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Wave Influences in Two-Media Photogrammetry

The deviations due to waves in assuming the boundary surface between air and water to be a horizontal plane are estimated, and a few correction methods of the approximation errors are proposed.

Introduction

Measurement of underwater features is becoming of greater importance to many fields such as coastal engineering and underwater geologic exploration. For this purpose, two-media photogrammetric mapping may be more effective than that by means of underwater photographs or traditional shipboard sounding methods, because two-media photographs cover far wider underwater areas.

Abstract: *In treating two-media photographs taken of coastal underwater areas, the boundary surface is usually approximated by a plane surface having the mean height of the wave crests and troughs. The objective of this paper is to estimate position errors due to this approximation and, further, to develop some techniques applicable to correction of the deviations, if they are not negligibly small in comparison with the photogrammetric accuracy usually attainable. First, the general approach to refraction calculation is described and simulation models were constructed for sinusoidal waves (long and short waves) and also for superposed waves. Then, the approximation errors for the numerical examples were calculated with the assumption of a plane air/water interface. The deviations for the actual sea surface were estimated from the characteristics of those for the waves employed, because the actual sea surface may be expressed by a Fourier series composed of many sinusoidal waves. It has been determined from the estimation that the approximation errors for the actual surface of the sea are not allowable in a deep underwater area (water depth > 10m). Also, the correction methods for those approximation errors are discussed briefly.*

The geometrical properties of two-media photographs have been investigated by Zaar (1948) and Rinner (1948) mainly in the case where the boundary surface is a plane. Also, the most interesting and important phenomenon has been determined to be the fact that the corresponding imaging rays in air, in general, do not intersect, even if the position and attitude at the exposure instant of a stereo pair of two-media photographs are reconstructed correctly. For this reason, many difficulties occur in optical-mechanical mapping with two-media pictures, which never appear in one-medium photogrammetry. Some practical and useful methods to find underwater points with optical-mechanical instruments have already been developed by Tewinkel (1963), Mori and Okamoto (1970a), Hoehle (1971), and Kreiling (1970). Further, the stereo pair has been tested and oriented with stereo instruments based on optical or mechanical projection by considering the y-parallax due to the bending of the imaging rays (Mori and Okamoto, 1970b, 1971).

Provided that the parameters describing the water surface and the orientations of the two-media photographs are known, an underwater point can be easily calculated from the corresponding image points on the stereo pair by use of the law of refraction. Thus, only the analytical orientation problem need be studied in the case where orientation points exist in water. This

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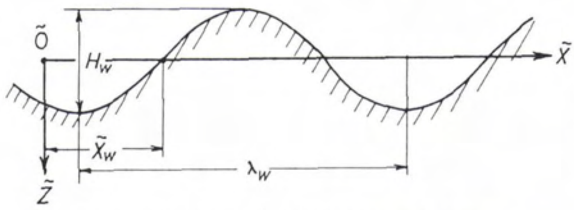

FIG. 1. The form of simple harmonic wave.

problem has been investigated by Rinner (1948, 1969), Schmutter and Bonfiglioli (1967), Hoehle (1971, 1972), Okamoto and Hoehle (1972), Okamoto and Mori (1973, 1974), Okamoto and Kuwahata (1973), and Girndt (1973). Also, many orientation techniques have already been developed for placid water conditions. The orientation theory derived by Okamoto *et al.* can be readily extended to an arbitrary boundary surface.

However, very little has been written that analyzes the photogrammetric influence of sea waves. Masry and MacRitchie (1980) investigated the practical characteristics of two-media photographs taken of coastal underwater areas. But the approximation errors (position errors due to approximating the sea surface by a plane) have not been fully studied and practical correction methods have not been discussed. In this paper, the general refraction calculation is described, the general approach to find underwater points with a stereo pair of two-media pictures is presented by means of direction cosines, the approximation errors are estimated for various types of waves, and the correction techniques of the deviations are discussed.

PREPARATIONS

The actual sea surface has generally a very complicated form and should be represented by the two-dimensional Fourier series. (Note: the conventional Fourier series is one-dimensional.) However, because the Fourier series is composed of many sinusoidal waves, we will begin with the investigation of a simple harmonic wave in estimating the approximation errors.

A simple harmonic wave can be expressed in the wave coordinate system $(\tilde{X},\tilde{Y},\tilde{Z})$ defined below in the form (see Figure 1)

$$\tilde{Z} = A \sin[-k(\tilde{X} - \tilde{X}_w)] \tag{1}$$

in which

$$A = H_w/2 \quad (H_w\text{: wave height})$$

$$k = 2\pi/\lambda_w \quad (\lambda_w\text{: wave length})$$

and $\tilde{X}_w$ indicates the $\tilde{X}$-coordinate of the point where the vertical displacement of the wave is considered to be zero. The ground coordinate system (X,Y,Z) is taken as a right-handed rectangular Cartesian system with its origin, O, at an arbitrary point over the sea surface. Also, a plane with the mean height of the wave crests and troughs is assumed to lie at the vertical distance, H, from the origin, O, in the ground coordinate system, which is considered to be parallel to the X-Y plane (see Figure 2). The origin, $\tilde{O}$, of the wave coordinate system $(\tilde{X},\tilde{Y},\tilde{Z})$ can be selected at an arbitrary point on the reference plane. Thus, it will be taken at the point where the Z-axis of the ground coordinate system intersects the plane with the mean height. Taking the wave coordinate system in such a way, the relationship can be described between the ground coordinate system (X,Y,Z) and the wave coordinate system $(\tilde{X},\tilde{Y},\tilde{Z})$ as

$$\begin{pmatrix} \tilde{X} \\ \tilde{Y} \\ \tilde{Z} \end{pmatrix} = \begin{pmatrix} \cos \tilde{\kappa} & \sin \tilde{\kappa} & 0 \\ -\sin \tilde{\kappa} & \cos \tilde{\kappa} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z - H \end{pmatrix} \tag{2}$$

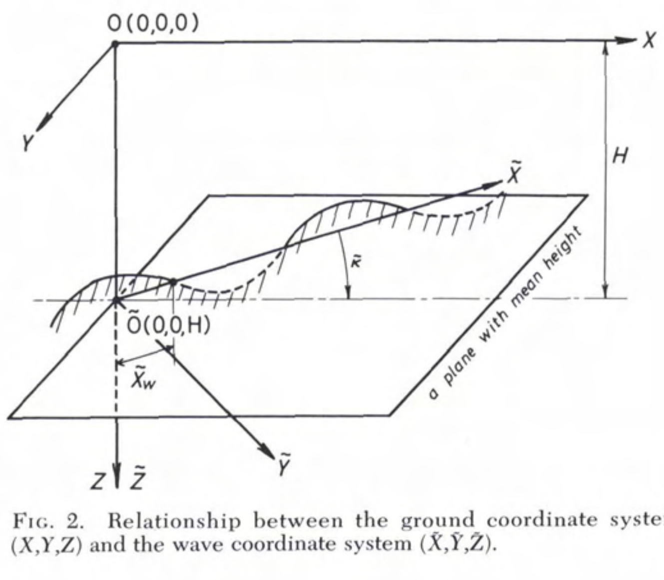

FIG. 2. Relationship between the ground coordinate system (X,Y,Z) and the wave coordinate system $(\tilde{X},\tilde{Y},\tilde{Z})$.

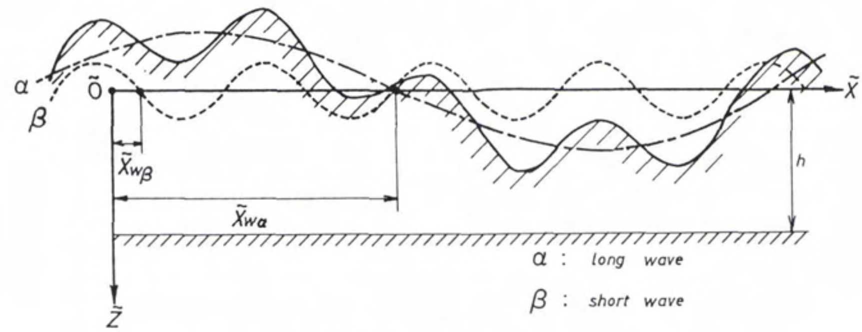

FIG. 3. Superposed wave profile.

where $\tilde{\kappa}$ denotes the direction of wave propagation with respect to the ground coordinate system. Equation 2 can be rewritten as

$$\begin{aligned} \tilde{X} &= X \cos \tilde{\kappa} + Y \sin \tilde{\kappa} \\ \tilde{Y} &= -X \sin \tilde{\kappa} + Y \cos \tilde{\kappa} \\ \tilde{Z} &= Z - H. \end{aligned} \tag{3}$$

By substituting Equation 3 into Equation 1, we get the equation of the simple harmonic wave expressed in the ground coordinate system (X,Y,Z) in the form

$$Z - H = A \sin[-k(X \cos \tilde{\kappa} + Y \sin \tilde{\kappa} - \tilde{X}_w)]. \tag{4}$$

Next, a superposed wave of a long wave and a short wave (a local wind wave) will be expressed in the ground coordinate system. The long wave has a much longer wave length than the water depth in coastal waters, while the wave length of the short wave is comparable to the water depth (see Figure 3). The equations of both waves are described in the wave coordinate system $(\tilde{X},\tilde{Y},\tilde{Z})$ as

$$\text{long wave:} \quad \tilde{Z}_\alpha = A_\alpha \sin[-k_\alpha(\tilde{X} - \tilde{X}_{w\alpha})] \tag{5}$$

$$\text{short wave:} \quad \tilde{Z}_\beta = A_\beta \sin[-k_\beta(\tilde{X} - \tilde{X}_{w\beta})]. \tag{6}$$

Thus, the equation of the superposed wave can be constructed in the form

$$\begin{aligned} \tilde{Z} = \tilde{Z}_\alpha + \tilde{Z}_\beta &= A_\alpha \sin[-k_\alpha(\tilde{X} - \tilde{X}_{w\alpha})] \\ &\quad + A_\beta \sin[-k_\beta(\tilde{X} - \tilde{X}_{w\beta})]. \end{aligned} \tag{7}$$

By substituting Equation 3 into Equation 7, we have

$$\begin{aligned} Z &= A_\alpha \sin[-k_\alpha(X \cos \tilde{\kappa} + Y \sin \tilde{\kappa} - \tilde{X}_{w\alpha})] \\ &\quad + A_\beta \sin[-k_\beta(X \cos \tilde{\kappa} + Y \sin \tilde{\kappa} - \tilde{X}_{w\beta})] + H \end{aligned} \tag{8}$$

under the assumption that the long and short waves propagate in the same direction.

Mathematically, the actual sea surface expressed by the two-dimensional Fourier series can also be described in the ground coordinate system (X,Y,Z). However, we will take an approach to estimate the approximation errors for the actual sea surface from the properties of those for the long and short waves, because the refraction calculation may prove too complicated and often impossible, due to the fact that many refraction points can exist for an underwater point in the case of a very complicated water surface.

GENERAL REFRACTION CALCULATION

Suppose that an underwater point $P(X,Y,Z)$, the water surface (the refractive interface), and the projection center $O_A(X_0,Y_0,Z_0)$ of a two-media photograph are given in the ground coordinate system. The refraction point $Q(\xi,\eta,\zeta)$, through which the imaging ray passes on the air/water interface, is sought as follows (see Figure 4): The optical path, L, is described in the form

$$\begin{aligned} L &= \sqrt{(\xi - X_0)^2 + (\eta - Y_0)^2 + (\zeta - Z_0)^2} \\ &\quad + N\sqrt{(X - \xi)^2 + (Y - \eta)^2 + (Z - \zeta)^2} \end{aligned} \tag{9}$$

where N indicates the refractive index of water. The imaging ray travels so that the optical path, L, approaches a minimum. By expressing the optical path, L, in the general form

$$L = G(\xi,\eta,\zeta),$$

the minimum condition can be described as

$$\begin{aligned} A &= \frac{\partial L}{\partial \xi} = \frac{\partial G}{\partial \xi} + \frac{\partial G}{\partial \zeta}\frac{\partial \zeta}{\partial \xi} = 0 \\ B &= \frac{\partial L}{\partial \eta} = \frac{\partial G}{\partial \eta} + \frac{\partial G}{\partial \zeta}\frac{\partial \zeta}{\partial \eta} = 0 \end{aligned} \tag{10}$$

because there is one constraint between the three unknowns ξ, η, and ζ, which is given from the equation of the boundary surface; i.e.,

$$\zeta = A \sin[-k(\xi \cos \tilde{\kappa} + \eta \sin \tilde{\kappa} - \tilde{X}_w)] + H \tag{11}$$

for a simple harmonic wave, or

$$\begin{aligned} \zeta &= A_\alpha \sin[-k_\alpha(\xi \cos \tilde{\kappa} + \eta \sin \tilde{\kappa} - \tilde{X}_{w\alpha})] \\ &\quad + A_\beta \sin[-k_\beta(\xi \cos \tilde{\kappa} + \eta \sin \tilde{\kappa} - \tilde{X}_{w\beta})] + H \end{aligned} \tag{12}$$

for a superposed wave, respectively. By solving Equation 10 with respect to two independent parameters ξ and η, we can find the refraction point

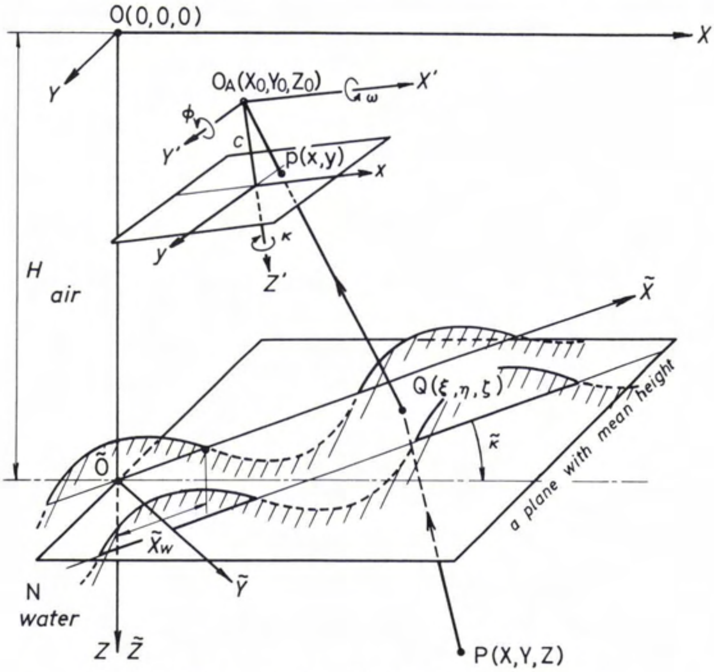

FIG. 4. Determination of refraction point $Q(\xi,\eta,\zeta)$.

$Q(\xi,\eta,\zeta)$. However, an iterative approach is necessary, because Equation 10 is non-linear with respect to ξ and η. The linearized form of Equation 10 becomes

$$
\begin{aligned}
A_0 + \left(\frac{\partial A}{\partial \xi}\right)_0 \Delta\xi + \left(\frac{\partial A}{\partial \eta}\right)_0 \Delta\eta &= 0 \\
B_0 + \left(\frac{\partial B}{\partial \xi}\right)_0 \Delta\xi + \left(\frac{\partial B}{\partial \eta}\right)_0 \Delta\eta &= 0
\end{aligned}
\tag{13}
$$

$$\xi = \xi_0 + \Delta\xi,\ \eta = \eta_0 + \Delta\eta$$

where the suffix 0 denotes approximation value.

The refraction point $Q(\xi,\eta,\zeta)$ and the exterior orientation elements $(\phi,\omega,\kappa,X_0,Y_0,Z_0)$ of the two-media photograph being given, the image point $p(x,y)$ can be obtained by means of the conventional collinearity equations

$$
\begin{aligned}
x &= c\frac{d_{11}(\xi - X_0) + d_{12}(\eta - Y_0) + d_{13}(\zeta - Z_0)}{d_{31}(\xi - X_0) + d_{32}(\eta - Y_0) + d_{33}(\zeta - Z_0)} \\
y &= c\frac{d_{21}(\xi - X_0) + d_{22}(\eta - Y_0) + d_{23}(\zeta - Z_0)}{d_{31}(\xi - X_0) + d_{32}(\eta - Y_0) + d_{33}(\zeta - Z_0)}
\end{aligned}
\tag{14}
$$

$$(\mathbf{D}_\phi\ \mathbf{D}_\omega\ \mathbf{D}_\kappa)^{\mathrm{T}} = \begin{pmatrix} d_{11} & d_{12} & d_{13} \\ d_{21} & d_{22} & d_{23} \\ d_{31} & d_{32} & d_{33} \end{pmatrix}$$

where $\mathbf{D}_\phi$, $\mathbf{D}_\omega$, and $\mathbf{D}_\kappa$ denote rotation matrices of the rotation elements ϕ, ω, κ about the Y, X, and Z axes, respectively, of the ground coordinate system (X,Y,Z).

Calculation of Underwater Point

The general approach to calculate underwater points analytically with a stereo pair of two-media pictures is described in this section. Also, we will assume that the two photos of the stereo pair are not taken simultaneously, because this situation is more general than simultaneous photography. By employing a superposed wave surface as an example of the water surface, this procedure will be precisely outlined as follows (see Figure 5):

At the exposure instant of the left picture, the superposed wave can be expressed as

$$
\begin{aligned}
Z = {} & A_\alpha \sin[-k_\alpha(X \cos \tilde{\kappa} + Y \sin \tilde{\kappa} - {}_1\tilde{X}_{w\alpha})] \\
& + A_\beta \sin[-k_\beta(X \cos \tilde{\kappa} + Y \sin \tilde{\kappa} - {}_1\tilde{X}_{w\beta})] + H.
\end{aligned}
\tag{15}
$$

However, the wave is propagating during the time interval between the two photos of the stereo pair of two-media photographs. Then, the equation of the superposed wave must be constructed for the right picture in the form

$$
\begin{aligned}
Z = {} & A_\alpha \sin[-k_\alpha(X \cos \tilde{\kappa} + Y \sin \tilde{\kappa} - {}_2\tilde{X}_{w\alpha})] \\
& + A_\beta \sin[-k_\beta(X \cos \tilde{\kappa} + Y \sin \tilde{\kappa} - {}_2\tilde{X}_{w\beta})] + H
\end{aligned}
\tag{16}
$$

where the propagation distance of the wave is

$$D_\alpha = {}_2\tilde{X}_{w\alpha} - {}_1\tilde{X}_{w\alpha}$$

for the long wave, and

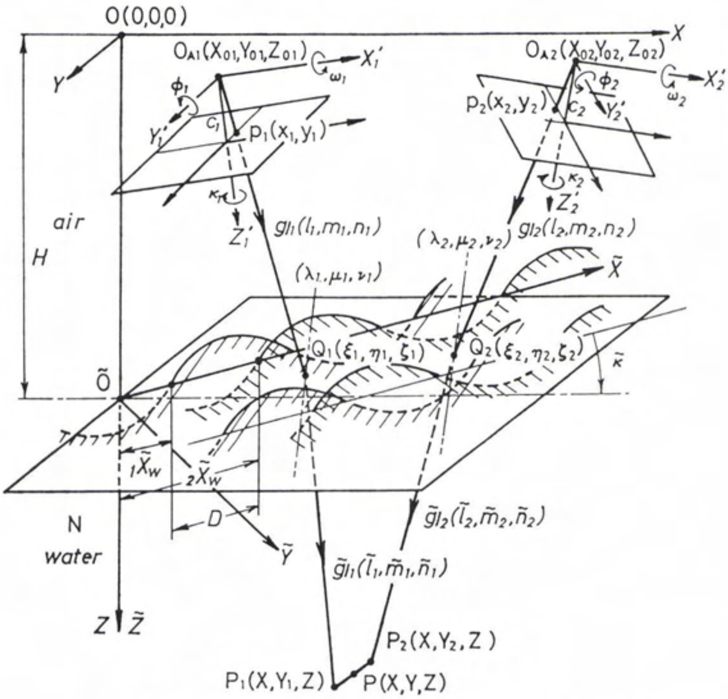

FIG. 5. Calculation of underwater point $P(X,Y,Z)$.

$$D_\beta = {}_2\tilde{X}_{w\beta} - {}_1\tilde{X}_{w\beta}$$

for the short wave, respectively.

The form of the water surface at the exposure instant of the stereo pair of two-media photographs being given in the ground coordinate system (X,Y,Z), we can calculate space coordinates of an underwater point from the corresponding image points $p_1(x_1,y_1)$ and $p_2(x_2,y_2)$ under the assumption that the exterior and interior orientation parameters are known. The corresponding rays $\mathbf{g}_1(l_1,m_1,n_1)$ and $\mathbf{g}_2(l_2,m_2,n_2)$ in air are expressed in the form

$$\mathbf{g}_1\colon \frac{X - X_{01}}{l_1} = \frac{Y - Y_{01}}{m_1} = \frac{Z - Z_{01}}{n_1} = \rho_1 \quad (17)$$

for the left picture, and

$$\mathbf{g}_2\colon \frac{X - X_{02}}{l_2} = \frac{Y - Y_{02}}{m_2} = \frac{Z - Z_{02}}{n_2} = \rho_2 \quad (18)$$

for the right one, respectively, where (l_1,m_1,n_1) and (l_2,m_2,n_2) denote direction cosines of the corresponding rays $\mathbf{g}_1$ and $\mathbf{g}_2$ in air, and also (X_{01},Y_{01},Z_{01}) and (X_{02},Y_{02},Z_{02}) are space coordinates of the projection center of the left and right pictures. For simplicity, a two-media photograph will be analyzed in the following discussions. For the derivation of the direction cosines (l,m,n), we will first find space coordinates of an image point $p(x,y)$ in the ground coordinate system (X,Y,Z). They are given in the form

$$\begin{bmatrix} X_p \\ Y_p \\ Z_p \end{bmatrix} = \mathbf{D}_\phi\, \mathbf{D}_\omega\, \mathbf{D}_\kappa \begin{bmatrix} x \\ y \\ c \end{bmatrix} + \begin{bmatrix} X_0 \\ Y_0 \\ Z_0 \end{bmatrix} \quad (19)$$

where c indicates the principal distance of the photograph. Further, the next expression will be introduced; i.e.,

$$\begin{bmatrix} \bar{X}_p \\ \bar{Y}_p \\ \bar{Z}_p \end{bmatrix} = \mathbf{D}_\phi\, \mathbf{D}_\omega\, \mathbf{D}_\kappa \begin{bmatrix} x \\ y \\ c \end{bmatrix} \quad (20)$$

which denotes the reduced transformed image coordinates of the image point $p(x,y)$ in the ground coordinate system. The direction cosines (l,m,n) can be described by means of Equation 20 as

$$l = \bar{X}_p \,/\, A,\ m = \bar{Y}_p \,/\, A,\ n = \bar{Z}_p \,/\, A \quad (21)$$

in which

$$A = \sqrt{\bar{X}_p^2 + \bar{Y}_p^2 + \bar{Z}_p^2}.$$

The refraction point $Q(\xi,\eta,\zeta)$ will be sought by using the following two equations:

- *the wave equation at the exposure instant of the photograph:*

$$\begin{aligned} Z = {} & A_\alpha \sin[-k_\alpha(X \cos \tilde{\kappa} + Y \sin \tilde{\kappa} - \tilde{X}_{w\alpha})] \\ & + A_\beta \sin[-k_\beta(X \cos \tilde{\kappa} + Y \sin \tilde{\kappa} - \tilde{X}_{w\beta})] + H \end{aligned} \quad (22)$$

- *the equation of the imaging ray* $\mathbf{g}$ *in air;*

$$\mathbf{g}: \frac{X - X_0}{l} = \frac{Y - Y_0}{m} = \frac{Z - Z_0}{n} = \rho. \quad (23)$$

By substituting Equation 23 into Equation 22, we have the equation with respect to ρ as

$$\begin{aligned}\rho n + Z_0 = A_\alpha \sin[-k_\alpha\{(\rho l + X_0)\cos\tilde{\kappa} \\ + (\rho m + Y_0)\sin\tilde{\kappa} - \tilde{X}_{w\alpha}\}] \\ + A_\beta \sin[-k_\beta\{(\rho l + X_0)\cos\tilde{\kappa} \\ + (\rho m + Y_0)\sin\tilde{\kappa} - \tilde{X}_{w\beta}\}] + H. \quad (24)\end{aligned}$$

By solving Equation 24 with respect to ρ, mathematically the refraction point $Q(\xi,\eta,\zeta)$ can be given in the form

$$\xi = \rho l + X_0,\ \eta = \rho m + Y_0,\ \zeta = \rho n + Z_0. \quad (25)$$

However, it is impossible to express the solution, ρ, in the explicit form. Thus, an iterative approach will be introduced. First, Equation 24 will be rewritten as

$$\begin{aligned}F(\rho) = A_\alpha \sin[-k_\alpha\{(\rho l + X_0)\cos\tilde{\kappa} \\ + (\rho m + Y_0)\sin\tilde{\kappa} - \tilde{X}_{w\alpha}\}] \\ + A_\beta \sin[-k_\beta\{(\rho l + X_0)\cos\tilde{\kappa} \\ + (\rho m + Y_0)\sin\tilde{\kappa} - \tilde{X}_{w\beta}\}] \\ + H - (\rho n + Z_0) = 0. \quad (26)\end{aligned}$$

Then, we will linearize Equation 26 with respect to ρ in the form

$$F_0 + \left(\frac{dF}{d\rho}\right)_0 \Delta\rho = 0 \quad (27)$$

$$\rho = \rho_0 + \Delta\rho.$$

The refraction point $Q(\xi,\eta,\zeta)$ being given, the equation of the imaging ray $\tilde{\mathbf{g}}$ in water can be constructed as follows. First, the direction cosines (λ,μ,ν) of the normal to the boundary surface will be sought. For this purpose, the water surface is expressed in the general form

$$Z - F(X,Y) = 0. \quad (28)$$

The normal direction to the air/water interface is given as

$$\operatorname{grad}(Z - F(X,Y)) = \left(-\frac{\partial F}{\partial X}, -\frac{\partial F}{\partial Y}, 1\right). \quad (29)$$

Also, the direction cosines (λ,μ,ν) of the normal becomes

$$(\lambda,\mu,\nu) = \left(-\left(\frac{\partial F}{\partial X}\right)/A, -\left(\frac{\partial F}{\partial Y}\right)/A,\ 1/A\right) \quad (30)$$

$$A = \sqrt{\left(\frac{\partial F}{\partial X}\right)^2 + \left(\frac{\partial F}{\partial Y}\right)^2 + 1}$$

For the case where the boundary surface is represented by the equation of the superposed wave, the normal direction can be described in the form

$$\operatorname{grad}(Z - F(X,Y)) = (e\cos\tilde{\kappa}, e\sin\tilde{\kappa}, 1) \quad (31)$$

where

$$\begin{aligned}e = A_\alpha k_\alpha \cos[-k_\alpha(\xi\cos\tilde{\kappa} + \eta\sin\tilde{\kappa} - \tilde{X}_{w\alpha})] \\ + A_\beta k_\beta \cos[-k_\beta(\xi\cos\tilde{\kappa} + \eta\sin\tilde{\kappa} - \tilde{X}_{w\beta})].\end{aligned}$$

Also, the direction cosines (λ,μ,ν) of the normal are

$$(\lambda,\mu,\nu) = \left[(e\cos\tilde{\kappa})/\sqrt{e^2+1}, (e\sin\tilde{\kappa})/\sqrt{e^2+1}, 1/\sqrt{e^2+1}\right]. \quad (32)$$

From the general law of refraction,

$$\begin{aligned}N\,\tilde{l} &= l - \lambda(\cos i - \sqrt{N^2 - 1 + \cos^2 i}) \\ N\,\tilde{m} &= m - \mu(\cos i - \sqrt{N^2 - 1 + \cos^2 i}) \quad (33) \\ N\,\tilde{n} &= n - \nu(\cos i - \sqrt{N^2 - 1 + \cos^2 i})\end{aligned}$$

where $(\tilde{l},\tilde{m},\tilde{n})$ denote the direction cosines of the imaging ray $\tilde{\mathbf{g}}$ in water and

$$\cos i = l\lambda + m\mu + n\nu.$$

The refraction point $Q(\xi,\eta,\zeta)$ and the direction cosines $(\tilde{l},\tilde{m},\tilde{n})$ of the imaging ray $\tilde{\mathbf{g}}$ in water having been given in the ground coordinate system (X,Y,Z), the corresponding imaging rays $\tilde{\mathbf{g}}_1(\tilde{l}_1,\tilde{m}_1,\tilde{n}_1)$ and $\tilde{\mathbf{g}}_2(\tilde{l}_2,\tilde{m}_2,\tilde{n}_2)$ in water can be described in the form

$$\tilde{\mathbf{g}}_1: \frac{X - \xi_1}{\tilde{l}_1} = \frac{Y - \eta_1}{\tilde{m}_1} = \frac{Z - \zeta_1}{\tilde{n}_1} \quad (34)$$

$$\tilde{\mathbf{g}}_2: \frac{X - \xi_2}{\tilde{l}_2} = \frac{Y - \eta_2}{\tilde{m}_2} = \frac{Z - \zeta_2}{\tilde{n}_2} \quad (35)$$

Then, the underwater point $P(X,Y,Z)$ can be calculated from Equations 34 and 35 by means of the following formulas:

$$\begin{aligned}Z &= \frac{\xi_2 - \xi_1 + [\tilde{l}_1\zeta_1/\tilde{n}_1 - \tilde{l}_2\zeta_2/\tilde{n}_2]}{\tilde{l}_1/\tilde{n}_1 - \tilde{l}_2/\tilde{n}_2} \\ X &= \xi_1 + \tilde{l}_1(Z - \zeta_1)/\tilde{n}_1 = \xi_2 + \tilde{l}_2(Z - \zeta_2)/\tilde{n}_2 \\ Y_1 &= \eta_1 + \tilde{m}_1(Z - \zeta_1)/\tilde{n}_1 \quad (36) \\ Y_2 &= \eta_2 + \tilde{m}_2(Z - \zeta_2)/\tilde{n}_2 \\ Y &= (Y_1 + Y_2)/2 \\ \Delta Y &= Y_2 - Y_1\end{aligned}$$

Estimation of Approximation Error

In usual mapping with a stereo pair of two-media photographs taken of coastal underwater areas, the air/water interface may be approximated by a plane surface having the mean height of the wave crests and troughs. We will estimate position errors due to this approximation for the actual sea surface which can be described by the two-dimensional Fourier series. For this purpose, the approximation errors of sinusoidal and superposed waves are first calculated for the following conditions (see Figure 6):

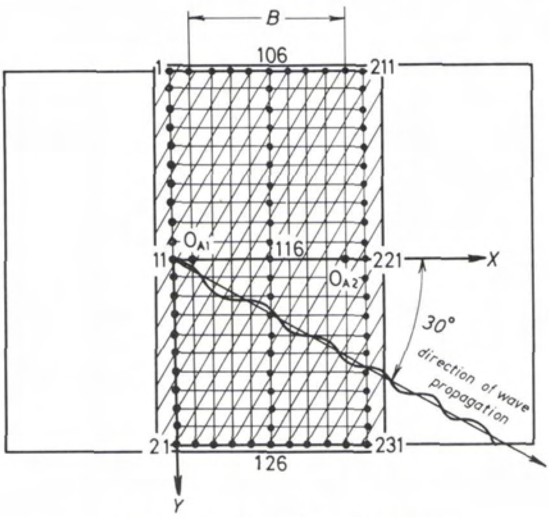

FIG. 6. Position of check points.

flying height	$H + h$ = 500m, 1500m
forward overlap	χ = 60%
base length	B = 264m, 800m
direction of wave propagation	$\bar{\kappa}$ = 30°
refractive index of water	N = 1.333
number of check points used	231
water depth	h = 5m, 10m, 20m
wave height in deep water	H_{w0} = 1m, 2m for long wave H_{w0} = 0.3m for short wave
period of wave	T = 10, 15 seconds for long wave T = 4 seconds for short wave
photography	simultaneous, non-simultaneous

Wave parameters (λ_w, H_w, v) can be determined from the water depth h, the wave height in deep water H_{w0}, and the period of wave T. The wave parameters used for the calculation are shown in Tables 1, 2, and 3. Furthermore, the value of $\bar{X}_w$ can be given arbitrarily for simultaneous photography. For the case of non-simultaneous photography, ${}_1\bar{X}_w$ for the left picture can have the arbitrary value, while the value of ${}_2\bar{X}_w$ for the right picture can be obtained from the wave speed, v, and the time interval between the exposure instants of the stereo pair of two-media photographs. The velocity of the aircraft is assumed to be about 90m/second.

In order to calculate the approximation errors for various types of simple harmonic and superposed waves, the refraction point $Q(\xi,\eta,\zeta)$ is first found by means of Equation 13. Also, the image point $p(x,y)$ is determined with Equation 14. Then, the underwater point $P(X,Y,Z)$ is sought by using Equation 36 in assuming the water surface by a plane. Finally, position errors due to the approximation are evaluated in the X, Y, and Z directions. The obtained values are shown in Tables 4 through 8. In these tables, the number i indicates the position of the underwater point where the maximum error, $E_{\max}$, occurred (see Figure 6). Also, the standard error, σ, is calculated by means of the following formulas:

$$\sigma_X = \sqrt{\frac{\sum_{i=1}^{231} \Delta X_i^2}{231}},\ \sigma_Y = \sqrt{\frac{\sum_{i=1}^{231} \Delta Y_i^2}{231}},\ \sigma_Z = \sqrt{\frac{\sum_{i=1}^{231} \Delta Z_i^2}{231}}$$

where

$$\Delta X_i = X_i(\text{calculated}) - X_i(\text{given})$$
$$\Delta Y_i = Y_i(\text{calculated}) - Y_i(\text{given})$$
$$\Delta Z_i = Z_i(\text{calculated}) - Z_i(\text{given})$$

TABLE 1. WAVE PARAMETERS FOR A LONG WAVE WITH H_{wo} = 1 M AND T = 10 SECONDS

wave parameters λ_w, v, H_w	water depth h 5 m	10 m	20 m
wave length λ_w	68 m	92 m	121 m
wave speed v	6.8 m/sec	9.2 m/sec	12.1 m/sec
wave height H_w	1.11 m	0.98 m	0.92 m

TABLE 2. WAVE PARAMETERS FOR A LONG WAVE WITH H_{wo} = 2 M AND T = 15 SECONDS

wave parameters λ_W, v, H_w	water depth h 5 m	10 m	20 m
wave length λ_w	103 m	144 m	197 m
wave speed v	6.9 m/sec	9.6 m/sec	13.2 m/sec
wave height H_w	2.65 m	2.28 m	2.00 m

TABLE 3. WAVE PARAMETERS FOR A SHORT WAVE WITH $H_{wo} = 0.3$ M AND $T = 4$ SECONDS

water depth h / wave parameters λ_w, v, H_w	5 m	10 m	20 m
wave length λ_w	22 m	25 m	25 m
wave speed v	5.6 m/sec	6.2 m/sec	6.2 m/sec
wave height H_w	0.28 m	0.29 m	0.30 m

From Tables 4, 5, and 6, the following properties of the approximation errors have been revealed for simple harmonic waves (long and short waves):

- The approximation errors for long waves are almost independent of the water depth.
- The approximation errors for short waves, however, increases drastically with the water depth.
- The fluctuation of the approximation errors due to wave propagation is comparatively large.
- The flying height has almost no influence on the magnitude of the approximation error.
- The maximum error occurs around the edge of the model.
- Simultaneous photography does not always give the minimum approximation errors.
- When the wave length becomes smaller than the water depth, many refraction points can exist, for an underwater point, which satisfy the law of refraction (see Figure 7).
- The approximation errors distribute almost symmetrically with respect to the zero-axis. However, they do not follow a normal distribution (see Figures 8 and 9).

Also, Tables 7 and 8 have clarified the following characteristics of the approximation errors for superposed waves:

- In shallow water areas (water depth < 10m), the approximation errors are mainly caused by a long wave.
- On the other hand, a short wave has a much greater influence on the approximation errors in deep water areas (water depth >10m).
- The maximum approximation error in height is nearly the sum of those of the long and short waves.

Further, we will estimate the approximation errors of a superposed wave (Table 8) in comparison

TABLE 4. APPROXIMATION ERRORS FOR A LONG WAVE WITH $H_{wo} = 1.0$ M AND $T = 10$ SECONDS

[m]

		flying height $H + h = 500$ m					flying height $H + h = 1{,}500$ m				
water depth h	photography	$\bar{X}_w$		NO.	E_{max}	σ	$\bar{X}_w$		NO.	E_{max}	σ
5	A	${}_1\bar{X}_w = 17$	X	85	0.08	0.05	${}_1\bar{X}_w = 17$	X	126	0.08	0.04
		${}_2\bar{X}_w = 17$	Y	211	0.07	0.03	${}_2\bar{X}_w = 17$	Y	21	0.06	0.03
			Z	231	0.28	0.15		Z	231	0.31	0.15
	B	${}_1\bar{X}_w = 17$	X	231	0.08	0.03	${}_1\bar{X}_w = 17$	X	146	0.08	0.05
		${}_2\bar{X}_w = 38$	Y	231	0.21	0.06	${}_2\bar{X}_w = 80$	Y	20	0.11	0.04
			Z	231	0.63	0.31		Z	231	0.25	0.11
10	A	${}_1\bar{X}_w = 23$	X	84	0.10	0.06	${}_1\bar{X}_w = 23$	X	84	0.10	0.06
		${}_2\bar{X}_w = 23$	Y	21	0.09	0.04	${}_2\bar{X}_w = 23$	Y	17	0.08	0.04
			Z	2	0.26	0.12		Z	227	0.23	0.12
	B	${}_1\bar{X}_w = 23$	X	21	0.11	0.05	${}_1\bar{X}_w = 23$	X	148	0.10	0.06
		${}_2\bar{X}_w = 50$	Y	1	0.21	0.07	${}_2\bar{X}_w = 104$	Y	64	0.12	0.06
			Z	1	0.61	0.35		Z	229	0.27	0.11
20	A	${}_1\bar{X}_w = 30$	X	147	0.15	0.09	${}_1\bar{X}_w = 30$	X	84	0.14	0.09
		${}_2\bar{X}_w = 30$	Y	20	0.14	0.06	${}_2\bar{X}_w = 30$	Y	20	0.14	0.06
			Z	2	0.26	0.10		Z	231	0.23	0.10
	B	${}_1\bar{X}_w = 30$	X	211	0.15	0.07	${}_1\bar{X}_w = 30$	X	126	0.14	0.08
		${}_2\bar{X}_w = 66$	Y	231	0.28	0.10	${}_2\bar{X}_w = 138$	Y	21	0.24	0.08
			Z	231	0.84	0.44		Z	230	0.36	0.17

A; simultaneous photography
B; non-simultaneous photography

TABLE 5. APPROXIMATION ERRORS FOR A LONG WAVE WITH H_{wo} = 2.0 M AND T = 15 SECONDS [m]

		flying height $H + h = 500$ m					flying height $H + h = 1{,}500$ m				
water depth h	photography	$\hat{X}_w$		No.	E_{max}	σ	$\hat{X}_w$		No.	E_{max}	σ
		${}_1\hat{X}_w = 26$	X	105	0.12	0.07	${}_1\hat{X}_w = 26$	X	128	0.11	0.07
5	A	${}_2\hat{X}_w = 26$	X	19	0.11	0.04	${}_2\hat{X}_w = 26$	Y	190	0.11	0.04
			Z	1	0.74	0.38		Z	211	0.68	0.39
		${}_1\hat{X}_w = 26$	X	19	0.14	0.05	${}_1\hat{X}_w = 26$	X	20	0.17	0.08
	B	${}_2\hat{X}_w = 47$	Y	22	0.25	0.08	${}_2\hat{X}_w = 89$	Y	231	0.45	0.15
			Z	21	1.03	0.57		Z	211	0.93	0.46
		${}_1\hat{X}_w = 36$	X	148	0.15	0.09	${}_1\hat{X}_w = 36$	X	84	0.15	0.09
10	A	${}_2\hat{X}_w = 36$	Y	41	0.13	0.05	${}_2\hat{X}_w = 36$	Y	213	0.12	0.05
			Z	2	0.61	0.32		Z	230	0.57	0.32
		${}_1\hat{X}_w = 36$	X	230	0.15	0.07	${}_1\hat{X}_w = 36$	X	2	0.16	0.09
	B	${}_2\hat{X}_w = 66$	Y	230	0.28	0.09	${}_2\hat{X}_w = 126$	Y	21	0.53	0.17
			Z	231	1.00	0.57		Z	2	1.05	0.50
		${}_1\hat{X}_w = 49$	X	84	0.19	0.12	${}_1\hat{X}_w = 49$	X	126	0.19	0.11
20	A	${}_2\hat{X}_w = 49$	Y	213	0.16	0.07	${}_2\hat{X}_w = 49$	Y	20	0.17	0.07
			Z	3	0.52	0.26		Z	230	0.51	0.26
		${}_1\hat{X}_w = 49$	X	2	0.18	0.10	${}_1\hat{X}_w = 49$	X	20	0.23	0.10
	B	${}_2\hat{X}_w = 88$	Y	1	0.30	0.11	${}_1\hat{X}_w = 166$	Y	43	0.51	0.21
			Z	210	0.99	0.58		Z	230	1.23	0.68

A; simultaneous photography
B; non-simultaneous photography

TABLE 6. APPROXIMATION ERRORS FOR A SHORT WAVE WITH H_{wo} = 0.3 M AND T = 4 SECONDS [m]

		flying height $H + h = 500$ m					flying height $H + h = 1{,}500$ m				
water depth h	photography	$\hat{X}_w$		No.	E_{max}	σ	$\hat{X}_w$		No.	E_{max}	σ
		${}_1\hat{X}_w = 5$	X	147	0.06	0.04	${}_1\hat{X}_w = 5$	X	168	0.06	0.04
5	A	${}_2\hat{X}_w = 5$	Y	63	0.05	0.02	${}_2\hat{X}_w = 5$	Y	42	0.06	0.03
			Z	1	0.07	0.03		Z	230	0.09	0.03
		${}_1\hat{X}_w = 5$	X	231	0.06	0.03	${}_1\hat{X}_w = 5$	X	147	0.06	0.04
	B	${}_2\hat{X}_w = 20$	Y	189	0.13	0.05	${}_2\hat{X}_w = 50$	Y	21	0.05	0.02
			Z	2	0.32	0.18		Z	1	0.09	0.04
		${}_1\hat{X}_w = 6$	X	146	0.10	0.06	${}_1\hat{X}_w = 6$	X	105	0.10	0.07
10	A	${}_2\hat{X}_w = 6$	Y	21	0.15	0.06	${}_2\hat{X}_w = 6$	Y	106	0.12	0.06
			Z	1	0.27	0.14		Z	231	0.23	0.12
		${}_1\hat{X}_w = 6$	X	231	0.12	0.05	${}_1\hat{X}_w = 6$	X	105	0.11	0.06
	B	${}_2\hat{X}_w = 24$	Y	231	0.28	0.10	${}_2\hat{X}_w = 60$	Y	169	0.08	0.04
			Z	231	0.61	0.35		Z	22	0.17	0.11
		${}_1\hat{X}_w = 6$	X	20	0.22	0.11	${}_1\hat{X}_w = 6$	X	1	0.22	0.12
20	A	${}_2\hat{X}_w = 6$	Y	21	0.44	0.17	${}_2\hat{X}_w = 6$	Y	140	0.40	0.17
			Z	23	0.97	0.56		Z	189	0.95	0.57
		${}_1\hat{X}_w = 6$	X	2	0.24	0.10	${}_1\hat{X}_w = 6$	X	42	0.21	0.13
	B	${}_2\hat{X}_w = 24$	Y	168	0.51	0.21	${}_2\hat{X}_w = 60$	Y	84	0.18	0.09
			Z	22	1.30	0.73		Z	221	0.36	0.21

A; simultaneous photography
B; non-simultaneous photography

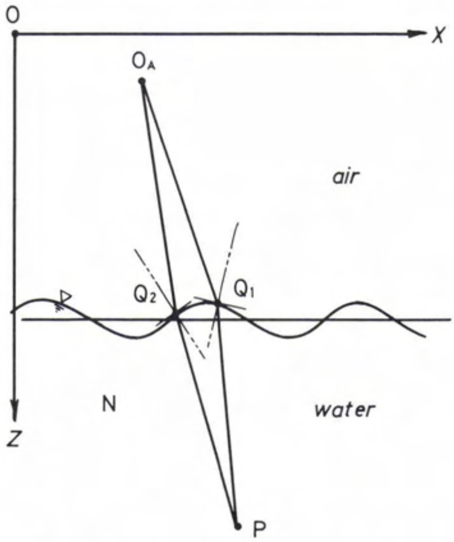

FIG. 7. Many refraction points for an underwater point.

with the accuracy usually attainable in photogrammetry. Suppose that the photogrammetric accuracy required is 1/10,000 in the X and Y coordinates and 1/5,000 in the Z coordinate: then,

$$\sigma_X/(H + h) \text{ or } \sigma_Y/(\mathrm{H} + \mathrm{h}) < 1/10{,}000$$
$$\sigma_Z/(H + h) < 1/5{,}000.$$

These indicate that the approximation errors must be in the range

$$\sigma_X \text{ or } \sigma_Y < 0.05\mathrm{m}$$
$$\sigma_Z < 0.10\mathrm{m}$$

for the flying height $H + h = 500$m, and

$$\sigma_X \text{ or } \sigma_Y < 0.15\mathrm{m}$$
$$\sigma_Z < 0.30\mathrm{m}$$

for the flying height $H + h = 1500$m, respectively. It is obvious from Table 8 that the approximation errors do not satisfy the above requirement for the case of the flying height $H + h = 500$m. On the other hand, the planimetric accuracy obtained is nearly the allowable for the case of $H + h =$

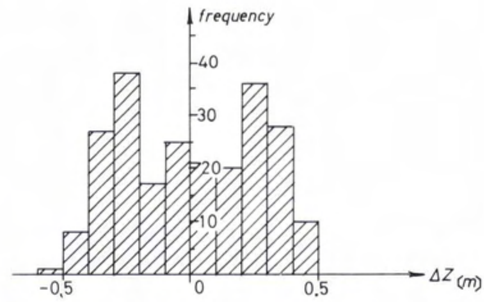

FIG. 8. Histogram of the approximation errors ΔZ for a long wave with $H_{w0} = 2.0$m and $h = 20$m in the case of the flying height $H + h = 1500$m (simultaneous photography).

1500m, while the approximation errors in height may be negligibly small only in shallow water areas (water depth < 10m) for simultaneous photography. The height accuracy obtained becomes worse than that required in deep water areas, even when simultaneous photography is taken. Also, it may not be tolerable even in shallow water areas for the case of non-simultaneous photography.

From the conclusions obtained above, we will estimate the approximation errors for the actual sea surface in the case of the flying height $H + h =$ 1500m, because photography from the low altitude is never effective in the analysis of two-media photographs taken of coastal water areas. Based on the following remarks:

- The form of a short wave may not deviate greatly from that of a sinusoidal wave even in shallow water areas (water depth ≃ 5m) (see Table 3).
- The form of a long wave may deviate greatly from that of a simple harmonic wave in coastal waters. However, it can be approximated with many sinusoidal waves.
- Ripples on the actual sea surface cause a disturbance of the image, because many refraction points may exist for an underwater point.
- Generally, the superposition of many sinusoidal waves may increase the approximation errors.

We can conclude regarding the approximation errors for the actual sea surface that

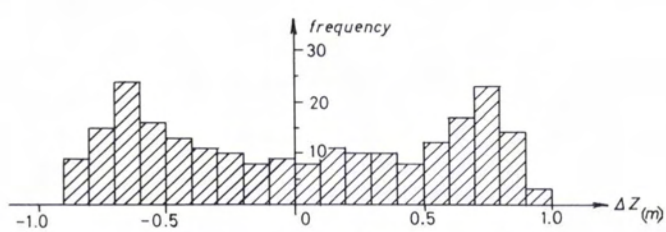

FIG. 9. Histogram of the approximation errors ΔZ for a short wave with $H_{w0} =$ 0.3m and $h = 20$m in the case of the flying height $H + h = 1500$m (simultaneous photography).

TABLE 7. APPROXIMATION ERRORS FOR A SUPERPOSED WAVE (LONG WAVE WITH $H_{wo} = 1.0$ M AND $T = 10$ SECONDS AND SHORT WAVE WITH $H_{wo} = 0.3$ M AND $T = 4$ SECONDS) [m]

		flying height $H + h = 500$ m					flying height $H + h = 1{,}500$ m				
water depth h	photography	$\bar{X}_w$		No.	E_{max}	σ	$\bar{X}_w$		No.	E_{max}	σ
	A	${}_1\bar{X}_{w\alpha} = 17$	X	125	0.13	0.06	${}_1\bar{X}_{w\alpha} = 17$	X	168	0.13	0.05
		${}_2\bar{X}_{w\alpha} = 17$	Y	63	0.11	0.04	${}_2\bar{X}_{w\alpha} = 17$	Y	42	0.12	0.04
		${}_1\bar{X}_{w\beta} = 5$	Z	43	0.35	0.16	${}_1\bar{X}_{w\beta} = 5$	Z	231	0.34	0.15
5		${}_2\bar{X}_{w\beta} = 5$					${}_2\bar{X}_{w\beta} = 5$				
	B	${}_1\bar{X}_{w\alpha} = 17$	X	212	0.13	0.04	${}_1\bar{X}_{w\alpha} = 17$	X	168	0.13	0.06
		${}_2\bar{X}_{w\alpha} = 38$	Y	212	0.25	0.08	${}_2\bar{X}_{w\alpha} = 80$	Y	42	0.15	0.04
		${}_1\bar{X}_{w\beta} = 5$	Z	44	0.80	0.36	${}_1\bar{X}_{w\beta} = 5$	Z	231	0.33	0.10
		${}_2\bar{X}_{w\beta} = 20$					${}_2\bar{X}_{w\beta} = 50$				
	A	${}_1\bar{X}_{w\alpha} = 23$	X	126	0.20	0.09	${}_1\bar{X}_{w\alpha} = 23$	X	84	0.20	0.09
		${}_2\bar{X}_{w\alpha} = 23$	Y	21	0.23	0.07	${}_2\bar{X}_{w\alpha} = 23$	Y	106	0.18	0.07
		${}_1\bar{X}_{w\beta} = 6$	Z	2	0.47	0.18	${}_1\bar{X}_{w\beta} = 6$	Z	8	0.42	0.17
10		${}_2\bar{X}_{w\beta} = 6$					${}_2\bar{X}_{w\beta} = 6$				
	B	${}_1\bar{X}_{w\alpha} = 23$	X	215	0.18	0.07	${}_1\bar{X}_{w\alpha} = 23$	X	84	0.20	0.09
		${}_2\bar{X}_{w\alpha} = 50$	Y	210	0.44	0.13	${}_2\bar{X}_{w\alpha} = 104$	Y	21	0.19	0.07
		${}_1\bar{X}_{w\beta} = 6$	Z	3	1.16	0.50	${}_1\bar{X}_{w\beta} = 6$	Z	1	0.39	0.15
		${}_2\bar{X}_{w\beta} = 24$					${}_2\bar{X}_{w\beta} = 60$				
	A	${}_1\bar{X}_{w\alpha} = 30$	X	213	0.32	0.14	${}_1\bar{X}_{w\alpha} = 30$	X	215	0.31	0.15
		${}_2\bar{X}_{w\alpha} = 30$	Y	212	0.42	0.18	${}_2\bar{X}_{w\alpha} = 30$	Y	106	0.47	0.18
		${}_1\bar{X}_{w\beta} = 6$	Z	189	1.05	0.57	${}_1\bar{X}_{w\beta} = 6$	Z	208	1.05	0.58
20		${}_2\bar{X}_{w\beta} = 6$					${}_2\bar{X}_{w\beta} = 6$				
	B	${}_1\bar{X}_{w\alpha} = 30$	X	9	0.34	0.12	${}_1\bar{X}_{w\alpha} = 30$	X	169	0.34	0.15
		${}_2\bar{X}_{w\alpha} = 66$	Y	231	0.71	0.23	${}_2\bar{X}_{w\alpha} = 138$	Y	106	0.35	0.12
		${}_1\bar{X}_{w\beta} = 6$	Z	231	1.85	0.86	${}_1\bar{X}_{w\beta} = 6$	Z	219	0.65	0.27
		${}_2\bar{X}_{w\beta} = 24$					${}_2\bar{X}_{w\beta} = 60$				

A; simultaneous photography
B; non-simultaneous photography

- The approximation errors in the X and Y coordinates may be negligibly small and need not be corrected.
- In the case of non-simultaneous photography, the approximation errors in height may not be in the allowable range and must be corrected.
- In the case of simultaneous photography, the approximation errors in height may be negligibly small only in shallow water areas (water depth < 10m). In deep water areas, they may become larger than the usual photogrammetric errors and must be corrected.
- The approximation errors are correlated on account of the great number of parameters required to determine the actual sea surface. Thus, they can be corrected by means of interpolation methods based on stochastic processes.

DISCUSSIONS AND CONCLUSIONS

A stereo pair of two-media photographs taken of coastal underwater areas are usually analyzed with the assumption of a horizontal, flat air/water interface. In this paper, the position errors due to this approximation have been estimated for the actual sea surface. As a result, it has been concluded that the approximation errors in height may not be always negligibly small, particularly in deep underwater area (water depth > 10m). In order to correct the deviations due to waves, the following techniques are to be considered:

(1) *Determination of the actual sea surface from stereo pairs.*

For this purpose, a stereo pair of two-media photographs must be taken simultaneously from two aircraft and, further, the water surface must be clearly imaged and observed. Moreover, the orientations of the stereo pair should be known. The determination procedure can be performed from portion to portion of the water surface. Thus, the air/water interface can be traced accurately, even if it has a very complicated form. Practically, however, the procedure may prove tedious. In addition, in areas where water penetration takes place, the water surface itself may not be clearly imaged (Masry and MacRitchie, 1980).

(2) *Orientation of two-media photographs with orientation points in water.*

Theoretically, the form of the boundary surface

TABLE 8. APPROXIMATION ERRORS FOR A SUPERPOSED WAVE (LONG WAVE WITH H_{wo} = 2.0 M AND T = 15 SECONDS AND SHORT WAVE WITH H_{wo} = 0.3 M AND T = 4 SECONDS)

[m]

		flying height $H + h$ = 500 m					flying height $H + h$ = 1,500 m				
water depth h	photography	$\bar{X}_w$		No.	E_{max}	σ	$\bar{X}_w$		No.	E_{max}	σ
	A	$_1\bar{X}_{w\alpha} = 26$	X	147	0.18	0.08	$_1\bar{X}_{w\alpha} = 26$	X	166	0.16	0.08
		$_2\bar{X}_{w\alpha} = 26$	Y	214	0.15	0.05	$_2\bar{X}_{w\alpha} = 26$	Y	42	0.16	0.05
		$_1\bar{X}_{w\beta} = 5$	Z	210	0.68	0.39	$_1\bar{X}_{w\beta} = 5$	Z	211	0.72	0.39
5		$_2\bar{X}_{w\beta} = 5$					$_2\bar{X}_{w\beta} = 5$				
	B	$_1\bar{X}_{w\alpha} = 26$	X	213	0.17	0.06	$_1\bar{X}_{w\alpha} = 26$	X	20	0.20	0.09
		$_2\bar{X}_{w\alpha} = 47$	Y	22	0.39	0.10	$_2\bar{X}_{w\alpha} = 89$	Y	211	0.48	0.16
		$_1\bar{X}_{w\beta} = 5$	Z	22	1.36	0.62	$_1\bar{X}_{w\beta} = 5$	Z	22	1.09	0.46
		$_2\bar{X}_{w\beta} = 20$					$_2\bar{X}_{w\beta} = 50$				
	A	$_1\bar{X}_{w\alpha} = 36$	X	189	0.24	0.11	$_1\bar{X}_{w\alpha} = 36$	X	146	0.25	0.11
		$_2\bar{X}_{w\alpha} = 36$	Y	211	0.24	0.08	$_2\bar{X}_{w\alpha} = 36$	Y	41	0.20	0.08
		$_1\bar{X}_{w\beta} = 6$	Z	1	0.81	0.35	$_1\bar{X}_{w\beta} = 6$	Z	21	0.79	0.35
10		$_2\bar{X}_{w\beta} = 6$					$_2\bar{X}_{w\beta} = 6$				
	B	$_1\bar{X}_{w\alpha} = 36$	X	13	0.23	0.08	$_1\bar{X}_{w\alpha} = 36$	X	63	0.23	0.11
		$_2\bar{X}_{w\alpha} = 66$	Y	230	0.47	0.14	$_2\bar{X}_{w\alpha} = 126$	Y	170	0.45	0.17
		$_1\bar{X}_{w\beta} = 6$	Z	230	1.44	0.68	$_1\bar{X}_{w\beta} = 6$	Z	21	1.15	0.52
		$_2\bar{X}_{w\beta} = 24$					$_2\bar{X}_{w\beta} = 60$				
	A	$_1\bar{X}_{w\alpha} = 49$	X	20	0.37	0.16	$_1\bar{X}_{w\alpha} = 49$	X	103	0.33	0.16
		$_2\bar{X}_{w\alpha} = 49$	Y	21	0.55	0.19	$_2\bar{X}_{w\alpha} = 49$	Y	20	0.52	0.18
		$_1\bar{X}_{w\beta} = 6$	Z	3	1.49	0.63	$_1\bar{X}_{w\beta} = 6$	Z	21	1.32	0.63
20		$_2\bar{X}_{w\beta} = 6$					$_2\bar{X}_{w\beta} = 6$				
	B	$_1\bar{X}_{w\alpha} = 49$	X	2	0.43	0.14	$_1\bar{X}_{w\alpha} = 49$	X	231	0.37	0.11
		$_2\bar{X}_{w\alpha} = 88$	Y	211	0.78	0.23	$_2\bar{X}_{w\alpha} = 166$	Y	84	0.67	0.23
		$_1\bar{X}_{w\beta} = 6$	Z	211	2.27	0.93	$_1\bar{X}_{w\beta} = 6$	Z	207	1.44	0.71
		$_2\bar{X}_{w\beta} = 24$					$_2\bar{X}_{w\beta} = 60$				

A; simultaneous photography
B; non-simultaneous photography

can be determined from the orientation theories (Okamoto and Hoehle, 1972; Okamoto and Mori, 1973; Okamoto, 1980). Also, this method may have the advantages that

- Simultaneous photography need not be taken,
- The water surface can also be determined without imaged surface points, and
- There is no problem of incomplete stereomodels (Masry and MacRitchie, 1980).

However, so many parameters to define the boundary surface in the orientation coordinate system cannot be practically determined from the orientation theories of two-media photographs because of correlations between the orientation unknowns Thus, some wave parameters must be given even for the case where the water surface can be represented by a simple mathematical function. In addition, depending on the shape of the coast and also that of the underwater bottom, the form of the sea surface may not be expressed with a mathematical function such as Equation 8. In such cases, the orientation method may not be applied. These problems will be precisely investigated in a later report.

(3) *Interpolation method based on stochastic process.*

If the actual sea surface has a very complicated form, the approximation error is considered to be random function with many (random) parameters to determine the air/water interface. Thus, least-squares interpolation (Kraus, 1972; Mikhail, 1976) may be effectively introduced into the correction of the approximation errors. This technique also has the advantages cited in (2). However, it has a disadvantage in that it requires comparatively many ground control points in water.

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