] &= 0 \\
[ac]dX_0 + [bc]dY_0 + [cc]dZ_0 + [cd]df + [ce]dc_k + [cf]db - [cl] &= 0 \\
[ad]dX_0 + [bd]dY_0 + [cd]dZ_0 + [dd]df + [de]dc_k + [df]db - [dl] &= 0 \\
[ae]dX_0 + [be]dY_0 + [ce]dZ_0 + [de]df + [ee]dc_k + [ef]db - [el] &= 0 \\
[af]dX_0 + [bf]dY_0 + [cf]dZ_0 + [df]df + [ef]dc_k + [ff]db - [fl] &= 0
\end{aligned}
\tag{4}
$$

In this system the constants are defined as follows:

$$
\begin{array}{lll}
a_{(y)} = 0 & b_{(y)} = 1 & c_{(y)} = 0 \\
a_{(x)} = 1 & b_{(x)} = 0 & c_{(x)} = 0 \\
a_{(z)} = 0 & b_{(z)} = 0 & c_{(z)} = 1
\end{array}
$$

* $[aa] = \sum_1^n aa$ (definition after C. F. Gauss).

$$d_{(y)} = \frac{c_{k_0}{}^2 \cdot b_0^2}{N^2} \qquad e_{(y)} = -\frac{f_0^2 \cdot b_0(b_0 + x_{\bar{II}} - x_{\bar{I}})}{N^2}$$

$$d_{(x)} = \frac{b_0{}^2 \cdot c_{k_0}(x_{\bar{II}} - x_{\bar{I}})}{N^2} \qquad e_{(x)} = -\frac{b_0}{N^2}$$

$$d_{(z)} = \frac{b_0{}^2 \cdot c_{k_0} \cdot z_{\bar{I}}}{N^2} \qquad e_{(z)} = \frac{b_0}{N^2}$$

$$f_{(y)} = -\frac{f_0{}^2 \cdot c_{k_0}(x_{\bar{II}} - x_{\bar{I}})}{N^2} \qquad -l_{(y)} = Y_0 + \frac{f_0 \cdot c_{k_0} \cdot b_0}{N} - Y$$

$$f_{(x)} = \frac{f_0^2(x_{\bar{II}} - x_{\bar{I}})^2}{N^2} \qquad -l_{(x)} = X_0 + \frac{f_0 \cdot b_0(x_{\bar{II}} - x_{\bar{I}})}{2N} - X$$

$$f_{(z)} = -\frac{z_{\bar{I}} \cdot f_0^2(x_{\bar{II}} - x_{\bar{I}})}{N^2} \qquad -l_{(z)} = Z_0 + \frac{z_{\bar{I}} \cdot f_0 \cdot b_0}{N} - Z$$

$$N = c_{k_0} \cdot b_0 - f_0\,(b_0 + x_{\bar{II}} - x_{\bar{I}})$$

Approximate Values

The derived mathematical model of Equation 2 applies to the geometrical representation of Figure 1. Here certain assumptions are made for the initial state of the camera and the correlation of object and camera:

- The optical axis of the camera is normal to the image plane (Figure 1),
- The two image planes for the stereoscopic pictures are parallel to each other (Figure 2),
- Both stereoscopic images produced have the same photo scale, and
- The coordinate origin for the measurement of the image coordinates is known in both image planes.

It is on principle possible to discard also these initial conditions. But then, of necessity a more general mathematical model and the determination of further process parameters with the help of additional reference points would be required. It is, however, assumed that the chosen initial conditions can be satisfied by the preceding adjustment of the camera and adherence to the described taking disposition. Finally, the fulfillment of the premises can be checked by taking photographs of suitable test objects. If there are deviations, an appropriate correction can be made on the measured image coordinates $x_{\bar{I}}, x_{\bar{II}}, z_{\bar{I}}, z_{\bar{II}}$, prior to their introduction into the determination Equations 1 or 1a. A suitable test object is a quadratically divided grid plate of narrow mesh width arranged normal to the axis of the optical system (Figure 4). With equal-scale imaging and square shapes of the image grid meshes (Figure 5), the conditions 1, 2, and 3 can be considered as fulfilled. Deviations are explained in Figure 6. The origin for the image coordinate measurements is determined by the imaging of an object point migrating in the optical axis (between successive exposures) (Figure 4). The centers of the distances $A_{1'} - A_{1''}$ and $A_{2'} - A_{2''}$ (see Figure 7) are required to coincide; this position, then, is the basis for the image coordinate measurement. If the symmetry points for $A_{1'} - A_{1''}$ and $A_{2'} - A_{2''}$ do not coincide, the migrating object point was not lying in the optical axis (grid line $A - A$ in Figure 4). Prior to the beginning of a new test series for the determination of the origin of the image coordinate system, the grid plate has therefore to be subjected to appropriate lateral displacement. With this adjusting procedure being finished, it is possible by taking repeated photographs of the test grid plate after defined displacement in the Y-direction (Figure 7) to derive the approximate values for Y_1, Y_2, c_k, f, and b according to the representation of Figure 7. With the quantities given there, it is

$$y_1 = \left| \frac{\Delta y \cdot x'_{(2)}}{x'_{(1)} - x'_{(2)}} \right| \qquad y_2 = \left| \frac{\Delta y \cdot x'_{(1)}}{x'_{(1)} - x'_{(2)}} \right|$$

$$c_{k_0} = \left| \frac{x'_{(2)} \cdot x'_{(1)} \cdot \Delta y}{X(x'_{(1)} - x'_{(2)})} \right|$$

$$f_0 = \frac{Y_1 \cdot Y_2 \cdot c_{k_0}\;(d_2 - d_1)}{c_{k_0}(Y_2 d_2 - Y_1 d_1) + Y_1 \cdot Y_2(d_2 - d_1)} \tag{6}$$

$$b_0 = \frac{Y_1 \cdot Y_2(d_1 - d_2)}{c_{k_0}(Y_1 - Y_2)}$$

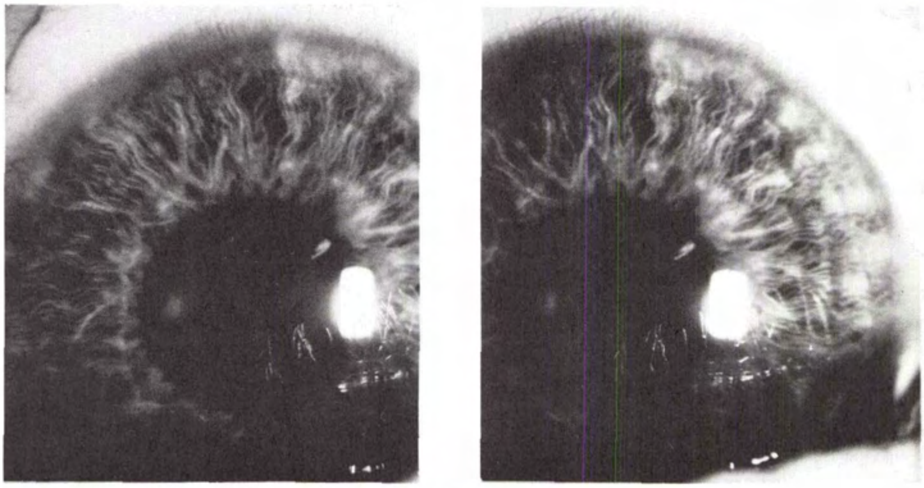

FIG. 9. Stereophoto.

SOLUTION OF THE NORMAL EQUATION SYSTEM

With the quantities calculated from Equations 5 and 6, it is possible to solve the normal Equations 4 in iterative arithmetic operations. Accordingly, one obtains after each calculating cycle:

$$\begin{aligned} c_{k_0(n)} &= c_{k_0(n-1)} + dc_{k(n)} & X_{0(n)} &= X_{0(n-1)} + dX_{0(n)} \\ b_{0(n)} &= b_{0(n-1)} + db_{(n)} & Y_{0(n)} &= Y_{0(n-1)} + dY_{0(n)} \\ f_{0(n)} &= f_{0(n-1)} + df_{(n)} & Z_{0(n)} &= Z_{0(n-1)} + dZ_{0(n)} \end{aligned} \tag{7}$$

The iteration is stopped when the determination of the unknowns in Equation 4 in an iterative step yields changes which lie below a chosen accuracy level. On the introduction into Equations 1 or 1a of the transformation constants so obtained, one now calculates from the measured image coordinates the unknown original position of their corresponding object points. Thus, through the determination of an arbitrary number of object points, a digital description of the measuring object is finally realized which, in case of need, can also be transferred into an analog representation (computer graphics).

APPLICATIONS

The derived imaging model is applicable to single-lens optical imaging systems which produce stereoscopic pictures by pupil division. New possibilities in very close range photogrammetric measurement of spatial objects and processes are opened by this photographic technique. The single-lens imaging system with pupil division can be used as an attractive complement of conventional photogrammetric camera instrumentation for close-range photogrammetry.

A very important application emerges in the medical sector. At the moment measurements on and in the eye complement the so far dominating more qualitative oriented diagnostic techniques. The fitting of contact lenses, the determination of the depth of the interior chamber, as well as the multitemporal topographic representation of the eye's fundus can be mentioned here as representative examples. The stereo opthalmoscope from Jena, which allows not only visual stereoscopic observation but also the production of stereoscopic pictures (Figures 8 and 9), can be calibrated in the way shown and used for the photogrammetric measurement in the eye. The measurement of the eye's interior requires, in addition to the previous representations, the inclusion of the dioptric power of the human eye.

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