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Phototriangulation: A Review and Bibliography*

Radial-line, mechanical, semi-analytical, and analytical methods are reviewed; the independent model and bundle adjustment methods are compared; and errors and accuracy are discussed.

Introduction

Aerotriangulation has witnessed a phenomenal development since its advent about half a century ago. It is probably true that no other single technological innovation has undergone so much change within so short a time. Although aerotriangulation methods can be classified into three major categories, there are no less than thirty variations of these methods, some of which are briefly described in Table 1. These variations constitute an expression of adaptability to the following factors: (1) availability and limitations of current instrumentations, (2) economic considerations, (3) preference for a particular methodological approach, (4) limitations imposed by computational (computer) facilities as well as computational (programming) abilities, and (5) accuracy and application requirements. It is the objective of this paper to review various aspects of phototriangulation methods and their applications in surveying and mapping.

Abstract: *The development of phototriangulation is reviewed. A comprehensive classification of phototriangulation methods found in the literature is made. The main concepts of phototriangulation are highlighted and two distinct but not mutually exclusive approaches to aerotriangulation today—bundle adjustment and independent models—are discussed. Representative accuracies from projects around the world are presented. Various applications are also enumerated. The paper contains a selected but classified bibliography.*

Radial-Line Triangulation

The practice of aerotriangulation probably started with radial-line methods, which are based on very simple geometric properties of the aerial photograph. For example, angles measured in the plane of a truly vertical photograph about the principal point are considered true horizontal angles, because radial lens distortions and relief displacements have no effect on such angles and tangential distortions are very small. The vertical photograph then becomes an angle-measuring device. This concept was first developed by Scheimflug and later expounded by Finsterwalder, von Gruber, Bagley, Crillier, and Hotine {92}. The primary principles involved in the practice of radial methods are those of resection and intersection by which a two-dimensional trigonometric network may be extended between control points graphically. Moreover, the equipment required is modest. The simplicity of the application of radial-line methods has encouraged further development of radial-line triangulation from the easy-to-do graphical radial-line plotting to the relatively more complicated procedure of mechanical slotted templates, stereo-templates, and Jerie's

* Presented paper, Commission III, 14th Congress of the International Society for Photogrammetry, Hamburg, July 1980.

0099-1112/82/4811-1733$02.25/0

TABLE 1. CLASSIFICATION OF PHOTOTRIANGULATION METHODS

Class	Sub. Class	Method	Description	Reference
Graphical/Mechanical	Radial—line Triangulation	1 Radial Triangulation	A graphical method in which directions from radial center (principal pt., nadir, or isocenter) of each overlapping photo are used for resection (to determine planimetric coords. of exposure station) and intersection (to determine planimetric coords. of a new prints).	Wolf [106b]
		2 Slotted Templates	Method is similar to the graphical approach except that slotted templates are used. Such templates contain long narrow slots which represent directions radiating from the center of photo. Positions of points are determined by resection and intersection.	Trorey [102]
		3 Mechanical Templates	Spider (mechanical) templates constructed kits such as Lazy Daisys which have dimensionally stable materials are used. The basic principle, as in other radial line methods, is that angles at the radial center in a vertical photo are true horizontal angles on the ground.	McComas [96]
		4 ITC-Jerie	Jerie designed two analog computers for block adjustment of horizontal and vertical points. For horizontal block adjustment the computer consists of section stereo templates (for conformal transformation), Multiplets (for introducing relative discrepancies at the points), and elastically connected studs for introducing zero discrepancies. The vertical analogue computer is designed to adjust a block of stereotriangulated strips by means of elastic rods which are also elastically connected to each other at the points.	Jerie [93] Moore [342]
		5 Stereo-template	Employs stereotemplates—a composite slotted templates which represent a stereo model in plan—constructed from a model in a stereoplotter not designed for precise bridging.	Scher [100]
	Analog	6 Multiple Projectors	Requires a bank of projectors for the formation of a strip consisting of relatively oriented models. Scale is transferred between successive models by imposing equal elevation constraints in the overlapping ground area. Misclosures, which are determined on the last model by means of plotted ground control, are adjusted by prorating them back through the strip.	Williams [255]
		7 Two Projectors	The first model is oriented relatively and absolutely. For successive models the orientation elements (φ, w, k) of the right projector are transferred to the left projector; φ and w are measured with spirit level and k is transferred through dial reading or by special device; otherwise, it is set at zero. Scaling is done by analog method as in (6).	Friedman [239] Holden [243]

TABLE 1—*Continued*

Class	Sub. Class	Method	Description	Reference
Mechanical	Analog (cont.)	8 Aeropolygon	First order instrument is used for co-orientation of successive models with the aid of "base in"/"base out" capabilities. Scale transfer is achieved by changing the base to attain equal elevation of points common to successive models as in (6). Only two projectors are required. Strip or block is adjusted by graphical or numerical methods.	Von Gruber [35] Adler [312]
		9 Aeroleveling	Exemplifies the use of auxiliary data (namely height obtained from statoscope data) for analog aerotriangulation. The flying height of each exposure station is used to preset b_z values during co-orientation of successive models. This gives smaller closing errors compared to aeropolygon. Angular elements of orientation may also be preset in stereotriangulation by use of data from solar periscope, horizon photo, and vertical gyros.	Strahle [252]
		10 Ind. Geodetic Control	Strip triangulation is the same as aeropolygon or aeroleveling. Adjustment is by using independent base lines whose lengths and azimuths on the ground are known.	Ghosh [189] Brandenberger [184]
Semi-Analytical	Independent Models	11 Ind. Models (Polynomial)	Emperical relative orientation of models are performed independently with first or second order instrument. Strip or block formation is done by three-dimensional similarity transformation. Adjustment of strip or block is by polynomial transformation on the computer.	Ayeni [180] Derenyi [186]
		12 Ind. Models (3-D Transformation)	Relative orientation, strip or block formation are the same as in (11). Adjustment of strip or block is by three-dimensional transformation formula with 7 parameters which may be solved iteratively in groups of 4 and 3, i.e., plan-height iteration as in PATM-43.	Parsic [248] Ackermann [176]
		13 Ind. Models (Anblock)	The procedure is the same as in (11): Strip on block adjustment is, however, accomplished by a two-dimensional transformation formula for planimetric adjustment as in PAT-M4.	Boniface [183] Ackermann [176]
		14 Ind. Models (Aux-Data)	Procedure for strip or block formation is the same as in (11) The X, Y, Z coordinates of projection center, and the φ and w elements of each photo, are determined from auxiliary airborne instruments are treated as control in the adjustment of strips or blocks.	Miles et al [34] Zarzycki [39]
	Bundle	15 Semi-Analytical Bundle	Model coordinates obtained from stereoplotter after relative orientation are transformed into photo coordinates which are corrected for systematic effects of film, lens, refraction, and Earth's curvature. Classical bundle adjustment is performed as in (25).	Maarek [193] Derenyi [186]

TABLE 1—*Continued*

Class	Sub. Class	Method	Description	Reference
Fully Analytical	Radial Triangulation	16 Numerical Radial Triangulation	Radial triangulation performed numerically. Photo coordinates are measured. A mathematical model which imitates the graphical procedures of resection and intersection is implimented with the aid of the numerical computation.	Wolf [106a] Hallert [91]
	Independent Models	17 Independent Horizontal Control	This is analogous to (10), which is a semi-analytical approach. This method, in contrast, is a fully analytical approach in which ground distances and azimuths are used as controls in aerotriangulation.	Kenefick *et al.* [192]
		18 IMT Polynomial	Measured comparator coords. are corrected for second under effects due to film, lens, refraction, and Earth's curvature. Numerical relative orientation is performed. Adjustment of strip, block, or section is by Polynomials as in (11).	Ackermann [176]
		19 IMT Direct Transformation	Procedures for coordinate measurement, refinement, relative orientation, strip, or block formation are the same as (15). Adjustment of strip, section, or block is by a three-dimensional transformation as described in (12).	Ackermann [311]
		20 IMT with Collocation	This method is the same as in (18) or (19) except that after adjustment an advanced least squares prediction method (collocation) is used to filter out systematic deformations of a block, thus reducing residual errors at control points. It is an alternative approach to the use of additional parameters in (23).	Ackermann [176]
		21 IMT with Triplet	Comparator coords. are measured from separate photos and these are refined as in (18). Separate units of 3 photos are relatively oriented with a forward overlap of one photo. The separate units (triplets) are assembled in strips or block. Strip or block adjustment is by polynomial (18) or by direct transformation (19).	Mikhail [219]
		22 IMT with Auxiliary Data	This method is analogous to (9) and (14). Auxilliary data obtained from APR, statiscope, or lake are all incorporated into the adjustment program as exemplified by PAT M-43	Faig *et al.* [31] Ackermann [256]
		23 IMT with Additional Parameters	The mathematical model for independent model in (19) is modified to incorporate additional parameters which will account for affine deformations, twisted models, perspective center errors, etc. Such parameters are treated as observations with appropriate weights.	Ebner *et al.* [189]
		24 IMT by Simultaneous 3-D Transformation	A method analogous to bundle adjustment in which independent models in a block are simultaneously transformed by a three-dimensional linear conformal transformation similar to collinearity equations.	Erio [188]

TABLE 1—*Continued*

Class	Sub. Class	Method	Description	Reference
Fully Analytical (cont.)	Simultaneous Bundle	25 Classical Bundle	Comparator coordinates are corrected for systematic effects as in (18). Collinearity equations are used in a least-squares solution to obtain spatial coords. of unknown points on the ground. Calibration data are given.	
		26 Bundle with SAPGO	Simultaneous adjustment of photogrammetric and geodetic observations is performed by extending the mathematical model for classical bundle adjustment.	Wong *et al.* [171]
		27 Bundle with L.S. Collocation	Advanced least-squares interpolation is used to filter the signals from the residuals obtained from classical bundle adjustment. An a-posteriori full covariance matrix is then used in a new math model for a second bundle adjustment.	Rampal [163]
		28 Bundle with additional Parameters	Analogous to (23). The mathematical model in (25) is expanded to include parameters which can take care of third-order systematic effects. These parameters, although unknown, are treated as observed quantities with weights attached to their approximate values.	Brown [147] Salmenpera *et al.* [165]
		29 Bundle with Self-Calibration	The mathematical model from collinearity equations is employed in a least-squares solution in which the camera constant, lens distortion constants, the principal point coordinates are carried. The exterior orientation elements and the ground survey coords. are treated as observed (unknown) quantities. Their approximate values are given appropriate weights. The only true observations are the photo coordinates.	Brown [148] Davis [153]
		30 Bundle with Self-Calibration and Additional Parameters	The procedures involved are the same as in (29). The mathematical model is, however, expanded to incorporate parameters of third-order systematic effects which may account for anomalous film deformation, atmospheric turbulence, etc. Additional parameters are also treated as observations with proper weights as in (28).	Brown [149] Brown [320]

analog computer methods, all of which are imitations of graphical procedures. Jerie's {93} analog computer method is more versatile and accurate than conventional methods. The stereotemplate technique (see Table 1) is an adaptation of third-order stereoplotters, such as the Multiplex, which are not designed for precise stereotriangulation, to accomplish radial triangulation by using stereotemplates constructed from stereomodels in plan. In 1955, Konecny {95} performed radial triangulation with convergent photography and by 1962, Reolofs {98} had executed radial triangulation in mountainous country.

Perhaps one of the most exciting developments in radial triangulation is the arrival of numerical methods, which make use of measured image coordinates in the solution of a mathematical model (a duplication of the principles of resection and intersection according to Hallert's {90} model). Turpin {103} and Wolf {105} improved on Hallert's model by eliminating the necessity to locate the line of flight as the axis of reference. Mikhail {97} introduced the least-squares approach to numerical radial triangulation by using redundant observations. Hallert {91} initiated the concept of numerical stereoradial method. Wolf and Bartlet

{106a}, however, advocated the use of triplets as a basic computational unit for numerical methods. Although numerical radial triangulation is no more widely practiced, its merit lies in the improved accuracy it provides over contemporary methods of radial line triangulation (see Table 1).

Mechanical Aerotriangulation

Another stage of development in aerotriangulation witnessed the use of stereoplotters with multiple banks of projectors such as the Multiplex or the Balplex. Adjacent models were successfully oriented to each other on the projectors to form a continuous stereo-triangulated strip. The two-projector type of stereoplotter such as the Wild B8 [Holden {243}] with some extra devices were also employed for triangulating a strip, but their use was short-lived owing to the development of universal instruments such as the Wild A7 stereo-autograph and Zeiss C8 Stereoplanigraph, each with base-in base-out capabilities (Zeiss parallelogram) and devices for precise recovery of exterior orientation. The introduction of such instruments gave birth to two stereotriangulation procedures, Aeropolygon [{232} {111}] and Aeroleveling [(see Table 1) and {312}], even though the theory behind these two methods was enunciated in 1935 by von Gruber {36}. The strip or block resulting from aeropolygon or aerolevelling may be adjusted graphically or computationally by polynomials. The concept of using distances and azimuth to control a strip was first introduced by Karara {190} and subsequently utilized by Bradenberger {184}, Colcord {185}, and Ghosh {189} in a method called Independent Geodetic Method for developing suitable polynomials for adjusting strips from aeropolygon or aerolevelling.

Semi-Analytical Methods

The 1950's also saw the development of the semi-analytical method of Independent Models as an imitation of the aeropolygon procedure, the only difference being that, in the semi-analytical method, independent models obtained from the stereoplotter are linked successfully one to another through a three-dimensional similarity transformation which represents the projective relation between the preceding model space system X, Y, Z and the subsequent model space system x, y, z in Equation 1 that is,

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = S\mathbf{M} \begin{bmatrix} X - X_0 \\ Y - Y_0 \\ Z - Z_0 \end{bmatrix} \quad (1)$$

where the unknown parameters are

S = scale factor;
$\mathbf{M}$ = orthogonal matrix of rotation K, Φ, Ω; and
X_0, Y_0, Z_0 = translatory elements.

The linkage operation to form a strip or block and the subsequent adjustment are performed on a high speed computer. This approach, which constitutes a compromise between the analog and the fully analytical method, is undoubtedly the most popular of all aerotriangulation methods. This is so by virtue of the fact that semi-analytical methods can be executed by a combination of relatively less expensive non-universal instruments such as the Wild A8 or Kern PG2 and a high speed computer or even a mini-desk computer with a basic memory of 10K words. The implication is that semi-analytical methods are within reach of small mapping organizations in terms of cost and ease of operations.

PERSPECTIVE CENTER

In semi-analytical methods of Independent Models it is customary to make use of the common perspective centers between adjacent models in conjunction with three or more pass points to provide sufficient geometric strength for the connection of successive models to form the strip or block. Accordingly, various methods for determining the perspective center have been developed [{212} {225} {226} {217}]. The necessity for accurate determination of the perspective center has also been established by Brazier {211} and Savage {224}.

STRIP OR BLOCK ADJUSTMENT

Polynomials were first used for the adjustment of strip, block or sections [{200} {202} {196} {204}]. In spite of theoretical objections, polynomials have remained popular with many mapping organizations because of their simplicity, ease, and economy, and also for their fairly good accuracy performance, which is adequate for topographic mapping.

Important innovations in the use of the similarity transformation may be enumerated as follows:

- the planimetric block adjustment approach [{183} {207}], which is exemplified in the PAT-M4 computer program {176a};
- the introduction of the Rodrigues Matrix to replace the orthogonal matrix **M** in Equation 1 [{206b} {176a}];
- the planimetric-height iteration demonstrated in PAT-M43 {176a}, in which a seven-parameter similarity transformation is solved for in groups of four and three parameters; this procedure is faster and requires less storage capacity than the classical seven-parameter approach; and
- the admission of observation on parameters, particularly known and unknown ground coordinates, and the perspective center coordinates.

SEMI-ANALYTICAL BUNDLE ADJUSTMENT

Semi-analytical methods were usually associated with Independent Models vis-a-vis the similarity transformation until Maarek {198} developed a procedure which this author chooses to call the Semi-Analytical Bundle. Model coordinates obtained through analog methods are transformed into photo coordinates which are in turn refined for systematic errors computationally, before the bundle type of adjustment which will be discussed in the appropriate sections.

Analytical Methods

The effective correction of measured image coordinates for systematic errors forms one of the strong foundations for achieving high accuracy in analytical methods. Out of the four distinct analytical methods described in Table 1, only numerical radial triangulation lacks this foundation. Kenefick and others {192} reported an experience with the analytical approach to the Independent Geodetic Method. Strips and blocks were formed after corrections for systematic errors were applied to the image coordinates. A block of four strips (67 photos at scale 1:6000) was bridged using *X-Y* horizontal control comprised of distances and azimuths. The same block was also adjusted using conventional method. The independent geodetic method not only produced accuracies comparable to the conventional method, but overall savings in providing mapping control amounted to 33 percent compared to the conventional method. Two other analytical methods, Independent Models and Bundle Adjustment, will now be discussed in some detail.

Analytical Method of Independent Models

The primary distinction between the semi-analytical and fully analytical approaches to Independent Models is that, in the fully analytical approach, relative orientation is performed analytically using the image coordinates corrected for systematic effects. The analytical relative orientation is traditionally based on the coplanarity concept, although Bender {144} and Keller and Tewinkel {191} have shown that relative orientation can also be performed with the collinearity concept. Strip or block formation, and adjustment operations are the same as those of the semi-analytical method of independent models. There are, however, four deviations from this classical procedure.

The first deviation relates to the concept of weight constraints discussed by Case {151} and utilized by Ackermann {176a} in the development of PAT M-43 and also by Blais {261} in the SPACE-M program. By this concept photogrammetric model coordinates, perspective center coordinates, as well as terrestrial coordinates (known and unknown) are regarded as observations and, therefore, weighted accordingly and simultaneously adjusted. There are three implications of this rigorous approach which in the author's opinion are only justified in a fully analytical method of Independent Models. The first is that the ground control is no longer regarded as perfect. Secondly, it is possible to detect gross errors at the ground control points. The third implication has to do with the addition of more observation equations to Equation 3 to complete the adjustment. This has given rise to a large system of normal equations in which the normal coefficient matrix is symmetric, patterned, sparse, positive definite, and banded; the band width is defined as the maximum distance from the diagonal to the last non-zero elements of any row in the normal coefficient matrix. By taking advantage of the structure of this matrix, a partitioning procedure of the matrix into submatrices yields the reduced normal equations. In PAT-M-43 the models, pass points, and control points are ordered in their optimum sequence to achieve a minimum band width in the reduced normal equations, the solution of which is obtained by an algorithm called HYCHOL (HYper-CHOLesky), a direct method for a solution of a system of equations using sub-matrices as units for a Cholesky solution. It is particularly efficient for banded or banded/bordered matrix) {176b}. SPACE-M {261}, however, uses Cholesky's square root algorithm for the direct solution of the reduced normal equation after obtaining an automatic minimum band width. Ackermann's {176a} experience with the poor performance of iterative solutions of systems of normal equations, particularly conjugate gradient methods which will be discussed under bundle adjustment, influenced his choice of the direct method. (See Table 2 for other programs for Independent Models.)

The second deviation from classical procedure is the use of additional parameters to compensate for systematic image errors (third order effects) in the block adjustment with Independent Models. The need for special treatment of systematic image errors effects in the block has been confirmed by Ackermann {311} {176a}, who has demonstrated the general effectiveness of least-squares collocation for interpolation in improving the accuracy obtained by block adjustment with independent models. However, Ackermann {311} has found that least-squares collocation is limited in effectively dealing with such systematic image errors in the block; for example, maximum residual errors were hardly reduced by collocation. Ebner and Schneider {187} used the concept of "additional parameters" to compensate for third order systematic image errors due to model affine deformation, twisted models, and perspective center errors. Although these image errors are regarded as unknown parameters which may be common to any groups of models or to all models in the entire block, they are also treated as observations with appropriate weights so as to obviate the possibility of ill-conditioned normal equations due to highly correlated unknown parameters. According to Ebner and Schneider {187}, the accuracy of adjusted block coordinates is said to improve up to a factor of three by using the concept of "additional parameters."

TABLE 2. SOME COMPUTER PROGRAMS FOR PHOTOTRIANGULATION

Name or Acronym	Description	Computer Requirement	Author (Country)
ALBANY	Adjustment of Large Blocks with ANY number of photos, points, and images, using ANY photogrammetric measuring instrument and on ANY computer. Bundle adjustment and three dimensional independent model adjustment incorporated in one computer program.	120K bytes on IBM 360 or 100K octal words on CDC6600, Fortran IV	Erio [188] (U.S.A.)
GIANT	General Integrated Analytical Triangulation program; performs a least-squares adjustment of arbitrarily arranged and unconstrained blocks of frame photographs. Can adjust max. of 460 photos, 400 ground controls, and 9329 ground points.	1 BM 260/270 with 340 K bytes in Fortran IV.	Elassal [270] USGS (U.S.A.)
IMT	Independent Models Triangulation program which forms a strip or block from independently oriented (relative) photogrammetric models. Can handle 300 photos with 20 points per model.	Fortran on IBM 360/75, 252K bytes.	Ghosh and Morgan [273] (U.S.A.)
FORT BLOCK	Fortran program for BLOCK adjustment of photographs. Can handle 100 photos.	252K, bytes on IBM 360	Ohio State [298] (U.S.A.)
SAPGO—mfl	Simultaneous Adjustment of Photogrammetric and Geodetic Observations. Multiple focal lengths. Geodetic observations include horizontal angles, azimuth, elevation, etc. Computes min. band width solution of normal equations by Gaussian elimination with recursive Partitioning.	Fortran 400K bytes on IBM 360/751, UNIVAC 1108 and CDC 6500	Wong and Elphingstone [171] (U.S.A.)
MUSAT	Multiple—Station Analytical Triangulation program for simultaneous block adjustment of up to 2000 photos. Features include blunder elimination, data edit, control verification, and statistical analysis.	Fortran, 32K memory on IBM 7094. UNIVAC 1108.	Matos [293] (U.S.A.)
URELO	Unit Relative Orientation. Performs relative orientation with units of pair, triplets, etc., up to maximum of 8 photos each, and assembles the units into one integral strip.	Xerox Sigma S Computer 24K Words.	[129] Purdue Univ. (U.S.A.)
COMBAT II	Commercial Block Analytical Triangulation program which executes bundle adjustment with self calibration and error model. Banded bordered form of recursive partitioning. Automatic minimum band width for normal equation which is solved by BSOR.	Xerox Sigma 5 Computer, 24K of memories, Four tape units and disk.	DBA [147] (U.S.A.)
SURBAT	Simultaneous Unlimited, Rigorous Block Analytical Triangulation. Can handle max. of 450 strips with automatic editing error propagational unlimited no. of photos per strip. Algorithm same as COMBAT.	GE 635	DBA [148] (U.S.A.)
LOSAT LOBAT	Lunar Orbiter Strip Analytical Triangulation, Lunar Orbiter Block Analytical Triangulation Algorithm same as COMBAT.	GE 635	DBA [153] (U.S.A.)
(TPA)	Three Photo Aerial triangulation for block adjustment consists of a set of programs for coord. refinement, strip formation, and resection which provide input for block adjustment. Normal equations solved by Gaussial forward and backward elimination	IBM 360/370 in Fortram	NOS [156] [191] (U.S.A.)
PAT-M4	Program Aerial Triangulation with independent models for planimetric adjustment Anblock method of block adjustment. Normal equations solved by Hyper-Cholesky method uses Rodrigues-Caylary matrix.	minimum core capacity of 64K words on CDC 6600 on 256K on IBM 360/370	Stuttgart University Ackermann [109], [176], [256], [311] (W. Germany)

PAT-M43	Program Aerial Triangulation with independent models with succession of 4 and 3 parameters transformation, plan-height iterative adjustment. Automatic minimization of band width and solution of normal equation by Hyper-Cholesky method.	with additional external disk storage. Also on UNIVAC.	
PAT-M43-APR STATOS-LAKE	Program Aerial Triangulation with Independent Models as M-43, With auxiliary data from APR, statoscope, and lakes.		
PAT-B	Program Aerial Triangulation with Bundle adjustment.		
BAP	Bundle adjustment with Additional Parameters and Self Calibration. Can handle 999 photos.	Fortran on IBM 370/158	Muller [348] (W. Germany)
ASP	Adjustment of Spatial Phototriangulation.	—	Mashimov [291] (U.S.S.R.)
HVB	Horizontal and Vertical Block adjustment of independent models using polynomial transformation. Can handle 200 or more models. Adjusts blocks consisting of strips or sections.	UNIVAC 1107 Algol	Holsen et al [280] (Norway)
SCHUT	Adjustment of strips and blocks by polynomial transformation. Iterative Gauss-seidel solution of the complete set of normal equations.	Fortran on IBM 360/370 256K bytes of core storage.	Schut [305] (Canada)
SPACE-M	Spatial Photogrammetric Adjustment for Control Extension, using independent models and auxiliary data. Very few restrictions about position and density of control. Can handle 1000 models. Normal equation solved by Choleskis square-root algorithm.	Fortran on CDC-CYBER system or IBM with 300 K octal words.	Blais [201] (Canada)
PABS	Polynomial Adjustment of Blocks of Strips formed from Independent models. Gauss-Jordan solution for normal equation. Can handle 120 models	Fortran on IBM 370/145 256K bytes of core storage	Ayeni [180] (Nigeria)
BATT-SAP	Block Adjustment by three-dimensional transformations—similarity or affine or projective transformation. Gaussi-Jordan solution of normal equation. Can handle independent models.		Ayeni [259] (Nigeria)
EMMBA	Extended Mathematical Model in Bundle Adjustment. Allows the handling of observations and unknown parameters in any order. Solution of normal equation by conjugate gradient method.	UNIVAC 1108	Haljala [278] (Finland)
BUEND	Adjustment by Bundle method. Program can also perform Calibration.	—	Schenk [164] (Australia)
STEREO	Swedish Block-Triangulation system with 7 parameters three-dimensional similarity Transformation. Iterative procedure for solving normal equation. Can handle up to 50 models.	CDC 6600 Fortran, 131K octal words.	Sigmark [203] (Sweden)
AN BLOCK	Anblock method of calculating blocks of aerial triangulation with special techniques of data storage, data ordering, and solution of normal equations.	—	Meulemester IGN [295] (France)
BUNDLE	Block adjustment by Bundle method; observations are reduced to quasi-observations. Normal equations solved by direct method—Gauss-Cholesky type	—	IGN [152a,b] (France)
IMT	Independent Model Triangulation by analytical method. Corrections for systematic distortion, relative orientation; strip or block formation and adjustment by similarity transformation using Thompson's method.	—	Ord. Survey Proctor [195] (G. Britain)

The third deviation from classical procedures in analytical independent models is the use of triplets, which was first demonstrated by Mikhail {218}. An overlap of one photograph is required in using the triplet, rather than the stereopair, as a basic unit in block triangulation. Mikhail holds that there is a reduction in the number of units and, therefore, in the number of parameters in the Block; for n photos in each of s strips, the number of parameters for the bundle method is $6ns$ while, for the conventional method of stereopairs, it is $7(n\text{-}1)s$ and for triplet method, $3.5(n\text{-}1)s$ or $3.5ns$, when n is odd or even, respectively {129}. The triplet method, which is also used at the National Ocean Survey {191}, has yielded accuracies comparable with those obtained by bundles and stereopairs {129}.

The fourth deviation from classical procedure relates to the use of auxiliary data in analytical Independent Models. This will be discussed in another section.

BUNDLE ADJUSTMENT

The evolution, application, and potential of the bundle adjustment has been fully discussed by Brown {147}. Only a brief description of the method is presented here with some particular emphasis on the problem of solving large systems of normal equations.

The classical bundle adjustment (see Table 1) is performed by making use of image coordinates, corrected for first and second order systematic effects, in the formation and solution of linear observation equation (Equation 3) and normal equations (Equation 4). This solution is conventionally obtained by minimizing sums of squares of weighted residuals, using Lagrange's multipliers. Note that both equations are based on the collinearity equations (Equation 2), that is,

$$x = -f\,\frac{(X - X_o)m_{11} + (Y - Y_o)m_{12} + (Z - Z_o)m_{13}}{(X - X_o)m_{31} + (Y - Z_o)m_{32} + (Z - Z_o)m_{33}}$$

$$y = -f\,\frac{(X - X_o)m_{21} + (Y - Y_o)m_{22} + (Z - Z_o)m_{23}}{(X - X_o)m_{31} + (Y - Z_o)m_{32} + (Z - Z_o)m_{33}} \qquad (2)$$

where x,y = photo coordinates in the image space;
m's = elements of the orthogonal rotation matrix in κ, ϕ, ω;
X, Y, Z = ground coordinates in the object space;
X_o, Y_o, Z_o = group coordinates of exposure station; and
f = camera constant.

then

$$\mathbf{V} + \overset{e}{\mathbf{B}}\overset{e}{\mathbf{\Delta}} + \overset{s}{\mathbf{B}}\overset{e}{\mathbf{\Delta}} + \mathbf{E} = 0 \qquad (3)$$

and

$$\begin{bmatrix} \dot{\mathbf{N}} & \bar{\mathbf{N}} \\ \bar{\mathbf{N}}' & \ddot{\mathbf{N}} \end{bmatrix} \begin{bmatrix} \overset{e}{\mathbf{\Delta}} \\ \overset{s}{\mathbf{\Delta}} \end{bmatrix} + \begin{bmatrix} \dot{\mathbf{u}} \\ \ddot{\mathbf{u}} \end{bmatrix} = 0 \qquad (4)$$

where $\mathbf{V}$ = residuals on photocoordinates

$\dot{\mathbf{N}} = \overset{e}{\mathbf{B}}{}'\mathbf{W}\overset{e}{\mathbf{B}}$ $\quad \dot{\mathbf{u}} = \overset{e}{\mathbf{B}}{}'\mathbf{WE}$

$\bar{\mathbf{N}} = \overset{e}{\mathbf{B}}{}'\mathbf{W}\overset{s}{\mathbf{B}}$ $\quad \ddot{\mathbf{u}} = \overset{s}{\mathbf{B}}{}'\mathbf{WE}$

$\ddot{\mathbf{N}} = \overset{s}{\mathbf{B}}{}'\mathbf{W}\overset{s}{\mathbf{B}}$ $\quad \mathbf{W}$ = weight matrix of photo coordinates x,y;

$\overset{e}{\mathbf{\Delta}}$ = correction vector to approx. exterior orientation elements;

$\overset{s}{\mathbf{\Delta}}$ = correction vector to approx. survey coordinates;

$\mathbf{E}$ = misclosure vector from Equation 1; and

$\overset{e}{\mathbf{B}}, \overset{s}{\mathbf{B}}$ = design coefficient matrices of $\overset{e}{\mathbf{\Delta}}, \overset{s}{\mathbf{\Delta}}$, respectively.

Equation 4 may be expressed concisely as

$$\mathbf{N\Delta} = \mathbf{U}. \qquad (5)$$

There is, however, a departure from the classical bundle adjustment. The concept of constraints [Case {151}], both functional and weight constraints, may be introduced to admit observations on exterior orientation parameters and ground survey coordinates. This yields two more observation equations (Equations 6 and 7), that is,

$$\overset{e}{\mathbf{V}} - \overset{e}{\mathbf{\Delta}} + \overset{e}{\mathbf{E}} = 0 \quad \text{(exterior orientation)} \qquad (6)$$

$$\overset{s}{\mathbf{V}} - \overset{s}{\mathbf{\Delta}} + \overset{s}{\mathbf{E}} = 0 \quad \text{(ground survey coordinates)} \qquad (7)$$

The resulting normal equations from Equations 3, 6, and 7 are given in Equation 8 by minimizing the sums of squares of weighted residuals using Lagrangian multipliers; that is,

$$\begin{bmatrix} \dot{\mathbf{N}} + \overset{e}{\dot{\mathbf{W}}} & \bar{\mathbf{N}} \\ \bar{\mathbf{N}}' & \ddot{\mathbf{N}} + \overset{s}{\dot{\mathbf{W}}} \end{bmatrix} \begin{bmatrix} \overset{e}{\mathbf{\Delta}} \\ \overset{s}{\mathbf{\Delta}} \end{bmatrix} = \begin{bmatrix} \dot{\mathbf{u}} - \overset{e}{\dot{\mathbf{W}}}\overset{e}{\mathbf{E}} \\ \ddot{\mathbf{u}} - \overset{s}{\dot{\mathbf{W}}}\overset{s}{\mathbf{E}} \end{bmatrix} \quad (8)$$

This development may be extended to the concept of self calibration by admitting observations on the unknown parameters of interior geometry. Equation 3 will, therefore, be replaced by Equation 9 plus one other observation equation (Equation 10), that is,

$$\mathbf{V} + \overset{e}{\mathbf{B}}\overset{e}{\mathbf{\Delta}} + \overset{s}{\mathbf{B}}\overset{e}{\mathbf{\Delta}} + \overset{i}{\mathbf{B}}\overset{i}{\mathbf{\Delta}} + \mathbf{E} \quad (9)$$

$$\overset{i}{\mathbf{V}} - \overset{i}{\mathbf{\Delta}} + \overset{i}{\mathbf{E}} \text{(for interior geometry)} \quad (10)$$

The normal equations for self calibration may be obtained from Equations 6, 7, 9, and 10 by minimizing sum of squares of weighted residuals; thus, we have Equation 11:

$$\begin{bmatrix} \overset{e}{\mathbf{B}}{}'\mathbf{W}\overset{e}{\mathbf{B}} + \overset{e}{\dot{\mathbf{W}}} & \overset{e}{\mathbf{B}}{}'\mathbf{W}\overset{s}{\mathbf{B}} & \overset{e}{\mathbf{B}}{}'\mathbf{W}\overset{i}{\mathbf{B}} \\ \overset{s}{\mathbf{B}}{}'\mathbf{W}\overset{e}{\mathbf{B}} & \overset{s}{\mathbf{B}}{}'\mathbf{W}\overset{s}{\mathbf{B}} + \overset{s}{\dot{\mathbf{W}}} & \overset{s}{\mathbf{B}}{}'\mathbf{W}\overset{i}{\mathbf{B}} \\ \overset{i}{\mathbf{B}}{}'\mathbf{W}\overset{e}{\mathbf{B}} & \overset{i}{\mathbf{B}}{}'\mathbf{W}\overset{s}{\mathbf{B}} & \overset{s}{\mathbf{B}}{}'\mathbf{W}\overset{i}{\mathbf{B}} + \overset{i}{\dot{\mathbf{W}}} \end{bmatrix} \begin{bmatrix} \overset{e}{\mathbf{\Delta}} \\ \overset{s}{\mathbf{\Delta}} \\ \overset{i}{\mathbf{\Delta}} \end{bmatrix} + \begin{bmatrix} \overset{e}{\mathbf{B}}\mathbf{W}\mathbf{E} - \overset{e}{\dot{\mathbf{W}}}\overset{e}{\mathbf{W}} \\ \overset{s'}{\mathbf{B}}\mathbf{W}\mathbf{E} - \overset{s}{\dot{\mathbf{W}}}\overset{s}{\mathbf{E}} \\ \overset{i'}{\mathbf{B}}\mathbf{W}\mathbf{E} - \overset{i}{\dot{\mathbf{W}}}\overset{i}{\mathbf{E}} \end{bmatrix} = 0 \quad (11)$$

It should be noted that the normal coefficient matrix from Equation 8 gives a sparse but banded-diagonal matrix while Equation 11 yields a banded-bordered matrix. The concept of bordering a normal coefficient matrix was proposed by Brown {147} for "block-invariant" parameters (that is parameters which are common to all photos in the block or strip) such as the interior orientation (projective) parameters which are recovered simultaneously along with the exterior orientation parameters and ground survey coordinates in self calibration.

There are two applications of the banded-bordered system in a bundle adjustment. The first concerns the concept of Simultaneous Adjustment of Photogrammetric and Geodetic Observations (SAPGO) {171}.

Geodetic observations, such as distances, azimuths, horizontal angles, latitudes, longitudes, etc., may be used to generate additional observation equations of the type shown in Equation 12:

$$\overset{e}{\mathbf{V}} + \overset{\theta}{\mathbf{G}}\overset{\theta}{\mathbf{\Delta}} + \overset{\theta}{\mathbf{E}} = 0 \quad (12)$$

It was observed by Brown {147} and Wong *et al.* {171} that the normal equations resulting from the simultaneous adjustment of Equations 3, 6, 7, and 12 (that is, SAPGO) do not conform to the banded diagonal structure in Equation 8, the "offending" parameters being the geodetic observed parameters introduced in Equation 12. The banded structure may be retained, however, by imposition of restrictions on the location of the geodetic observations in the block—a solution which in practice may not be feasible. Brown's {147} solution to this problem is that the "offending" parameters may be relegated to the border of the normal coefficient matrix (without causing ill-conditioning) by the method of "augmented bordering" with recursive partitioning, in which the bandwidth of the normal coefficient matrix is retained and the borderwidth is equal to the number of geodetic observation equations.

Another application of "bordering" relates to the concept of "Additional Parameters," which are intended to accommodate unknown systematic image errors in the block. If these errors are common to all photos, they may be treated as "block-invariant" parameters and the border of a banded-bordered system of normal equations may be used to accommodate these parameters. The bundle adjustment with additional parameters is discussed in {169} and {155}.

SOLUTION OF NORMAL EQUATIONS IN BUNDLE ADJUSTMENTS

It should be noted that there are certain special procedures, such as "cross-strip ordering" for photo numbering, appropriate "reordering of unknowns" in the normal equations, and the concept of "collapsing" (see {147} for details), that are important in generating a normal coefficient matrix which is diagonal dominant, of smallest possible dimension, with automatic minimum band width. It is also important to note that none of the normal equations discussed in this section is solved without deriving the reduced normal equations. The procedure for this may be illustrated by using Equation 4, which may be solved by eliminating $\overset{s}{\mathbf{\Delta}}$ to obtain the reduced normal equation Equation 13; that is,

$$(\dot{\mathbf{N}} - \bar{\mathbf{N}}\ddot{\mathbf{N}}^{-1}\bar{\mathbf{N}}')\,\overset{e}{\mathbf{\Delta}} = \bar{\mathbf{N}}\ddot{\mathbf{N}}^{-1}\ddot{\mathbf{u}}. \quad (13)$$

In order to perform a rigorous adjustment of large photogrammetric blocks with the bundle method, the problems of finding a solution for large systems of normal equations is a formidable task. The conventional solution of normal equations usually takes one of two forms; the direct methods and iterative methods.

Direct methods. Three of the best known methods in this group are the Gauss-Cholesky method {152a}, the Gauss-Jordan method {270}, and the Gaussian elimination with recursive partitioning method {307}. These methods adopt the "block elimination" procedure whereby the whole vector of unknowns $\boldsymbol{\Delta}$ in Equation 5 is partitioned into sub-vector $\boldsymbol{\Delta}_1, \boldsymbol{\Delta}_2, \boldsymbol{\Delta}_3, \ldots, \boldsymbol{\Delta}_n$ and their associated coefficient sub-matrices which are used as units in the computation. The on-diagonal sub-matrix **S** associated with $\boldsymbol{\Delta}_1$ is used as a pivot in the elimination procedure. After the elimination of one group of parameters, the normal coefficient matrix must retain a banded or banded-bordered structure. The Gauss-Cholesky block-elimination procedure {152a} involves only the decomposition of **S** into the product of a lower triangular matrix and its transpose, whereas in the Gauss-Jordan method the inverse of pivotal matrix **S** is computed. A good example of an efficient program constructed according to the principle of Gaussian elimination is SAPGO {171} in which the banded portion along the diagonal of the full inverse matrix is solved in the backward process. By adopting a recursive partitioning algorithm combined with direct access input/output techniques for storage and retrieval of data, a solution which is faster than Cholesky's square root method and Gauss' methods is achieved {307}. The major disadvantages of all direct solution method is that external storage requirements are high. In any case, the procedure is very difficult to program and cumbersome as well as time consuming to operate on account of the fact that the submatrices of varying orders in the normal equations are transfered very many times between internal and external storage devices. Some direct methods are also known to collapse due to the excessive amount of round-off error introduced in the solution.

Iterative methods. Among the iterative methods for the solution of normal equations in a large photogrammetric block which can be found in the literature, perhaps the best known include the Gauss-Seidel method, the method of Successive Over Relaxation (SOR), and the conjugated gradient (CG) method.

The Gauss-Seidel method may be achieved by expressing the coefficient **N** in Equation 5 in terms of three matrices as

$$\mathbf{N} = \mathbf{C} - \mathbf{E} - \mathbf{F} \qquad (14)$$

where $\mathbf{C}$ = diagonal matrix of **N** (the off-diagonals elements are zero);
$\mathbf{E}$ = lower triangular matrix of **N**, with zero diagonal; and
$\mathbf{F}$ = upper triangular matrix with zero diagonal.

Equation 5 becomes

$$(\mathbf{C} - \mathbf{E} - \mathbf{F})\boldsymbol{\Delta} = +\mathbf{U} \qquad (15)$$

It can be shown that the final solution is given by

$$\boldsymbol{\Delta}^{(k+1)} = \mathbf{C}^{-1}(\mathbf{E} + \mathbf{F})\boldsymbol{\Delta}^{(k)} + \mathbf{C}^{-1}\mathbf{U} \qquad (16)$$

Since **C** is a diagonal matrix, Equation 16 may be written for ease of computation as

$$\boldsymbol{\Delta}_i^{(k+1)} = \frac{1}{n_{ii}} \sum_{\substack{j=1 \\ j\neq i}}^{m} n_{ij}\boldsymbol{\Delta}_j^{(k)} + \frac{1}{n_{ii}}(\mathbf{U}_i);\ 1 \leq i \leq m. \qquad (17)$$

Equation 17 is Jacobi's method for $(k + 1)^{\text{th}}$ iteration. The expression for Gauss-Seidel may be obtained from Equation 16 as

$$\boldsymbol{\Delta}^{(k+)} = (\mathbf{C} - \mathbf{E})^{-1}\mathbf{F}\boldsymbol{\Delta}^{(k)} + (\mathbf{C} - \mathbf{E})^{-1}\mathbf{U}. \qquad (18)$$

For $(k + 1)^{\text{th}}$ approximation and

$$\boldsymbol{\Delta}_i^{(k+1)} = \frac{-1}{n_{ii}} \sum_{i=1}^{i=1} n_{ij}\boldsymbol{\Delta}_j^{(k+1)} + \sum_{j=i+1}^{m} n_{ij}\boldsymbol{\Delta}_j^{(k)} - \mathbf{U}_i;\ 1 \leq i \leq m \qquad (19)$$

which is Gauss-Seidel formula for $(k + 1)$ approximation.

Since Equation 17 uses only $\boldsymbol{\Delta}$ in computing $\boldsymbol{\Delta}^{(k+1)}$ in contrast to Equation 19, which uses $\boldsymbol{\Delta}_j^{(k+1)}$, it can be deduced that the Gauss-Seidel method will converge faster than Jacobi's method.

Unfortunately, the rate of convergence of the Jacobi and Gauss-Seidel methods for large systems of normal equations may still be slow. Among the various methods developed to accelerate Gauss-Seidel, perhaps the most popular is the Block Successive Order Relaxation (BSOR) method. The concept of Successive Order Relaxation (SOR) may be illustrated by Equation 20; that is,

$$\bar{\boldsymbol{\Delta}}^{(k+1)} = \boldsymbol{\Delta}^{(k)} + w(\boldsymbol{\Delta}^{(k+1)} - \boldsymbol{\Delta}^{(k)}) \qquad (20)$$

where $\bar{\boldsymbol{\Delta}}^{(k+1)} = (k + 1)^{\text{th}}$ approximate solution vector by SOR, $\boldsymbol{\Delta}^{(k+1)}$ and $\boldsymbol{\Delta}^{(k)}$ are the same as in Equation 18, and w is a suitable constant acceleration parameter.

For BSOR the coefficient matrix **N** and corresponding vectors $\boldsymbol{\Delta}$ and **U** in Equation 5 are partitioned into submatrices (blocks). Equations 19 and 20 may be combined to obtain the computational form of BSOR, Equation 21, that is,

$$\boldsymbol{\Delta}_i^{(k+1)} = \boldsymbol{\Delta}_i^{(k)} + w\mathbf{N}_{ii}^{-1}\left[\sum_{J=1}^{i=1} \mathbf{N}_{ij}\boldsymbol{\Delta}_j^{(k+1)} - \sum_{j=i+1}^{m} \mathbf{N}_{ij}\boldsymbol{\Delta}_j^{(k)} + (\mathbf{U}_i) - \mathbf{N}_{ii}\boldsymbol{\Delta}_i^{(k)}\right] \qquad (21)$$

(see {263b} for details).

Carlson and Haljala {264} recommend that, for a stable block with moderate geometric structure, the value of w should be between 1.8 and 1.9, whereas in very weak blocks w should vary between 1.9 and 1.94. Note that, if all the submatrices of **N** in Equation 21 are of dimension 1 by 1, BSOR degenerates into SOR method. It is also important to note that Gauss-Seidel is essentially a particular case of SOR when $w = 1$.

Varga {306b} has proved that the optimum acceleration parameter w can be computed from the explicit formula.

$$w = \frac{2}{1 + \sqrt{1 + \mathbf{G}}} \quad 0 < w < 2 \tag{22}$$

where $\mathbf{G} = (\mathbf{I} - \mathbf{C}^{-1}\mathbf{E})^{-1}\mathbf{C}^{-1}\mathbf{F}$.

From Equation 21 it is not difficult to see how the point iterative Gauss-Seidel process can be extended to the Block iterative Gauss-Seidel (BGS) process. In a two-photo simulated strip, Brown *et al.* {263b} found that the convergence rate of BSOR far exceeded that of BGS on the order of 40 to 50 times faster at 100 iterations.

Unlike the above methods, the conjugate gradient (CG) method does not require a device for accelerating the iteration. It also allows the handling of observations and unknowns in any order and there is no need for a special preliminary determination of the approximate values for unknown parameters. The normal coefficient matrix is not computed and therefore not stored in core. These advantages make it possible to solve for large systems of equations in core, without using slow and expensive external storage devices. The mathematical development for this method may be found in {278}. In the experiment performed by Carlson and Haljala {264} involving 5000 unknowns, the conjugate gradients method converged faster than BSOR in spite of the fact that the CG system was built up without using external storage during iteration. This is in contrast to BSOR which was built up with a large amount of external storage operations (disk and drum).

Iterative methods in general tend to be easier to program and require much less storage than direct methods. The only basic disadvantage in iterative methods is that it is difficult to determine the convergence criteria {134}. Table 2 contains a brief description of some program which make use of iterative and direct methods.

ORTHOGONAL TRANSFORMATIONS IN BUNDLE ADJUSTMENT

Yassa {310} has introduced yet another innovation to the solution of the problem of bundle adjustment in which the formation of normal equations is avoided by reducing very large systems of observation equations to smaller systems through repeated partitioning of a vector space into subspace and its orthogonal complements. Two variants of this method were discussed by Yassa {310}; the Gram-Schmidt ortho-normalization process and the Householder orthogonal transformation in which the estimates of parameters, their accuracies, and covariance matrix can be evaluated without any matrix inversion associated with the classical least-squares approach by means of normal equations.

RELATIONSHIP BETWEEN BUNDLE ADJUSTMENT AND INDEPENDENT MODELS

Some elements of the projective equations in Equation 1 may be redefined as follows: x,y,z = image coordinates i^{th} point in the image space; X,Y,Z = the corresponding ground coordinates in the object space; and X_o,Y_o,Z_o = the object space coordinates of some discrete point in the image space. If we linearize Equation 1, we have

$$\mathbf{V} = \overset{e}{\mathbf{A}}\overset{e}{\boldsymbol{\Delta}} + \overset{s}{\mathbf{A}}\overset{s}{\boldsymbol{\Delta}} + \quad \mathbf{E} = 0. \tag{23}$$

Equation 23 is essentially the same as Equation 3. This can be similarly demonstrated by substituting the calibrated focal length, $-f$, for z in Equation 1 to obtain Equation 24; that is,

$$\begin{bmatrix} x \\ y \\ -f \end{bmatrix} = S_i \; m_i \begin{bmatrix} X - X_o \\ Y - Y_o \\ Z - Z_o \end{bmatrix} \tag{24}$$

By multiplying the first and second rows of Equation 24 by $-f$, we obtain the collinearity Equation 2, which is linearized to obtain Equation 3 or Equation 23.

Erio {188} has incorporated both the three-dimensional similarity transformation, for independent model adjustment, and the bundle adjustment into one versatile computer program, ALBANY (see Table 2). In a comparative experiment using the International Society for Photogrammetry simulated test block, Erio {188} obtained improved results with ALBANY compared to those obtained by the bundle adjustment, independent models, and sequential type of adjustment.

IMAGE ERRORS

Five major sources of image errors which cause a departure from the idealized collinearity concept are depicted in Figure 1. These errors may be classified into three distinct types. The first types are of the first order and are due to instrumental fault and or operator's blunders. They are usually detected by calibration of the instrument and by repeated measurements. There are, however, a few computer algorithms for automatic detection of gross errors or blunders [{266} {299} {11} {262}].

Systematic errors due to lens distortion, film distortion, Earth's curvature, and atmospheric refraction are second order effects. Notable contributions in the mathematical modeling of second

TABLE 3. REPRESENTATIVE ACCURACIES OF SOME AEROTRIANGULATION METHODS

Method	Photo Scale	Photo Type	Camera Type	Areal Coverage (km^2)	F/S Over lap (%)	Instrument	No. of photo or Models
Fully Analytical IMT (Type 19, 20)**	(1/7800)	Wide Angle	Zeiss RMKA (15/23)	94.6	—	Zeiss PSK Comp	112
Fully Analytical IMT Add. Parameters (Type 23)**	(1/28000)	Wide Angle	Zeiss RMKA (15/23)	1250	—	Zeiss PSK Comp.	100
Semi-Analytical IMT (Type 12)**	(1/28000)	Wide Angle	Zeiss RMKA (15/23)	1500	(60/20)	Wild A10	175
Bundle with L.S. Collocation (Type 27)**	(1/4000)	Wide Angle	Zeiss RMK (15/23)	0.7	(60/40)	Zeiss PSK	8
Bundle with Add. Parameters (Type 28)**	(1/4000)	Wide Angle	RMKA (15/23)	4	(60/60)	Zeiss PSK	47
Classical Bundle (Type 25)**	(1/8000)	Wide Angle	RMKA (15/23)	104	(60/30)	Zeiss PSK	84
Self Calibration with Add. Parameters (Type 30)**	(1/17,500)	Super Wide Angle	RMKA (8.5/23)	32	(60/60)	B.B.A Comparator	27

order systematic errors have been made by Betram {41}, Saastamoinen {62} {63}, and Schut {66} on refraction; Brown {44} {45} {46}, Conrady {48}, and Washer {67} on lens distortion; and Lampton {56} and Ziemann {68} {69} {70} on film distortion. The research performed by Brown {46} and Merchant {59} are also invaluable contributions to the understanding of image geometry in the aerial camera system.

Third order effects are exemplified by lack of film flatness, anomalous image deformation through film transport, atmospheric turbulence, and printing and processing effects (see Figure 1). This group of image errors is very difficult to account for computationally owing to lack of adequate mathematical models. Emperical formulas commonly referred to as "additional parameters" are normally adopted for compensating third order effects, which are usually presented as residual systematic errors after block adjustment. The use of reseau marks has also been adopted as an effective means for removing local anomalous film distortion around the measured image point [{61} {64} {68}]. An attempt has also been made by Andrade {40} to model atmospheric refraction due to air turbulence around the entrance node of the aerial camera by using the "boundary layer theory." Furthermore, El Hakim and Faig {272} have experimented on the compensation for systematic errors by using spherical harmonics while Kraus and Mikhail {287} have successively used advanced least-squares collocation for predicting residual systematic errors.

THE USE OF AUXILIARY DATA

Auxiliary data used in aerial triangulation may be defined as data, acquired at the instant of exposure, which provide useful information about the position, scale, or orientation of the aerial photograph in space. Such data, which are useful in controlling the strip or block, may be acquired by means of auxiliary airborne "external sensors" such as the Airborne Profile Recorder (APR) (b_z, Z, scale); statoscope (b_z); horizon camera (ϕ,ω); solar periscope (ϕ,ω); gyro systems (ϕ,ω); inertial guidance system (ϕ,ω,κ, scale); Doppler navigation (scale); Aerodist, Shiran, Hiran Autotape, etc. (scale, X,Y); and the versatile ANQ-28 (ϕ,ω,κ,b_z, X,Y,Z, scale). Vertical auxiliary data may also be obtained from a lake which spreads across the photogrammetric block, thus providing equal elevation constraints or a "block-invariant" parameter.

There are four different ways of using auxiliary data to control a strip or block. The first is by an analog method, e.g., aeroleveling which was discussed earlier. Auxiliary data may also be introduced to the adjustment of a strip or block obtained through the semi-analytical method of independent models. Auxiliary data, for example, the X, Y, Z of projection center and the ϕ,ω of each photo, have been used in the adjustment of a block {34}. The fully analytical approach to Independent Models, described earlier, provides remarkable versatility for incorporating additional observation equations related to auxiliary data. Auxiliary vertical controls from Statoscope and APR have been incorporated with PAT-M-43 and tested in some aerotriangulation projects [{31} {27}]. The fourth method of introducing auxiliary data is through bundle adjustment through the use of weight constraints on the observed parameters, e.g., orientation elements from ANQ-28 which yields Equation 6. The greatest merit of auxiliary

TABLE 3—*Continued*

No. of Controls Hor.	No. of Controls Ver.	No. of Hor. per Photo or Model	No. of Ver per Photo or Model	No. of Check Pts. Hor.	No. of Check Pts. Ver.	R.M.S.E. at Check Pts. Hor cm	R.M.S.E. at Check Pts. Ver cm	Project Title	Investigator
27	—	4.2	—	77††	—	5.3 (4.4)*	—	Appenweier Project, W. Germany	Ackermann [311]
32	—	3.1	—	226†	—	17.6 (28) •	—	Block Frankfurt, W. Germany	Ebner and Schneider [187]
28	61	6.3	2.9	238†	240	42.2	46.7	Zurich Block, Switzerland	Parsic [248]
6	6	1.3	1.3	7	7	2.2 (3.2) ••	1.6 (2.9) ••	Ph.D. Thesis Casa Grande, U.S.A.	Rampal [163]
8	16	5.9	2.9	100	100	2.3 (6.1) •	3.0 (4.3) •	Jamijarvi Test Field, Finland	Salmenpera Anderson and Salvolainen [165]
—	—	—	—	72	—	5.0	—	Aalbong, Finland	Hvidegaard [333]
6	6	4.5	4.5	18	19	7.0	3.0	Atlanta Project, (U.S.A.)	Brown [320]

* Result after least squares collocation for interpolation
** See Table 1
† Perimeter control distribution at 26 (5km) interval
†† Perimeter (plus points inside) distribution
• Result obtained without Additional Parameters
•• Result without L.S. Collocation

data lies in the drastic reduction of control requirements both in terms of density and distribution. This relates to planning and economic consideration vis-a-vis accuracy requirements for the phototriangulation project. They are also useful in achieving a solution for Camera calibration in a relatively flat terrain {46} as an alternative to the method of "mixed ranges" proposed by Merchant {59}.

ACCURACY CAPABILITY OF PHOTOTRIANGULATION

Many investigations, notably by Ackermann {1} {2}, Kubik and Kure {19}, and Ebner {9} {10}, have been made into the theoretical accuracy of photogrammetric triangulation. The merit of such studies lies in the predetermination of expected accuracy under certain specified influencing conditions (factors) relating to (i) density, distribution, accuracy, and type of controls; (ii) quality, type, and scale of photographs; (iii) type and quality of camera lens; (iv) length of strip or size of block; (v) percentage forward and side overlaps; (vi) roughness and configuration of terrain; (vii) accuracy of photogrammetric instrument as well as observations; and, (viii) the method of aerotriangulation. (All of these factors must be considered along with project specifications in planning phototriangulation projects {76} {78}). The results from such theoretical accuracy studies have been a guide to photogrammetrists in detecting the presence of uncompensated systematic errors in the practical block adjustments whose accuracies did not conform with theoretical expectations. The outcome of the application of theoretical accuracies has led recently to an intensive investigation of additional parameters (see Table 1) to account for uncompensated systematic image errors in the block, as exemplified by work done by Ebner and Schneider {187}, Schut {168} {169}, Brown {147}, Bauer {142}, Bauer and Muller {143}, and Salmenpera *et al.* {165} (see Table 3 for discussion of improved results obtained by the use of additional parameters). The alternative approach to additional parameters for improving the accuracy of block adjustment is the method of Least Squares Collocation, first applied by Kraus and Mikhail

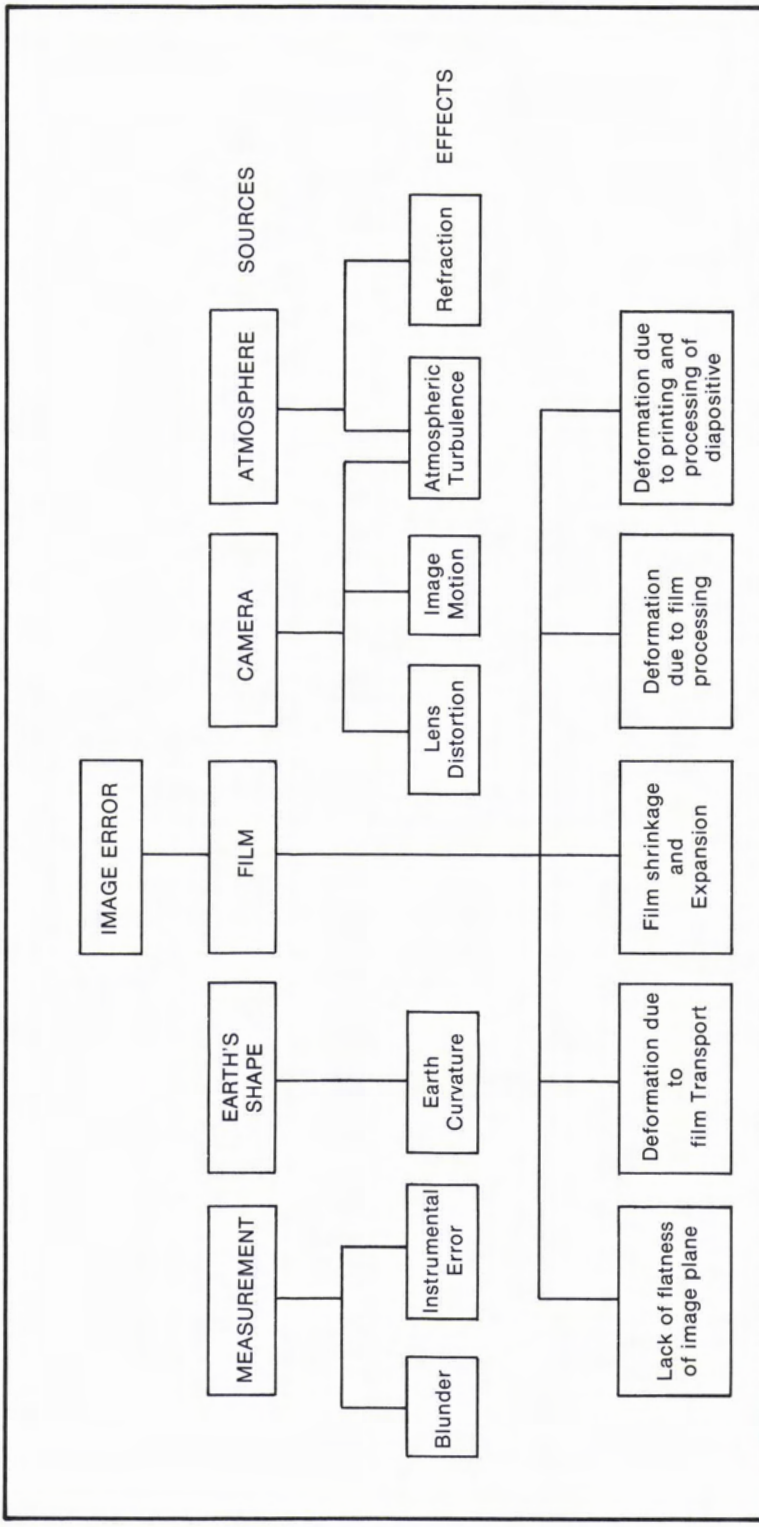

FIG. 1. Image errors—sources and effects.

{287}, in which the signal is filtered from the residual errors resulting from practical block adjustment (see Table 1). Ackermann {311} and Rampal {163} have successfully improved the block adjustment accuracies by using Least Squares Collocation (see Table 3). Some representative accuracies displayed in Table 3 confirm the capability of aerotriangulation as a geodetic tool (see Table 3). No attempt is made to compare the relative accuracies of the methods shown in Table 3 because aerotriangulation methods were not performed under identical conditions. For comparison of relative accuracies of some of these methods, see {188}.

Table 4. Applications of Phototriangulation

Application		Reference
1. Rectification		[323]
2. Controlling mosaic construction and planimetric map revision		[106b]
3. Control extension for topo mapping	Small Scale	[343]
	Medium Scale	[314]
	Large Scale	[324][328] [329][330]
4. Structural geology (estimation of fractures, slope stability, etc.)		[313]
5. Orthophoto products		[315]
6. Densification of trigonometric or triangulation or urban geodetic networks		[311] [320], [329] [329], [333], [341] [348], [352]
7. Glacier movement		[312]
8. Land-use mapping		[316]
9. Cadastral and large scale surveys		[176a] [318], [319] [338], [80] [81], [164]
10. Highway design and construction		[321], [331] [332], [347]
11. Ship building		[327]
12. Engineering structures (close-range)		[334] [344] [345] [345]
13. Three-dimensional microscopic mapping		[341] [343]
14. Convergent photo		[336], [95]
15. Lunar mapping		[335] [355], [322]
16. Side looking airborne radar (SLAR) imagery		[339]
17. Missile trajectory		[346]
18. Underwater mapping		[349]
19. Traffic accident and police work		[350]
20. Relief displacement and miscellaneous		[337], [351], [354]

Applications

For quite a long time the traditional application of phototriangulation has been in control extension for topographic mapping. However, the past few decades have witnessed new applications of phototriangulation (due to improved accuracy, capability, and economy) notably for intensification of trig- and geodetic networks {311} {320}, close-range applications for the calibration of engineering structures {344} {345}, lunar mapping {335}, ship building {327}, and microscopic mapping {341} {343}. Other applications of phototriangulation are listed in Table 4, with corresponding references.

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ACCURACY AND ERROR ANALYSIS

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(Received 29 May 1980; revised and accepted 1 March 1982)

Forthcoming Articles

R. G. Best, R. Fowler, D. Hause, and *M. Wehde,* Aerial Thermal Infrared Census of Canada Geese in South Dakota.

P. D. Carman, Aerial Camera Vibration.

Thomas L. Erb and *Warren R. Philipson,* Producing Stereo Teaching Aids from Aerial Photographs.

Charlotte M. Gurney and *John R. G. Townshend,* The Use of Contextual Information in the Classification of Remotely Sensed Data.

M. A. Hardisky, V. Klemas, and *R. M. Smart,* The Influence of Soil Salinity, Growth Form, and Leaf Moisture on the Spectral Radiance of *Spartina alterniflora* Canopies.

J. L. Heilman and *D. G. Moore,* Evaluating Depth to Shallow Groundwater Using Heat Capacity Mapping Mission (HCMM) Data.

John R. Jensen and *Michael E. Hodgson,* Remote Sensing Brightness Maps.

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Manmohan M. Trivedi, Clair L. Wyatt, and *David R. Anderson,* A Multispectral Approach to Remote Detection of Deer.

Kam W. Wong and *Youssuf Mustafa Siyam,* Accuracy of Earthwork Calculations from Digital Elevation Data.

Daniel K. Zigich and *Kenneth E. Kolm,* Evaluating the Effectiveness of Landsat Data as a Tool for Locating Buried Pre-Glacial Valleys in Eastern South Dakota.

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