Fred C. Billingsley
Jet Propulsion Laboratory
California Institute of Technology
Pasadena, CA 91109

Modeling Misregistration and Related Effects on Multispectral Classification

Misregistration causes field borders in a given (set of) band(s) to be closer than expected to a given pixel, causing additional pixels to be misclassified.

Introduction

Spectral analysis generally takes the form of multispectral classification in which the classification is done by comparing the sample measurement vector to the statistics of the set of known material vectors (training statistics) representing all possible classes, and by using one of several decision methods, determining which of the knowns it most nearly matches.

The problem pursued will be the effects of misregistration on the accuracy of multispectral classification. Misregistration is but one of a group of parameters (noise, class separability, spatial transient response, field size) which must all be considered simultaneously. The thread of the argument (which will be discussed in detail below) is this: any noise in the measurements (due to the scene, the sensor, or the analog to digital process) causes a finite fraction of measurements to fall outside of the classification limits. For field boundaries, where the misregistration effects are felt, the misregistration causes the border in a given (set of) band(s) to be closer than expected to a given pixel, so that the mixed materials in the pixels causes additional pixels to fall outside of the class limits. Considerations of the transient distance involved in the difference in brightness between adjacent fields, when scaled to "per pixel," allows the estimation of the width of the border zones. The entire problem is then scaled to field sizes to allow estimation of the global effects.

This approach allows the estimation of the accuracy of multispectral classification which might be expected for field interiors, the useful number of quantization bits, and one set of criteria for an unbiased classifier.

Abstract: *Misregistration is but one of a group of parameters (noise, class separability, spatial transient response, field sizes) affecting the accuracy of multispectral classification. The entire group must be considered simultaneously. Any noise in the measurements (due to the scene, to the sensor, or to the analog/digital conversion) will cause a finite fraction of the measurements to fall outside of the classification limits, even within nominally uniform fields. For field boundaries, where the effects of misregistration are felt, additional pixels will be misclassified due to the mixture of materials in the pixels. Misregistration causes field borders in a given (set of) band(s) to be closer than expected to a given pixel, causing additional pixels to be misclassified. Simplified models of the various effects are used to gain conceptual understanding and to estimate the performance to be expected.*

Conclusions

The following briefly stated observations are developed in detail in the body of the report:

- There is no firm cutoff to the registration accuracy required.

0099-1112/82/4803-0421$02.25/0
© 1982 American Society of Photogrammetry

- Unless overlay registration precision in the 0.1 to 0.2 pixel range can be obtained by the delivery system, precision users may have to re-register image segments before analysis.
- If the delivery system cannot meet this precision, registration overlay precision to 0.5 to 0.7 pixel should meet most users' needs without further registration.
- There is a grey area in registration precision of one-half to one to two pixels in which the requirements for high precision are not well justified.
- System registration to one to two pixels should satisfy users of film products.
- Interpolation algorithm choice is relatively unimportant, provided a higher-order interpolator is used.
- If small fields are important, small pixels are more important than sensor noise contributions.

The following parameters are found to be representative, and are used in the analysis:

r = average field shape ratio = 2 (long side/short side) τ = transient distance, 10 to 90 percent response = 1.5 pixels

$$\beta = \text{signal/noise} = \frac{\text{classification class size}}{\sigma \text{ of random noise}} \text{ (in same units)}$$

$$3 < \beta < 7$$

The Basic Model

The expected effect of misclassification may be estimated by a simple first-order approach, because the differences in classification accuracy between the many classification schemes and conditions that have been tested are overshadowed by the vagaries in the data and assumptions in the classification process. Therefore, higher order analysis will contribute little additional understanding.

Consider first the probability of correct identification of a field interior pixel. Field interiors are nonuniform because of the combined effects of sensor noise, scaled to equivalent reflectivity (NE$\Delta\rho$), and inherent nonuniformities in the field itself. The overall brightness distribution is considered to be Gaussian: This is approximately true for field interiors, although the distribution deviates considerably toward bimodal for mixed materials at field borders.

The combined effect of these various noise sources produced a finite probability of misclassification of a given single pixel (Figure 1). The first-order estimate considers the total variance caused by the scene, sensor, and quantization as compared to the defined class size limits, however these are determined. Similar, but relatively second-order, effect may be expected with a higher order analysis. Proper classifier training, resulting in accurate limits, is essential (Hixson *et al.*, 1980). The improvement provided by the various contextual classifiers is recognized, but not treated here.

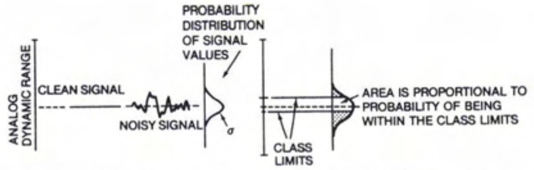


Fig. 1. Effect of noise on the probability of correct multispectral classification.

For simplicity, and because of the later desire to misregister one (or more) of the bands, the discussion will assume that spectral bands as sensed will be used, and that for recognition the unknown pixel must fall between appropriate limits in every band tested. Therefore, brightness outside of limits in any one band is sufficient for rejection, so that we need to consider only one band at a time. The limits in the band being considered will have been determined by the chosen classification algorithm at the observed brightnesses in the other analysis bands. Thus, in Figure 2, the limits in λ_1 (the band being considered) will be dependent on the brightness, λ_2, and the particular classifier.

The probability of a sample being within the class limits can be derived by assuming that an ensemble of noise-free signals from a series of areas of the same material can be anywhere within the quantizing range with uniform probability, but that individual samples are perturbed by the

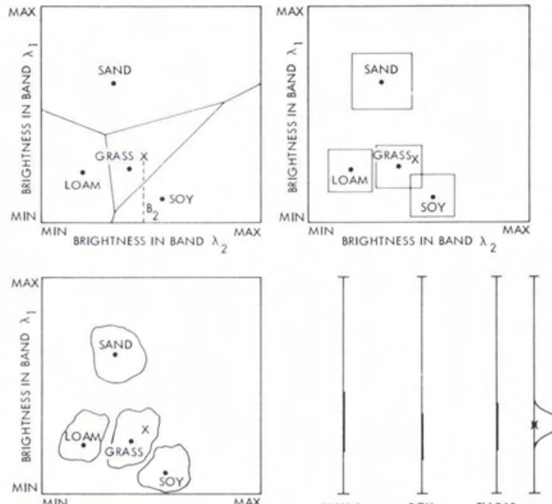


Fig. 2. Classification limits in band λ_1 will depend on brightness λ_2 and the chosen classifier. The lower right figure shows classification limits as seen by the three classifiers, and the distribution for grass.

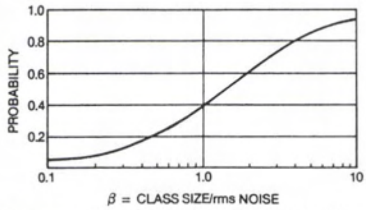


FIG. 3. Given a signal uniformly probable over the dynamic range, and Gaussian noise with standard deviation = σ. The curve shows the probability of correctly recognizing a class corresponding to the noise-free signal as a function of the ratio β = class size/σ.

Gaussian noise with a distribution equal to σ. The probability distribution of the signal plus noise is found by convolving the probability distribution of the signal with that of the noise. The probability of correct class assignment (i.e., the pixel is within the class limits) is then found by integrating the probability distribution between appropriate class limits (Friedman, 1965). The result of this calculation is shown in Figure 3. In the useful range of ($3 < \beta < 7$), the curve can be approximated by

$$\beta \log P = -0.40$$

where P = probability of correct classification, and

$$\beta = \frac{\text{class size}}{\sigma_{\text{scene}}},$$

with class size and σ_{scene} in the same units.

Sources of noise will be the scene itself and the sensor, both assumed to be random for this analysis. The root-mean-square (RMS) sum is taken to give the total effective noise. A number of pixel measurements may be averaged together to reduce the noise before classification. This final noise figure may be compared to the width of the class to give β, from which the probability, P, of correct classification may be estimated. This leads to the Classification Error Estimator, Figure 4.

As an example, consider a scene having a field-interior variation of 3 percent,[1] to be viewed with a sensor having a total noise figure of 1 percent.[2] The total effective noise seen by the classifier (upper left) will be the RMS sum of these, or 3.16 percent, which for a total 0 to 255 digital number (dn) range, would be 8.1[3] dn. If the class width (determined by the classifier algorithm) is 25 dn[4] (right center), β = 3.1,[5] giving P = 0.742[6] (right lower). If this P is not accurate enought for the analysis, several pixels must be averaged (right upper): a 2×2^7 averaging will raise β to 6.2,[8] giving a new P = 0.86.[9]

EDGE EFFECTS

To this point, the analysis is based on pixels well inside uniform fields and well away from field boundaries. A number of experimenters have spent appreciable time discovering that classification accuracy falls off at boundaries due to what has become known as the mixed-pixel effect. We will start at that point and attempt to model the effect to allow us to quantify our expectations.

We assume as a starting point that all the spectral bands used in classification, whether obtained from one date or a series of dates, are in perfect registration. This means that when the pixel grids from each band are aligned the data contents (field borders, roads, all features) are also aligned: Note that this is more than simply having all internal distortions removed, which is all that many geometric rectifications accomplish. Misregistration will (later) be considered as the lack of alignment of the pixel grids; because the computer can only work with pixel grids, aligning these pixel grids appears to the computer as a shift in the boundaries. We will assume that training samples are accurate and that class limits have been set from these by the classifier chosen. The classification is modelled as follows: signature shifting in any individual band due to mixed pixels will tend to cause misclassification, so that the situation may be treated one band at a time. The effects of pixel mixture in all bands may then be RMS'd (added in the RMS sense) together if desired. The entire analysis simplifies to the consideration of the transient intensity shift across field boundaries as compared to the class limits and the noise components of the measurements.

The first step in analyzing the spatial extent of pixel mixing across borders is to estimate the shape and extent of the transient intensity shift. If the impulse response functions or the modulation transfer functions (MTFs) of the various components (and, hence, the entire system) are known, a precise transient response may be calculated. For example, the specification for the Thematic Mapper for Landsat D call for a 2 percent to 98 percent time equivalent of about 2 pixels, implying a 10 percent to 90 percent transient response of about 1.5 pixel. The practical result of this is that the sharp edges of the real scene will be softened by the filtering effect of the scanning aperture (assumed to be rectangular and having uniform response) and it is this softened transient response which is sampled. Interpolation required for registration will cause some further softening. The use of any of the competent higher-order interpolation functions (sin$x$/x, TRW cubic convolution, modified cubic convolution, other splines) will have some effects on the rise time whose importance must be judged relative to the finesse of the classification being attempted. A total τ_{10-90} (transient response from 10 percent to 90 percent) of 1.5 pixels with no ringing will be used as a surrogate global value.

The transient situation across a border is

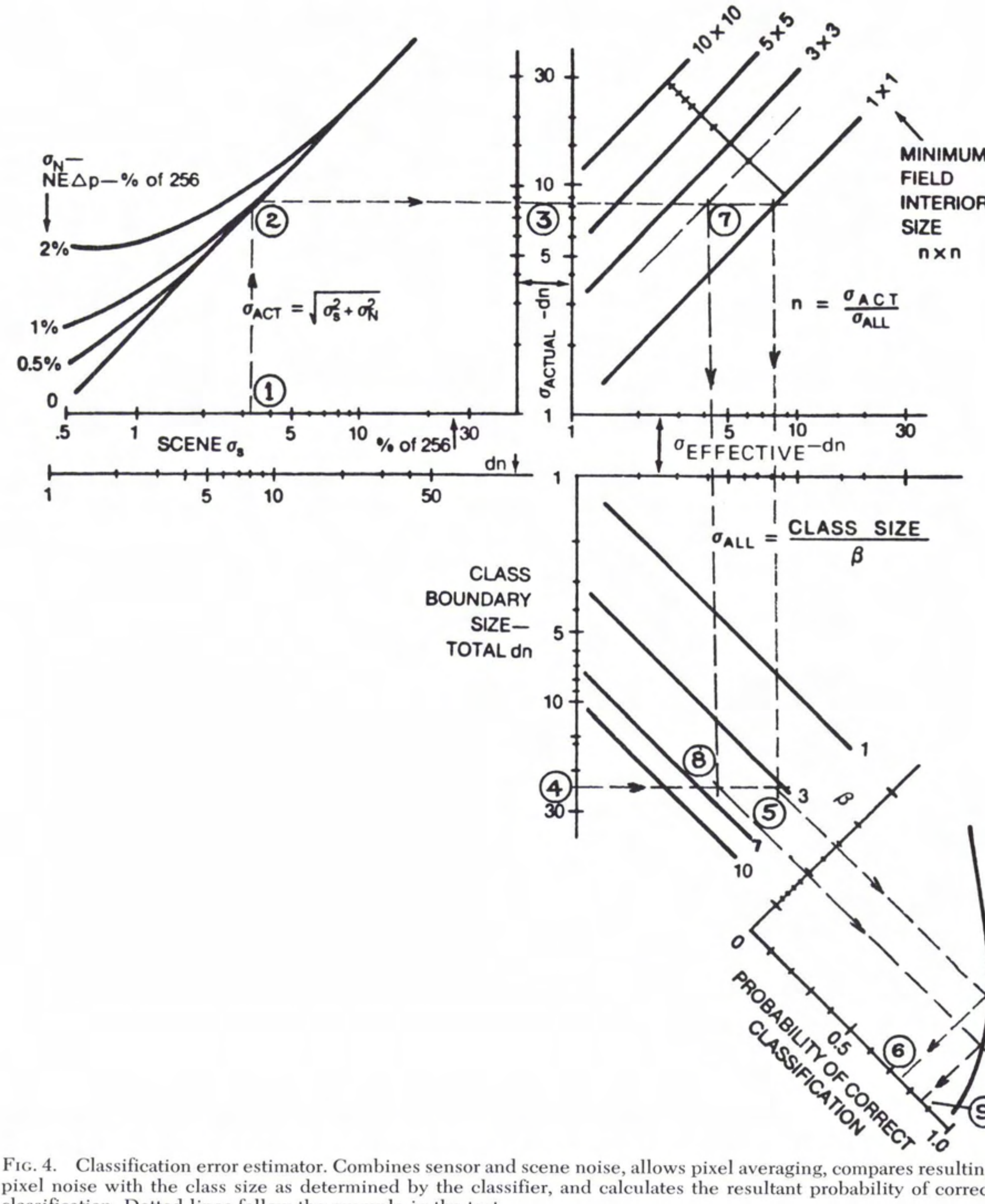


FIG. 4. Classification error estimator. Combines sensor and scene noise, allows pixel averaging, compares resulting pixel noise with the class size as determined by the classifier, and calculates the resultant probability of correct classification. Dotted lines follow the example in the text.

sketched in Figure 5. We are concerned here with the decrease in probability that a given pixel will have a value within the class limits as that pixel moves toward the boundary, as shown in Figure 6. The analysis only needs to determine the area under the normal curve (assuming the noise is Gaussian) between the limits as determined by the classification class size and the brightness shift from the "field interior value" caused by the mixture. The amount of brightness shift is propor-

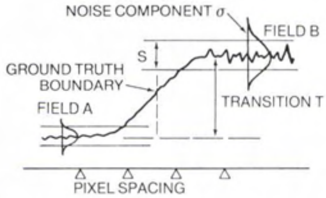


FIG. 5. Cross section of brightness trace across a boundary, showing the distance required for the brightness transition.

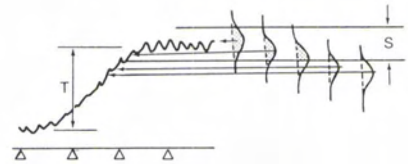


FIG. 6. The distribution of "field" pixels moves down the transition curve as the measurement point moves toward the boundary. The shaded area is the proportion which will be correctly classified.

tional to the difference between the brightness of the field under consideration and the adjacent field which is causing the shift. The left portion of Figure 7 reflects this shift in brightness (vertical axis) scaled to units of class size S as it affects the area within the class (the probability of recognition).

The important intensity relation is the magnitude of the total brightness difference, T, as related to the size, S, of the class being tested by the ratio T/S. This transient total difference, T, is spread across the field border by the transient response distance, τ, of the sensor. The curves of the shift for various T/S, as a function of the distance of the pixel from the border are combined with the probability curves of the previous discussion in Figure 7. From this may be estimated the loss in probability in classification of pixels near borders.

BIAS IN FIELD SIZE ESTIMATION

It can be appreciated that several things are happening simultaneously: If the lower limit of field B and the upper limit of field A have a gap between, pixels "lost" by field B will not be picked up by field A, and will be considered unknowns and not be counted in either field. The lost pixels will be some interior pixels, due to insufficient β, and a large number of near-border pixels, resulting in apparent field size loss. Only if the lower limit of field B and the upper limit of field A are coincident will pixels lost from one field be picked up by the other, and vice versa, to give complete account of all pixels. For the field size estimator to be unbiased, the loss-and-pickup in both directions must cancel; that is, on the average the true border must be located. The total ef-

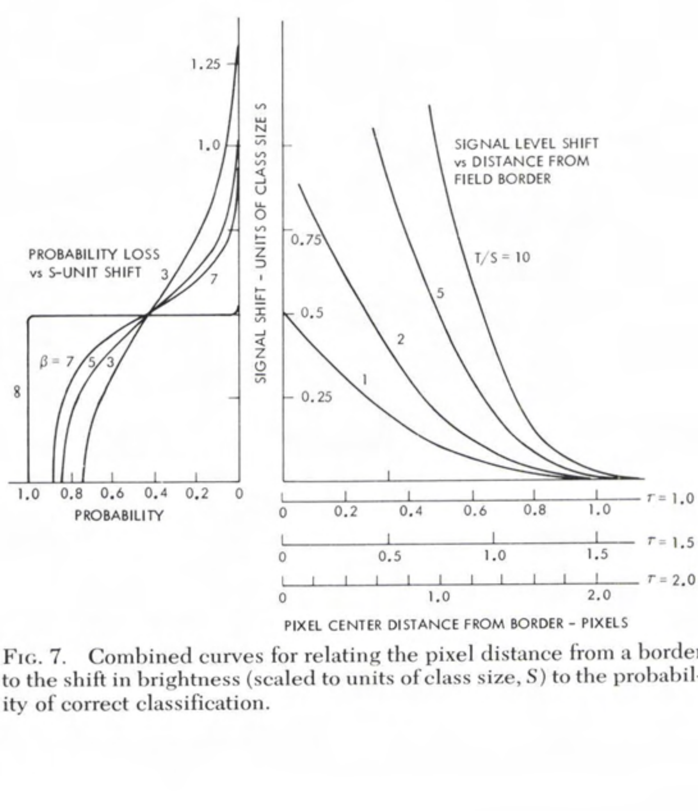


FIG. 7. Combined curves for relating the pixel distance from a border to the shift in brightness (scaled to units of class size, S) to the probability of correct classification.

fect will depend on the ratio of the number of border pixels to the number of field-interior pixels, and hence is a function of the field shape and size.

This leads directly to the required algorithm for field size estimation: First divide the scene into blobs, each of which is sufficiently uniform, and with closed boundaries. Then for each blob (field) determine the average brightness for all the interior pixels which are safely away from the border. For each segment of the border, the correct field edge decision level is midway (in σ's) between the average brightness of the two fields on either side. After the borders are located using this criterion, the field interiors may be reclassified using the classification limits as determined from the training samples.

Effects of Misregistration

In preparation for estimation of the misregistration effects, an analysis will first be made of the expectations of registered data and the sensitivity to the various parameters estimated. The starting model used has rectangular fields aligned with the pixel grid. Pixels are grouped into four zones: (1) Interior (i)—those with centers two or more pixels inside borders, (2) Inner border (ib)—pixels with centers one-and-one-half pixel inside borders, (3) Outer border (ob)—pixels with centers one-half pixel inside borders, and (4) Exterior border (xb)—pixels outside the borders, with centers one-half pixel outside. Estimates of classification accuracy for each zone are obtained from Figure 7. The total estimate of classification accuracy is the sum of pixels in each zone multiplied by the corresponding zone accuracy estimate. Later, the field will be misregistered, changes in the number of pixels in each zone calculated, and the probabilities again summed. The following parameters are required:

- r = the field shape ratio, i.e., length of long side/length of short side;
- T = transient brightness difference between field being considered and its neighbor;
- S = decision class size;
- τ = transient distance for 10 percent to 90 percent response; and
- β = class size S/σ of Gaussian noise

The following global values selected for the parameters are considered to be representative:

$r = 2$
$T/S = 1$ to 5
$\tau = 1.5$ pixels
$\beta = 3$ to 5

After the parameters r, T/S, τ, and β are selected, the resultant (from Figure 7) probabilities are substituted for the brightness in the various zones to produce a "probability image" aligned with the desired output pixel grid. The probability assigned to a pixel at a given location represents the probability that that pixel will have a brightness falling within the classification limit determined by the classifier, for the given spectral band. The total probability of correct classification is given by

$$P = \frac{1}{rn_1^2} - (p_i n_i + p_{ib} n_{ib} + p_{ob} n_{ob} + p_{xb} n_{xb})$$

where n_1 is the field width (short side) in pixels, and n_i, n_{ib}, n_{ob}, n_{xb} are the number of pixels in the various zones. Using these values, the global estimate of the probability of correct classification with no misregistration is given Figure 8 for three values of T/S. The predominant effect is the pixel mixture (the effect of T/S). As expected, this is worst for small fields (n_1 small) because of the larger percentages of border pixels for these fields. Note that for $T/S = 1$, decision level midway between brightnesses of adjacent fields, no probability loss occurs, even with small fields. Unfortunately, this desirable condition cannot be systematically obtained.

Misregistration of Congruent Fields

The initial model for misregistration is a displacement of d pixels, equal in both x and y. The result of this misregistration is that some area is lost from the external border, causing a further classification accuracy decrease. The misregistration loss as seen by the external border loss is given by

$$P = p_{xb}\left[d\,\frac{r+1}{r}\,\frac{1}{n_1} + (4d - d^2)\frac{1}{n_1^2}\right]$$

The basic character of this misregistration loss term is $1/n_1$, so that it will have a slope approximately equal to -1 on a log-log plot vs n_1. The precise results depend critically on the values of p_{xb} estimated for the p_{xb} from Figure 7:

T/S	β	$\tau = 1$	$\tau = 1.5$	$\tau = 2$
	3	0.10	0.14	0.20
1	5	0.02	0.025	0.07
	7	0	0.01	0.04
	3	0	0	0
2	5	0	0	0
	7	0	0	0

Using these values, the loss ΔP due to displacement misregistration is plotted in Figure 9 for various parameter combinations.

Misregistration Due to Noncongruence

Size and Ratio (Aspect) Changes

Size and aspect ratio changes can come about from several causes such as scan velocity or altitude changes, and if uncompensated can cause additional misregistration errors. Progressive mis-

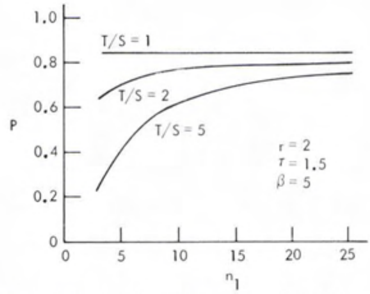


FIG. 8. Probability of correct classification for fields having the short side dimension of n_1 pixels, for registered pixels.

registration from a point of accurate registration will occur from both causes (Figure 10); the modeling of this effect considers first that size changes $N = n'/n$ will cause a shift in points n to points n' both vertically and horizontally, and then that changes in aspect ratio will cause further shifts in the horizontal position of vertical borders by changing the field shape ratios by the factor $R = r'/r$. The resulting shifts are

$$\Delta n_v = (N - 1)\, n_v \text{ and } \Delta n_h = (NR - 1)\, rn_v$$

For analysis, this shift will be divided around the borders symmetrically as optimum field registration is accomplished (Figure 10). Two cases must be distinguished (using scan velocity as a surrogate cause):

Case I: A slow scan decreases pixel spacing and puts more pixels into a given field. When these are placed into the output grid, the field appears stretched. The field as defined by the other (correct) bands (or comparison image) now covers only part of the stretched field, so that the classification tends to see only interior pixels, and the accuracy will increase, ultimately reaching the field-interior accuracy. The sizes of the border errors are (in pixels)

$$e_1 = (N - 1)\, n_1 \text{ and } e_2 = (NR - 1)\, rn_1$$

Case II: A fast scan has the opposite effect, causing the field to appear smaller and the analysis pixels defined by the other bands now include more exterior pixels. The classification accuracy will decrease.

For fast scan, the smaller apparent field covers an area expressed as a fraction f_i of the total:

Fractional Areas:

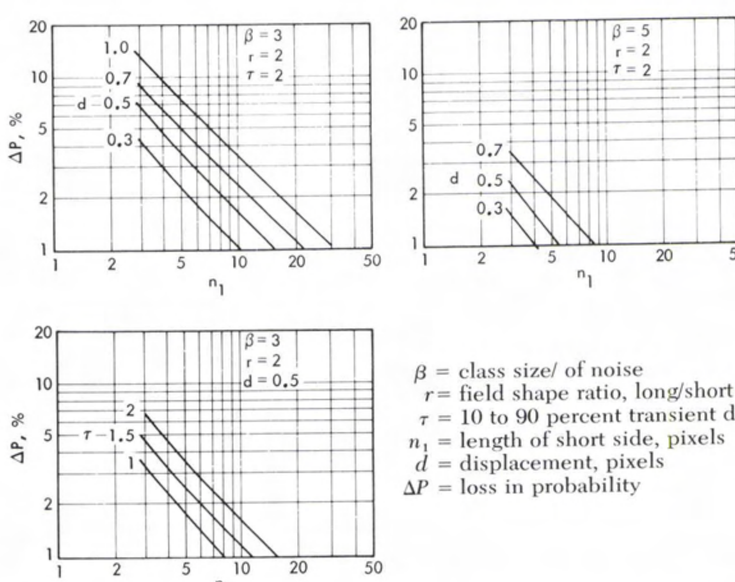
$$\text{Interior:} \qquad f_i = \frac{r'(n_1')^2}{rn_1^2} = RN^2$$

$$\text{External Border} \quad f_{xb} = \frac{2Nn_1 + 2NRn_1r + 4}{rn_1^2}$$

The total expected probability is

$$P_{tot} = f_i p_i + f_{xb}\, p_{xb}.$$

Because the external border pixels included within the analyzed field have a low probability, the fractional area RN^2 represents approximately the fraction of the basic field-interior accuracy to be expected. Also, because the total size shrinkage (in pixels) is small for small n_1, only larger n_1 need



β = class size/ of noise
r = field shape ratio, long/short sides
τ = 10 to 90 percent transient distance
n_1 = length of short side, pixels
d = displacement, pixels
ΔP = loss in probability

FIG. 9. Loss of classification accuracy due to misregistration of one band, for various parameter combinations.

be considered, and the $1/n_1^2$ term may be dropped. This allows P_{tot} to be approximated for $r = 2$ by

$$P_{tot} \approx RN^2 p_i + \frac{3}{n_1} p_{xb}$$

For large fields, the probability is seen to be independent of field size, and only weakly dependent (because of low p_{xb}) for small sizes.

WAVY BORDERS AND MULTIPLE ACQUISTIONS

For single-band analysis, with borders distorted so that there are pixels both inside and outside of the analyzed area, some pixels will have increased probabilities of correct classification and some will have less. The decrease in probability across border is (very) approximately linear, so that the (signed) average displacement will model the effect.

For multiband analysis, those pixels having a low probability of classification will have the largest effect as the net probability at each pixel location is the product of the probabilities obtained for each acquisition (band). In this case the RMS displacement will produce a better model of the effects.

SOME OBSERVATIONS

ON BASIC CLASSIFICATION

- The total noise figure (compared to the class size in a given determination) controls β, which in turn controls the maximum attainable classification accuracy. However, for a practical range of $3 < \beta < 7$, increasing β has only a moderate effect.
- Because of this, if small fields are most important, the reflected energy might more profitably be divided into smaller pixels, even at the expense of $NE\Delta\rho$. As this will cause an increase in data rate, optimum coding should be investigated. The possible noise introduced in reconstructing the data will cause some further decrease in the overall effective $NE\Delta\rho$ and so decreases β. But since there is smaller sensitivity to β than to $1/n_1$, there should be a net gain in utility (see also Landgrebe *et al.* (1977)).
- Increasing the number of bits of quantization produces improvements which asymptotically approach zero as each successive bit reduces the step size by a factor of 1/2.
- A scene having as little as 2 percent variation is a very uniform scene. Since this noise is RMS'd with the sensor noise, it will overwhelm any but a very noisy sensor. Therefore, for purposes of multispectral classification, an extreme number of bits would seem to be unnecessary.

ON EDGE EFFECTS

- For accurate field size estimation, the decision brightness must be halfway between the brightnesses of the fields on either side of a given boundary. This means that classifiers set for material identification will in general produce errors in field size. But the field-interior brightness is increasingly hard to estimate for small fields because of the fewer interior pixels.
- It is important to keep the transient response distance and the accompanying sample spacing small in order to get as many pixels into a given ground distance as possible. Field area errors become large at $n_1 = 5$ or less. The transient distance must also be matched between spectral bands.

ON MISREGISTRATION

- For large *T/S* (i.e., 2 or more) the edge effects are so great that the base probability is drastically affected, and the external border pixels have zero probability of being within the class limits. For this reason, there is no misregistration effect for large *T/S*.
- Square fields show the most misregistration loss, when scaled to n_1.
- A shape ratio $r = 2$ is believed to be representative.
- Misregistration loss decreases with higher β. However, these losses in general are small to begin with, and the discussion calling for sacrifice of β to gain smaller IFOV (more pixels n_1 into a given field) would seem to override.
- Decrease in τ increases the basic accuracy of edge pixels and also decreases the misregistration losses. However, this decrease increases the potential of aliasing (the generation of errors during interpolation). The minimum possible τ as set by the scanning aperture size combined with the 1/frequency nature of the signal content would seem to reduce the aliasing to an acceptable value.
- Geometric rectification and registration procedures must not only remove the internal distortions but must also produce pixels on a defined (preferably ground-referenced) grid. Many current procedures do not do this. Without this reference grid, users will have to reinterpolate before multitemporal data can be compared.
- Scale and aspect ratio errors will have only minor effects on moderate-area problems, but they will cause problems in correlating over large distances.
- Altitude relief displacement will require users to use many control points to register images in areas of high relief. Alternatively, a digital terrain model may be used to correct inter-control-point pixels.

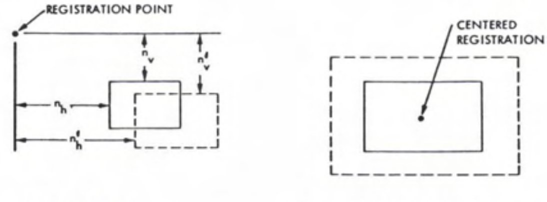


Figure 10. Construction for Estimating Misregistration Caused By Size and Aspect Errors.

- Unless standard reference grids are established, users requiring registration will have to interpolate every image, even in low relief areas.
- For single-band analysis, the algebraic average of the displacement may be used. For multiband analysis, with erratic errors in location among the bands, the lowest probability of correct classification controls and the RMS of the displacements is appropriate.

An Unanswered Question

This report models the potential misregistration effects on multispectral classification accuracy. It may allow the comparison of the various tests and simulations, and points out the variables which must be reported for those simulations to allow their validation. It does not answer the following question: Given a certain loss in accuracy due to misregistration, how does that damage the ability to use the data analysis results? These evaluations will be discipline dependent, and must be sought separately.

Acknowledgment

This paper is a condensation of JPL Publication 81-6, having the same title, dated April 1981. It presents the results of one phase of research performed at the Jet Propulsion Laboratory, California Institute of Technology, sponsored by the National Aeronautics and Space Administration under Contract NAS7-100.

References

Friedman, H. D., 1965. On the Expected Error in the Probability of Misclassification, *Proc IEEE,* Vol. 53, p. 658.

Hixson, M., D. Scholz, N. Fuhs, and T. Akiyama, 1980. Evaluation of Several Schemes for Classification of Remotely Sensed Data, *Photogrammetric Engineering and Remote Sensing,* Vol. 46, No. 12, pp. 1547-1553.

Landrebe, D. A., L. L. Biehl, and W. R. Simmons, 1977. An Empirical Study of Scanner System Parameters, *IEEE Trans on Geoscience Elects,* Vol. GE-15, No. 3, pp. 120-130.

Tucker, C. J., 1980. Radiometric Resolution for Monitoring Vegetation: How Many Bits are Needed?, *Int. Jour. of Remote Sensing,* Vol. 1, No. 3, pp 241-254.

(Received 9 April 1981; accepted 10 September 1981; revised 1 October 1981)

Appendix

Considering β as in the text allows an estimation of the total noise permissible as it affects the attainable classification accuracy. If the amount of scene noise to be encountered in a given classification task can be estimated, the allowable extra noise from the sensor and quantization can be specified by estimating the loss of accuracy of the classification caused by quantization error. This

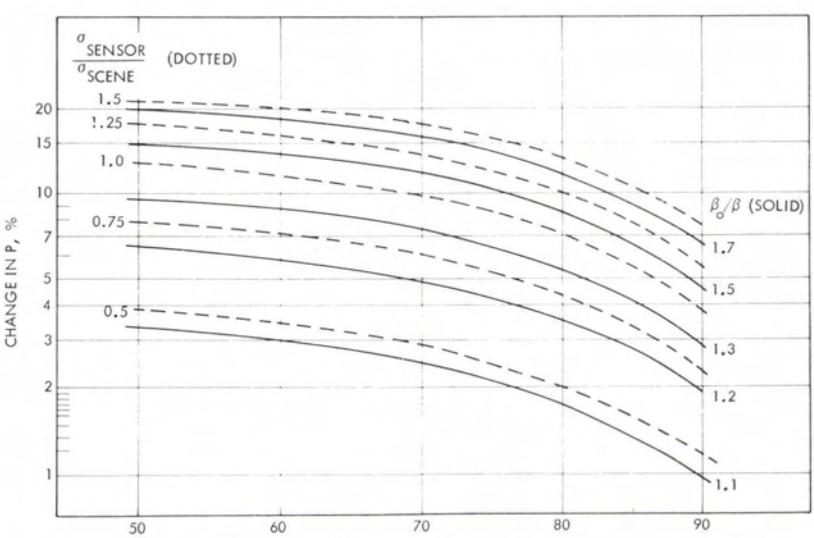


Fig. A-1. Loss in classification accuracy due to additional noise. Example: given an initial desired probability of 85 percent and no more than a 2 percent loss due to sensor noise, find $\sigma_{sensor}/\sigma_{scene} = 0.6$, or the corresponding $\beta_0/\beta = 1.15$.

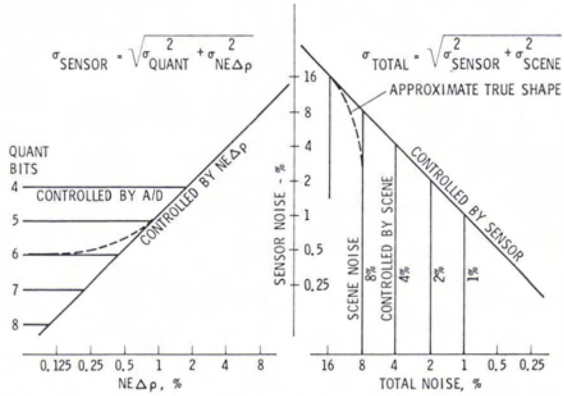


FIG. A-2. Noise contribution breakpoints. The left side calculates the total sensor noise. This is combined with the scene noise (right side) to calculate the total effective noise.

leads to an estimate of the number of bits which will be useful.

Define the perfect sesor as having no random noise nor quantization error (i.e., an infinite number of bits). This will define (for n by n pixels averaged)

$$\beta_0 = \frac{\text{class size} \cdot n}{\sigma_{\text{scene}}} \text{ and } P_0 = 10^{-0.4/\beta}$$

For the real sensor, $\beta < \beta_0$ because of the finite τ_{sensor} and $\sigma_{\text{quantization}}$. The new probability of correct classification P is related to P_0 by

$$P = P_0^{(\beta_0/\beta)}$$

A plot of the loss in classification accuracy versus P_0 is given in Figure A-1, for the parameter families β_0/β and $\tau_{\text{sensor}}/\tau_{\text{scene}}$. Noise allocation starts with defining of the desired P_0 and ascertaining that the required β_0 can be obtained. Definition of the allowed ΔP determines (e.g., from the graph) the allowed $\tau_{\text{sensor}}/\tau_{\text{scene}}$. An estimation of the scene noise for which the other conditions apply allows the calculation of the total sensor noise allowed. The final step is to partition this noise between sensor random noise and quantization noise.

For example, let the desired $P_0 = 85$ percent and allow no more than 2 percent loss due to the total sensor noise. The no-sensor-noise β_0 must be ≥ 5.7 to give P_0. Then, from Figure A-1, the allowed $\tau_{\text{sensor}} = 0.6 \times \tau_{\text{scene}}$. If the scene has a $\tau_{\text{scene}} = 2$ percent, the allowable $\sigma_{\text{sensor}} = 0.6 \times 2$ percent $= 1.2$ percent, which must be partitioned between $NE\ \Delta\rho$ and the quantization noise. For $NE\ \Delta\rho = 1$ percent, the allowable $\sigma_{\text{quant}} = \sqrt{1.2^2 - 1^2} = 0.66$ percent, which can be met by six-bit quantization.

Two observations are important here: (1) Increasing the number of bit of quantization produces improvements which asymptotically approach zero, as each successive bit reduces the step size by a factor of 1/2; and (2) a scene having as little as 2 percent variation is a very uniform scene. Since this noise is RMS'd with the sensor noise, it will overwhelm any but a very noisy sensor. Therefore, for purposes of multispectral classification of field-interior pixels, more than six bits would seem to be unnecessary (see also Tucker (1980)). The relative contributions of the various noise sources is shown in Figure A-2.

DENVER UPDATE

Colorado Governor Richard Lamm (second from left) has proclaimed the week of March 14-20 as "Surveying, Mapping, Photogrammetry and Remote Sensing Week." During that week the combined membership of the American Congress on Surveying and Mapping (ACSM) and the American Society of Photogrammetry (ASP) will meet in joint session in Denver for their 28th annual convention. Lamm presented the proclamation to 1982 convention director James Plasker. Looking on are Al Letey, left, technical advisor for the convention, and John Enos, right, assistant director of this year's convention, representing ASP. More than 4,000 members of the two professional societies are expected to attend the convention.