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Dip Determinations in Photogeology

A rigorous method is developed and is compared with previously published methods.

Abstract: *Conceptual errors, often present in photogeological methods of dip determination, make the results of those methods unreliable.*

A new method, which the writer believes to be error free and considers rapid and accurate, is first presented; it is based on parallax difference measurements and on the use of a trigonometric formula which incorporates the perspective correction. The derivation of the formula and its significance are briefly explained and the method is then illustrated step by step.

In the second part of the paper, the most serious errors present in other published methods are indicated and the ensuing unreliability of the attainable results is shown.

Introduction

During photointerpretation work, it is often of particular interest to determine the angles of slope of inclined planar features from vertical aerial photographs using simple and rapid procedures. Particularly in photogeology the possibility of accurate measurements of the dip of structural surfaces, such as bedding, fracture, or contact planes, often plays an important role.

Thus, in the same way as a correct and handy use of the compass with clinometer is expected from a field geologist, it should be logical to expect from a photogeologist a fluent application of correct methods for the determination of the attitude of inclined surfaces when employing a stereoscope.

Obviously, the choice of the methods to be applied is very important, so that reliable results may always be achieved. Several methods have been proposed in internationally circulated publications and they may be found also quoted, or even related in detail, in textbooks and manuals of photointerpretation. Turner (1977) compared ten of these methods, indicating their relative advantages, limitations, and level of accuracy. However, the present writer is not aware of any publication which calls attention to some serious conceptual errors which invalidate some of them. As it will be shown, because of these errors it is possible to go from relatively accurate results to completely unacceptable ones, according to the values of certain parameters. When this fact is not realized, any comparison among the different methods becomes meaningless.

First, a new method which the writer believes free of errors and considers easy and rapid to apply and accurate in the results is illustrated. For the explanation of the method it will be necessary to focus the geometric relationships among the points of a real inclined plane on the ground and its photographic images. This will turn out useful in the second part of the paper, where the most serious errors often present in other methods are indicated and analyzed.

The method described here is based on measurements of parallax differences and on the use of a trigonometric formula, while graphic operations are reduced to a minimum. Basically, the formula, for which a simple derivation is given, is not new. In fact it could also be obtained, with proper transformations and simplifications, as well as symbol variations, either from the formulas presented by Fichter (1954) and by Raasveldt (1956) and revived by Howard (1968) for the geometry of a stereomodel, or from the formula presented by Threet (1956) for the geometry of a photograph, or, at last, from the formula introduced by Ricci and Petri (1965, p. 125) and valid in both cases.

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Until a few years ago this method would perhaps have been regarded as rather unattractive by those photointerpreters who are not proficient in the use of trigonometric formulas and tables, or more or less complicated nomograms. Today this problem is completely overcome by the availability on the market of those electronic minicalculators which incorporate trigonometric functions and may be considered within everybody's price range. On the other hand, it is obvious that the results obtained by employing a correct mathematical method will always be more accurate than those obtained by other methods, i.e., trigonometric-graphic methods, purely graphic methods, or methods which use visual estimation in the stereomodel.

Preliminary Assumptions

It will be assumed that the reader is familiar with those basic notions of photogrammetry which are commonly used in photointerpretation, such as (1) the principles which regulate the geometry of a perspective (conic) projection, either a photograph or a stereomodel, compared with those of an orthographic projection, that is, a topographic map; (2) the procedure for measuring, on a stereopair, the photobase adjusted to the photographic scale of the datum plane passing through a certain point *P* of the ground and briefly referred to as *photobase adjusted to P;* and (3) the method for measuring the parallax difference between two points by using, for instance, a parallax bar.

The equation of the parallax difference for the calculation of the difference in elevation between two points *A* and *K* can be written

$$h = \frac{f|\Delta p|}{b \pm |\Delta p|} \qquad (1)$$

where h = the absolute difference in elevation between *A* and *K*, reduced to the photo scale of the datum plane containing *A*,
f = camera focal distance,
$|\Delta p|$ = absolute value of the parallax difference between *A* and *K*,
b = photobase adjusted to *A*,

and where a plus sign is employed when *K* is higher than *A* and a minus sign is used when *K* is lower than *A*.

Hereafter, for convenience, instead of considering generic inclined planes, we shall refer to bedding planes of rock layers, without any prejudice, in so doing, to the general validity of what it is said.

The strike of a bed is defined as the azimuth of the intersection line of the bed surface with a horizontal plane. The horizontality of this line guarantees the invariability of the strike through a perspective projection. On the other hand, the angle of dip, that is the zenithal angle at which a line of maximum slope of a bed surface is inclined from the horizontal, will generally become altered through a perspective projection.

Actually, the image of an inclined bed, as it appears under the stereoscope on a stereomodel correctly observed in the center of the visual field when looking straight down, will generally be affected by two distinct types of deformations, the first due to vertical exaggeration and the second to the perspective projection. Therefore, the methods of dip determination which are based on visual estimations in the stereomodel will have to take into account both these deforming factors, while vertical exaggeration will not come into play with those methods based only on the geometry of single photographs, such as the purely graphic or trigonometric-graphic methods or the trigonometric method to be illustrated here.

Throughout this paper an absence of photographic tilt is assumed, as is the horizontality of the air base.

The Formula for Dip

Taking into consideration the two images of an inclined bed on a stereopair, we intend to find a formula by which to calculate the angle of dip from quantities measurable on both photographs.

In Figure 1 the bed is schematically represented by the right-angle triangle *EHP*, reduced to the photo scale of the datum plane through *P*. $E\hat{P}H = \alpha$ is the real dip angle, in which *HP* is the horizontal distance and *HE* is the vertical distance. Horizontal line *AB*, normal to *HP* at *P*, represents the strike of the bed.

In a photograph with center *N* and focal length $\overline{ON} = f$, point *K* represents the perspective projection of vertex *E* of triangle *EHP*. *KR* is drawn from *K* normal to strike *AB*, and *FK* is erected parallel and equal to *EH*. Because distance *KR* corresponds, in the perspective projection, to horizontal distance *HP*, angle $F\hat{R}K = \beta$ of the right-angle triangle FKR represents the perspectively deformed dip of the bed, which may be called *photographic perspective dip.*

Let us derive, first of all, the geometric relationship between the real dip, α, and the perspective dip, β. From the two pairs of similar triangles *KGH, KUN* and *EHK, ONK* we may write

$$\frac{\mathrm{HP - KR}}{\mathrm{NU}} = \frac{\mathrm{EH}}{\mathrm{ON}}$$

from which we derive

$$\frac{\mathrm{HP}}{\mathrm{EH}} = \frac{\mathrm{KR}}{\mathrm{EH}} + \frac{\mathrm{NU}}{\mathrm{ON}}.$$

Because

$$\cot \alpha = \frac{\mathrm{HP}}{\mathrm{EH}} \text{ and } \cot \beta = \frac{\mathrm{KR}}{\mathrm{EH}}$$

(where $0° \leq \alpha \leq 90°$ and $0° \leq \beta \leq 90°$),

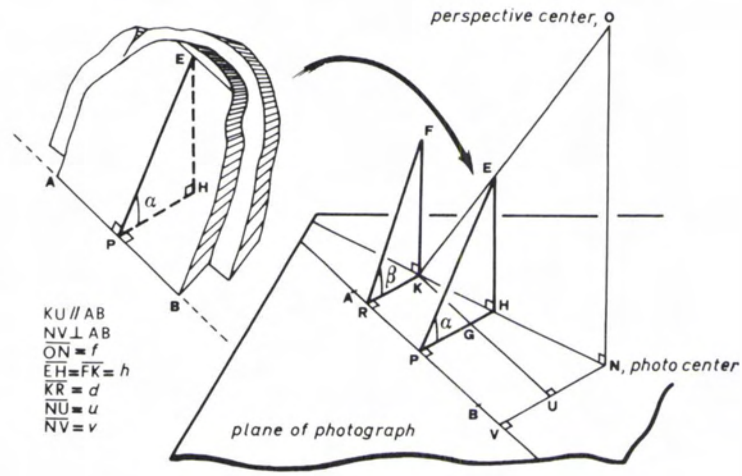

FIG. 1. Isometric illustration of the spatial relationships between real angle α of dip of a bed and its corresponding photographic perspective angle β.

by letting $\overline{NU} = u$, we obtain

$$\cot\alpha = \cot\beta + \frac{u}{f}$$

It must be noted now that Figure 1, from which the above equation was deduced, corresponds to just one of the possible different cases which may occur when the position and orientation of the bed image on the photo, as well as the value of the real angle of dip, change. Moreover, the two different cases of point E being above or below the datum plane through AB should have to be considered. Changes in these conditions may determine changes in the algebraic signs of the two terms on the right side of the aquation. It is easy, although rather time-consuming, to see that the various possible cases may be summarized in the following formula:

$$\cot\alpha = \left|\cot\beta \pm \frac{u}{f}\right| \qquad (2)$$

where we shall select a plus sign for perspective angles β dipping in the same direction as the oriented segment $\overrightarrow{NU}$ and a minus sign for perspective angles β dipping in the direction opposite to $\overrightarrow{NU}$.* In the latter case, when

$$\cot\beta - \frac{u}{f} > 0,$$

the real dip direction will result opposite to $\overrightarrow{NU}$, that is concordant with the perspective dip direction. On the other hand, when

$$\cot\beta - \frac{u}{f} < 0,$$

the real dip direction will result concordant with $\overrightarrow{NU}$, that is opposite to the perspective dip direction. In this case the image of the bed on the photo appears perspectively overturned and the real dip direction will be found reversing the photographic one.

For a more thorough analysis concerning perspectively overturned dips, the reader is referred to Raasveldt (1959).

Obviously, perspective angle β is the angle which would be assumed as real angle of dip, should we not take into consideration that aerial photographs are perspective projections. Therefore, in Equation 2 the term $\pm\, u/f$, which characterizes the position and orientation of the bed image on the photo, represents the *perspective correction term*. This means that, on a photo of given focal length f, the perspective correction will be the same for all those beds whose strike lines have a same distance $\overline{NU} = u$ from the photo center N and whose dip directions are coherent with reference to the directions of their respective oriented segments $\overrightarrow{NU}$ (that is, all concordant with or all discordant from $\overrightarrow{NU}$). In particular, when these beds present the same value of perspective dip β, they will have also the same value of real dip α. This may happen, for instance, in the case of stratigraphic levels belonging to elongated plicative structures, which are sometimes outcropping for long rectilinear stretches.

The perspective correction will be zero (or negligible) and therefore $\alpha = \beta$, when $u = 0$ (or u small enough); that is, when, independently from the position of the bed image on the photo, its strike line results radial (or approximately radial) from the photo center. Obviously, a bed image which falls in the proximity of the center represents an even more particular case inside this case.

* The perspective dip direction will be $\overrightarrow{KR}$ for K higher than AB and $\overrightarrow{RK}$ for K lower than AB.

Now, by putting (see Figure 1)

$$\overline{KR} = d, \ \overline{NV} = v, \text{ and } \overline{EH} = \overline{FK} = h$$

and taking into consideration Equation 1, it is possible to transform Equation 2 into the following form:

$$\cot \alpha = \frac{1}{f} \left| \frac{db}{|\Delta p|} \pm v \right| \quad (3)$$

where it is easy to see that, with regard to the double sign, the same rules introduced for Equation 2 are still valid here, with the only condition of considering the oriented segment $\overrightarrow{NV}$ instead of $\overrightarrow{NU}$ and the two possible cases

$$\frac{db}{|\Delta p|} - v \gtrless 0 \text{ instead of } \cot \beta - \frac{u}{f} \gtrless 0.$$

It is worth noting that, as the dip angle is a trigonometric function of the *ratio* between two distances, neither of the two cumbersome interdependent parameters, flight altitude and photographic scale, appears in Equation 3. The only parameter which we need to know initially is the camera focal length. Its value can be usually found in the information printed along an edge of the photographs.

Description of the Method

The method can be illustrated step by step as follows:

(1) Mark, with a sharp pin-priker, the center and the transferred center on each photograph of the stereopair and encircle them with a grease pencil. Orient the photographs carefully under the stereoscope with the correct separation and the four points exactly in line; fix the photographs securely.

(2) Locate stereoscopically on the bed surface two points A and B at the same elevation and a third point K at an elevation sufficiently different from that of A and B. However, K may be chosen, at will or according to convenience, either higher or lower than A and B. Carefully mark these three points on both photographs with the pin-pricker and encircle them with the grease pencil. Draw a line through A and B on both photographs with a sharp 3H pencil point so that a very fine furrow results on the photographic emulsion. This furrow will appear well visible when illuminated from proper angle.

To ensure that A and B are chosen at exactly the same elevation, a parallax bar may be used, in which the same setting is maintained for the two points.

Line AB represents the strike of the bed.

(3) With the parallax bar, measure the parallax difference Δp between any point on line AB (for instance, A) and point K, to the nearest 0.01 to 0.05 millimetre.

(4) With a linen tester (tube magnifier) measure on either of the two photographs the distance $KR = d$ from K to line AB to the nearest 0.05 to 0.1 millimetre.†

By laying the zero mark of the measuring scale of the linen tester reticule over line AB, it will be possible to measure the effective segment KR, normal from K to AB, without needing to draw it on the photo.

Note that, depending on the position of K with respect to A and B, point R may fall either internal or external to the two end points of segment AB.

(5) With a millimetric scale ruler, measure the photobase b, adjusted to the photo scale of the datum plane through AB, to the nearest 0.5 millimetre. This measurement will be obtained by subtracting the distance between the two corresponding images of any point of line AB (for instance, A) from the distance between the two photo centers.

In the particular case where line AB falls, even only approximately, at the same elevation as the center of one of the two photos, b may be obtained, more simply, by measuring, on the other photograph, the distance between the center and the transferred center.

(6) On the same photograph on which d has been measured (cf. step 4, above), measure distance $NV = v$ between photo center N and line AB to the nearest 2 to 3 millimetres. This measurement can be carried out rapidly by using an acetate overlay on which two fine ink lines are ruled, forming a figure having the appearance of the capital letter T, as shown in Figure 2. The vertical part of the T is scaled in half centimetres, starting from point V where it meets the horizontal part of the T. This simple device, which may be called the *T distance finder*, is laid over the photo, with the horizontal part of the T passing through A and B and the vertical part passing through the photo center N. The value v can be read at the point of coincidence with N.

(7) Compute the real angle of dip α from Equation 3, which can be written in the form

$$\alpha = \tan^{-1} \frac{f}{\left| \frac{db}{|\Delta p|} \pm v \right|} \quad (4)$$

where, as we have already seen,

- f = focal length;
- d = distance, on one of the two photos, of K from AB (cf. step 4);
- b = photobase adjusted to A (cf. step 5);
- $|\Delta p|$ = absolute value of the parallax difference between K and A (cf. step 3);
- v = distance of the photo center N from

† A parallax bar and linen tester should be among the trade tools of any photointerpreter. However, for less accurate determinations, a simple scale ruler may be used, from which measurements to the nearest 0.2 mm can usually be made.

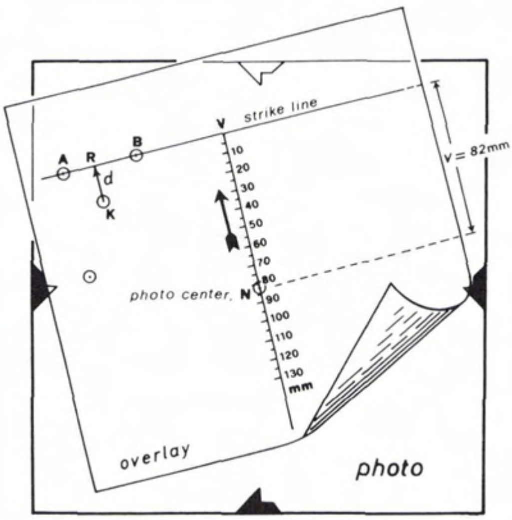

FIG. 2. The "T distance finder" for measuring distances v on a photo. A, B, K, and R are points of the photographic image of the bed taken into consideration; in this specific case $\overrightarrow{NV}$ is concordant with dip direction $\overrightarrow{KR}$.

AB (cf. step 6), measured on the same photo on which d has been measured; and

where all the above measures will be expressed in millimetres and fractions.

In Equation 4, select a plus sign when, on the same photo on which d and v have been measured, the oriented segment $\overrightarrow{NV}$ results concordant with the perspective dip direction, while select a minus sign when $\overrightarrow{NV}$ results opposite to the perspective dip direction. In the latter case, when

$$\frac{db}{|\Delta p|} - v < 0,$$

this means that the bed is perspectively overturned and the real dip direction will be obtained by reversing the perspective one.

(8) Finally, as a partial check, repeat steps 4 and 6 on the second photograph of the stereopair and compute α again by entering the new values of d and v in Equation 4.

As we have already said, the computation with Equation 4 may be carried out very rapidly using an electronic minicalculator equipped with trigonometric functions.

When selecting A, B, and K on the photographs, it will not be necessary that, besides these, other points of strike line AB and of segment KR correspond to actual image points of the bed surface. Think, for instance, of bedding traces outcropping along the opposite walls of canyons or along the opposite slopes of hogbacks.

When it will not be possible to identify, on a bed surface outcropping discontinuously, two points A and B at the same elevation, locate, instead of, say, B, another point C so that A results at an elevation intermediate between K and C. Draw a fine line through K and C on the two photos and, by using a parallax bar, locate on it point B at the same elevation as A. Then proceed as described above.

With regard to the attainable accuracy of the results, when using this method, considerations analogous to those presented in other papers could be made (see, for example, Threet (1956) or Hemphill (1958)). The many non-systematic field checks made by the present writer over the course of several years have led him to the conclusion that an experienced photointerpreter can generally obtain measures within approximately $\pm 2°$ of the true values.

ERRORS RECURRENT IN OTHER METHODS

The most frequent error which can be found in published articles on methods of dip determination consists in the assumption that a line, which on the stereomodel of an inclined surface *appears* as a maximum slope line, actually corresponds to a real maximum slope line on the ground. In fact, because of perspective distortion, this is not generally true. Among the methods containing this wrong assumption, we may list those originally proposed by Desjardins (1943 and 1950), Elliot (1952), Hemphill (1958), Mekel *et al.* (1964, pp. 8-13), Danial (1966), and Allum (1969).

In order to best understand this error, let the inclined plane be schematized by one of the sides of a square-base, regular pyramid, resting upon a horizontal plane. Figure 3 shows a properly aligned stereopair of photographs on which the images of the pyramid appear. Moreover, in the cental part of the figure, the virtual stereoscopic image of the pyramid, as it would appear when observed from directly above, is represented in orthographic projection and at the same scale as that of the two photographic images. This representation of the stereomodel corresponds to a perspective projection of the ground object, analogous to that of the two photos. In it, the position of each image point lies midway between the homologous image points on the two photos, while point S, midway between photo centers M and N, represents the so-called *virtual projection center* of the stereomodel.

Let us consider side AKB of the pyramid. Because point K is the image of the vertex of the pyramid and point Q is the foot of the perpendicular from K to strike line AB on the stereomodel, segment KQ corresponds to an *apparent* line of maximum slope of surface AKB. Nevertheless, the *real* line of maximum slope through point K is apothem KP (not drawn in the figure for sake of clearness).

It is worthwhile to note that in the present case the individuation of line KP of real maximum

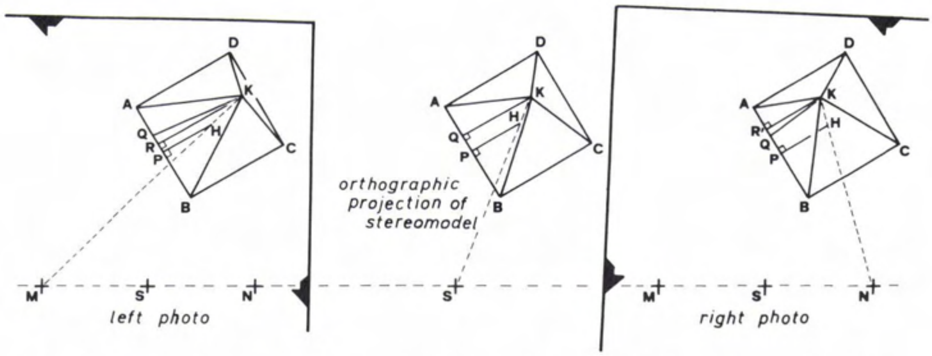

FIG. 3. Representation of the photographic images of a pyramid on a stereopair and of its corresponding stereoscopic image.

slope is immediate because of the particular geometric solid chosen to represent inclined surfaces. In practice, unfortunately, we cannot hope to meet such favorable conditions as would be represented, for example, by photographs of Egyptian Pyramids.

Only in the particular case in which, on the stereomodel, a line of apparent maximum slope results radial from the virtual projection center S, does it correspond to a line of real maximum slope.

If we assume as the vertical distance of the dip angle α the altitude of the pyramid, the corresponding horizontal distance is evidently HP, where point H represents the position of point K when corrected for the relief displacement with respect to the datum plane through strike line AB. In our case, H is coincident with the center of the square base of the pyramid. The erroneous assumption that points K and Q belong to the same line of real maximum slope leads to erroneously consider HQ, instead of HP, as the datum-corrected horizontal distance. Labeling δ the angle corresponding to HQ, then because $HQ \geq HP$, $\delta \leq \alpha$, where the equal sign represents the particular case considered above.

As it could be easy to see, the relationship between "putative" dip angle δ and real dip angle α is

$$\cot^2 \delta = \cot^2 \alpha + \left(\frac{t}{f}\right)^2 \qquad (5)$$

where f = camera focal length, and
t = distance of virtual projection center S of the stereomodel from the line containing segment KQ of apparent maximum slope.*

* This distance t could be obtained, in practice, from measurements on either of the two photographs. For instance, in the case of areas with not too great differences in topographic relief, t corresponds, with a sufficient degree of accuracy, to the distance (see Figure 3) of midpoint S between center and transferred center from the perpendicular to line AB through point Q.

From Equation 5 we may deduce the degree of inaccuracy which can be incurred by assuming KQ as a line of real maximum slope. In fact, let us consider, for example, two different stereopairs of photos of focal length f equal to 88 mm and 210 mm, respectively, and compute the values of angle δ for different values of t (0, 20, 40, 60, 80, 100 mm) and α (15, 30, 45, 60, 75, 90°) as shown in Table 1. In the case, for instance, of an inclined bed of real dip angle $\alpha = 60°$, for which, on a stereopair of focal length $f = 88$ mm, $t = 80$ mm, we have $\delta = 43°$, that is, a value 17° lower than the true one.

Note that, although the selection of points K and Q does not allow a correct determination of real dip angle α, it does allow the determination of the plunge of the straight line through K and Q. Therefore, the methods which follow this procedure remain valid for determinations of plunge of lines through any two given points on the ground surface.

A correct procedure to follow in order to find the datum-corrected horizontal distance HP relative to dip angle α consists in selecting first, on the bed surface, a strike line AB and a point K external to it, as was done in our trigonometric method. Then the correct planimetric position H of point K with respect to the datum plane through AB has to be determined by graphical or analytical-graphical methods. The distance of point H from line AB will be the desired horizontal distance HP.

A second error which can be found in articles on dip determinations consists in assuming, as a line of real maximum slope, segment KR or segment KR' (see Figure 3) drawn from K normal to strike line AB on the left-hand or right-hand photograph, respectively (cf. Mekel *et al.*, 1964, p. 13; A.S.P., 1960, p. 280; A.S.P., 1975, p. 1191). Because of space limitations, we shall omit a detailed examination of this case, for which considerations analogous to those of the previous case would be valid. Just note that R and R' are the photographic images of two distinct points on line AB; and that only in the particular case in which either KR lies

TABLE 1. COMPARISON BETWEEN CORRESPONDING VALUES OF α AND δ ACCORDING TO EQUATION 5

	f = 88 mm						f = 210 mm					
	t (mm)						t (mm)					
α (°)	0	20	40	60	80	100	0	20	40	60	80	100
15	15	15	15	15	15	14	15	15	15	15	15	15
30	30	30	29	28	27	26	30	30	30	30	29	29
45	45	44	42	40	37	34	45	45	45	44	43	42
60	60	58	54	48	43	38	60	60	59	57	55	53
75	75	71	62	54	47	41	75	74	72	69	65	61
90	90	77	66	56	48	41	90	85	79	74	69	65
	δ (°)											

radial from center M on the left-hand photo, or KR' lies radial from center N on the right-hand photo, KR or KR', respectively, will correspond to a line of real maximum slope.

A third serious error which can be found is that of not taking into account that both the aerial photographs and the stereomodel generated by them are to be considered perspective projections. To omit the perspective correction, both in methods which take into consideration the geometry of the single photos (A.S.P., 1960, p. 280; A.S.P., 1975, p. 1191; Allum, 1969) and in methods of visual estimation on the stereomodel (Mekel *et al.*, 1964, p. 7; van der Bent, 1969), can lead to very unreliable results.

From Equation 2 we may realize the degree of inaccuracy which can be incurred by omitting the perspective correction, that is, by assuming β as the real dip angle. This equation, which we derived for the geometry of a photo, is valid also for a stereomodel when (1) β stands for the so-called *stereoscopic perspective angle,* that is, the angle whose value is obtained by applying only the correction for the vertical exaggeration to the apparent angle estimated on the stereomodel; and (2) instead of the oriented distance $\overline{NU}$, we consider (see Figure 3) the oriented distance from the virtual projection center S of the stereomodel to the strike line through K (parallel to AB). For the sake of simplicity we shall label $\vec{u}$, in both cases, this oriented segment.†

Table 2, compiled using Equation 2, gives the values of real angle α for different values of β (15, 30, 45, 60, 75, 90°) and u (0, 20, 40, 60, 80, 100 mm) and for two distinct values of focal length f (88 and

† The magnitude u of $\vec{u}$, in the case of a stereomodel, could be measured, in practice, on either of the two photographs. For instance, in the case of areas with not too great differences in topographic relief, u corresponds, with a sufficient degree of accuracy, to the distance of midpoint S between center and transferred center from the line parallel to AB through K.

TABLE 2. COMPARISON BETWEEN CORRESPONDING VALUES OF β AND α ACCORDING TO EQUATION 2

		When $\vec{u}$ concordant with apparent dip direction						When $\vec{u}$ opposite to apparent dip direction					
		u (mm)						u (mm)					
	β (°)	0	20	40	60	80	100	0	20	40	60	80	100
f = 88 mm	15	15	14	13	13	12	12	15	16	17	18	20	21
	30	30	27	24	23	21	19	30	34	38	44	51	59
	45	45	39	35	31	28	25	45	52	61	67	85	*82**
	60	60	51	44	38	34	30	60	71	54	*84*	*72*	*61*
	75	75	64	54	46	40	35	75	88	*79*	*73*	*57*	*49*
	90	90	77	66	56	48	41	90	*77*	*66*	*57*	*48*	*41*
f = 210 mm	15	15	15	14	14	14	13	15	15	16	16	17	17
	30	30	29	27	26	25	24	30	31	33	35	37	39
	45	45	42	40	39	36	34	45	48	51	54	58	62
	60	60	56	52	49	46	44	60	64	69	74	79	84
	75	75	70	65	61	57	53	75	80	86	*89*	*84*	*78**
	90	90	85	79	74	69	65	90	*85*	*79*	*74*	*69*	*65*
		α (°)											

* The values of α in italics are those for which the dip direction is perspectively overturned.

210 mm). In the case, for instance, of a stereopair of focal length $f = 88$ mm, an inclined bed for which, either on one of the two photos or on the stereomodel, $\beta = 60°$, $u = 80$ mm, and $\vec{u}$ is opposite to the perspective dip direction, we have $\alpha = 72°$ and the real dip direction opposite to the perspective one; therefore,

$$\alpha - \beta = (180° - 72°) - 60° = 48°.$$

It is evident, from the above, that confusing angle δ or angle β with angle α is perhaps justifiable in methods proposed to foresters, soil scientists, and botanists (see, for example, Robbins, (1949); Spurr (1960) p. 149; Moessnes and Choate (1966); Arles (1969)) who usually deal with relatively low values of slope angles, but it is not admissible in methods to be applied by photogeologists who are expected to carry out accurate determinations of dip angles of structural surfaces ranging from the horizontal to the vertical.

Conclusions

The trigonometric method described in this paper is considered by the writer quite satisfactory when accurate determinations of dip angles are required, independent of the position and orientation of the images of the inclined beds on the photographs and for any value of the angles themselves.

However, should a photogeologist intend to use some other method, it is advisable that he be certain that he does not incur any of the errors pointed out above.

References

Allum, J. A. E., 1969. Proofs and uses of some formulae for the measurement of height differences and dips on aerial photographs by elementary methods. *Trans. Instn. Min. Metall.*, 78:B:108-113.

American Society of Photogrammetry, 1960. *Manual of Photographic Interpretation.* Washington, 868 p.

———, 1975. *Manual of Remote Sensing.* Falls Church, Va., 2 vol., 2196 p.

Arles, Cl., 1969. La mesure des pentes sur les photographies aériennes. *Soc. Française de Photogrammétrie, Bull.* 35:36-44.

Bent, E. Th. van der, 1969. Dip estimation for photogeology. *Photogram. Eng.* 35(12):1225-7.

Danial, N. F., 1966. A simplified graphical method for measuring vertical angles from aerial photographs. *Photogrammetria* 21(2):57-61.

Desjardins, L., 1943. Measurement of dip angles on aerial photographs. *Bull. Am. Ass. Petr. Geol.* 27(11):1534-8.

———, 1950. Techniques in photogeology. *Am. Ass. Petr. Geol. Bull.* 34(12):2284-2317.

Elliot, D. H., 1952. *Photogeologic interpretation using photogrammetric dip calculations.* Special Report no. 15, Dept. Natural Resources, Div. Mines, California, 20 p.

Fichter, A. J., 1953-54. Geometry of the imaginary stereomodel. *Photogrammetria* 10(4):134-139.

Hemphill, W. R., 1958. Determination of quantitative geologic data with stereometer-type instruments. *U.S. Geol. Survey Bull.* 1043-C:35-56.

Howard, A. D., 1968. The Fichter equation for correcting stereoscopic slopes. *Photogram. Eng.* 34(4):386-390.

Mekel, J. F., J. F. Savage, and H. C. Zorn, 1964. *Slope measurements and estimates from aerial photographs.* I.T.C. Publications B 26; International Training Center for Aerial Survey, Delft, 32 p.

Moessner, K. E., and G. A. Choate, 1966. Terrain slope estimation. *Photogram. Eng.* 32(1):67-75.

Raasveldt, H. C., 1956. The stereomodel, how it is formed and deformed. *Photogram. Eng.* 22(4):708-726.

———, 1959. Determination of angle of dip of seemingly vertical strata on vertical aerial photographs. *Photogram. Eng.* 25(1):49-53.

Ricci, M., and S. Petri, 1965. *Principios de Aerofotogrametria e Interpretação Geologica.* Companhia Editora Nacional. S. Paulo, 226 p.

Robbins, A. R., 1949. A method of finding gradients from air photographs with no control. *Photogram. Eng.* 15(4):636-7.

Spurr, S. H., 1960. *Photogrammetry and Photo-Interpretation.* Ronald. New York, 472 p.

Threet, R. L., 1956. Graphical template for determination of dip from aerial vertical photographs. *Bull. Am. Ass. Petr. Geol.* 40(5):1009-1016.

Turner, H., 1977. A comparison of some methods of slope measurement from large-scale air photos. *Photogrammetria* 32:209-237.

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