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# Multiple Scene Precision Rectification of Spaceborne Imagery with Very Few Ground Control Points

A pseudo-physical model of the spacecraft position and attitude parameters is employed.

#### INTRODUCTION

THE SUCCESS of the Landsat series of satellites has advanced the mapping, surveying and management of earth resources and environment during the 1970s. As a result, new, more advanced satellites, such as Landsat-D (Hughes Aircraft Co., 1979) and SPOT (CNES, 1978), will provide improved imagery data during the 1980s. However, the utility of the Multispectral Scanner (MSS) data, provided by Landsat, has been severely limited due to the 1980) in a geographic information system (Williams, 1980).

Spaceborne imagery suffers from a variety of geometric errors due to the satellite motion (orbit and attitude) and the sensor scanning mechanism (Stein and Van Wie, 1976). Since the launch of Landsat-1 in 1972, geometric correction has been included as part of the ground data processing (Sloan *et al.*, 1977; Wong *et al.*, 1978). The majority of products generated have been geometrically corrected for *a* 

ABSTRACT: The success of the Landsat series of satellites has greatly advanced remote sensing and its applications. However, the utility of the imagery data has been limited in application areas requiring high precision due to the expensive and time consuming process of precision geometric correction. Present correction methods require for each scene the identification of an average of 15 features whose geographic location is known. This paper discusses a method which reduces the number of features which must be identified by more than an order of magnitude. Results presented here show that ten scenes can be rectified by identifying only four features. This method, in combination with the latest resampling technology, will make the operational production of geocoded precision imagery products feasible and economical even in poorly mapped areas.

expensive, and in some cases impractical, methods of precision geometric correction.

Precision corrected products are a necessity in applications requiring high accuracy such as mapping, map updating, and change detection. Furthermore, precision geocoded (satellite independent, map-like) products are a necessity in applications where imagery data must be integrated with other Earth resource data (Friedmann and Orth, *priori*-known errors (mostly due to the scanning mechanism).

These bulk products suffer from two major drawbacks: they are relatively inaccurate (1 km) (Sloan *et al.*, 1977; Wong *et al.*, 1978) and they are in a satellite-specific projection instead of a standard map projection. Precision products with errors smaller than the instantaneous field-of-view of the sensor (80 m) have also been produced (usually in a

PHOTOGRAMMETRIC ENGINEERING AND REMOTE SENSING, Vol. 49, No. 12, December 1983, pp. 1657-1667. 0099-1112/83/4912-1657\$02.25/0 © 1983 American Society of Photogrammetry satellite-specific projection), but in much smaller quantities (Bernstein, 1973; Caron and Simon, 1975; Colvocoresses and McEwen, 1973; Forrest, 1974; Konecny, 1977; Kratky, 1974; Landsat User Handbook, 1979; Rifman, 1974; Wong, 1976).

Geometric correction consists of two separate steps. First, a transformation between the raw imagery projection and the corrected imagery projection is obtained. This process, referred to as auxiliary data processing (see Figure 1), involves the integration of spacecraft sensor models, telemetry, imagery, and ground truth to establish the precise location of each imagery pixel. The geometric accuracy of the final product depends amost entirely on this step. The second step, resampling, consists of obtaining the pixel intensities on a regular grid in the corrected projection.

With the advent of array processors and the development of special purpose hardware, the resampling step can now be done in a few minutes. Furthermore, with the development of a one-dimensional technique for rotating images, the resampling step can now include the generation of geocoded or map-like products (Friedmann, 1981). Until now, however, auxiliary data processing has required the identification of between ten and 40 features, whose geographic location is known (usually obtained from an existing map), per scene (185 km by 185 km). The process of identifying these ground control points on the imagery and obtaining their geographic coordinates from a map can be very time consuming, especially in sites remote from civilization, with featureless terrain or with poorly defined land/water interfaces. The process can be very expensive in areas not covered with maps. As a result of this ground control point (GCP) problem, precision rectification, especially where no maps are available, has not been wide spread.

This paper discusses a method of auxiliary data processing which yields an order of magnitude re-



FIG. 1. The geometric correction process. Auxiliary data are processed to obtain the transformation between the raw and corrected projections. The imagery (raw) is then resampled.

duction in the required number of ground control points and allows rectification of imagery over areas with no ground control points. This method, together with advances in resampling technology, will make the operational (high volume) production of precision geocoded products from satellite borne imagery practical and economical, even in areas with no maps.

The auxiliary data processing problem is discussed first, followed by a review of the methods presently employed. The new method is then explained and illustrated with results obtained by correcting Landsat-2 imagery. Finally, the applications and possibilities offered by the method are summarized.

## THE AUXILIARY DATA PROCESSING PROBLEM

In order to find the exact location of each imagery pixel, four sets of variables must be determined (Rifman, 1974):

- the position of the spacecraft as a function of time (orbit model);
- the orientation of the spacecraft as a function of time (attitude model);
- the view angle of the sensor as a function of time (sensor model); and
- the shape of the Earth (Earth shape model).

The four models which estimate these sets of variables can be classified according to their frequency of variation from nominal as follows:

- Low frequency of variation from nominal, when the model oscillation period is much greater than the scene imaging time (typically 25 seconds). The orbit and Earth shape\* models are low frequency.
- Mid-frequency of variation from nominal, when the model oscillation period is of the order of the scene imaging time. The attitude model is mid-frequency.
- High frequency of variation from nominal, when the model oscillation period is smaller than the scene imaging time. The sensor model is high frequency.

The auxiliary data processing problem consists in estimating these models and using them to predict the position of imagery pixels. Spacecraft telemetry, *a priori* measurements, and knowledge of the scan mechanism can be used in the estimation process. In practice, there are always deficiencies in the modeling process, and thus it is necessary to utilize ground truth to refine any or all models. A prediction of the location of a feature (ground control points) is made using the models, the actual location of the feature is measured, and the difference is used to improve the models.

\* The present discussion assumes that terrain relief effects are minimal. When this is not the case, terrain relief distortion (which is high frequency) must be corrected during resampling (Wong *et al.*, 1981).

#### PRESENT METHODS

Different auxiliary data processing procedures utilize varying degrees of modeling. Invariably, most present methods use a sensor model (Bernstein, 1973; Caron and Simon, 1975; Forrest, 1974; Kratky, 1974; Wong *et al.*, 1978; Rifman, 1974). This model is fairly constant with time, and any deviations are transmitted as spacecraft telemetry (e.g., line length in the case of Mss). The sensor must be modeled because correcting for its high frequency nature would require hundreds of ground control points. Some methods also model the Earth shape (Caron and Simon, 1975). Present methods, however, do not attempt to physically model the spacecraft attitude and orbit but instead rely on GCPs to remove these errors.

Polynomial fit procedures (Bernstein, 1973; Kratky, 1974) model the orbit and attitude by fitting a polynomial to line and pixel errors at each GCP. Due to the mid-frequency of the attitude variation, these procedures may require between 25 and 40 GCPs per scene to achieve root mean square (RMS) errors under one instantaneous field-of-view (IFOV). Other more successful procedures utilize a mathematical attitude-orbit model (Caron and Simon, 1975). Typically, a polynomial in time is used. Because of its more physical nature, this procedure requires between ten and 20 GCPs per scene for the equivalent errors. In all cases, the GCPs should be evenly distributed over the scene for best results.

Mathematical orbit-attitude models have been widely used in most ground stations since the mid-70s, yielding accurate results at the expense of having to identify an average of 15 GCPs per scene (Wong *et al.*, 1978). Recently, a more optimized model has been used at ERIM to produce maps (Wilson, 1981). If precisely located, only six GCPs are required per scene. Typically, however, when using good 1:25,000 scale maps, more like ten GCPs are required.

In some cases it has been possible to process

scenes with only three GCPs (Savarde, 1981); however, as pointed out by Caron and Simon (1975), these are special situations in which the spacecraft attitude happens to be varying slowly (which is not the usual case). It has also been possible to interpolate over one or two GCP-poor scenes with limited success (Rifman *et al.*, 1978).

## A New Correction Method Using Physical Spacecraft Models

In order to reduce and eventually eliminate GCPs, good physical models (especially for attitude, the main source of error) that can characterize the behavior of the spacecraft over the time frame of interest (multiple scenes or even a whole orbit) must be used. Not only can the number of GCPs be reduced, but the model also can be used to interpolate and/or extrapolate over areas with *no* GCPs.

A sophisticated physical model has been attempted for NIMBUS-6 (Lefferts and Markley, 1976) (very similar to the Landsat spacecraft) with inconclusive results. Simpler models which incorporate wheel speed telemetry have also been attempted for Landsats-1, -2, and -3 but have not achieved high precision (Hall and Waligora, 1978).

The results presented here have been obtained by using a new physical attitude model (Friedmann, 1980), an existing analytical Brouwer-Lyddane orbit model (Brouwer, 1959; Lyddane, 1963), and existing Earth and sensor models (Caron and Simon, 1975). A mathematical time series is used to refine the physical orbit-attitude model with GCPs.

Figure 2 shows an overview of the correction process. A given image line and pixel (L, P) are transformed to map coordinates. These predicted map coordinates are compared with those measured from a map, and the difference is used to improve the prediction. In order to transform the line and pixel, the orbit, attitude, and Earth shape models are used in a geometric transformation which includes the known geometry. The pseudo-physical attitude-



FIG. 2. Overview of the correction process, including physical orbit-attitude models.

orbit model (i.e., the mathematical time series) is updated with GCPs to improve the transformation accuracy.

The emphasis of this paper is on the physical attitude model, the pseudo-physical orbit-attitude model, and the results obtained after refinement with GCPS.

#### ATTITUDE MODEL DESCRIPTION

Spacecraft attitude (or the motion of the spacecraft about its center of mass) is governed by the applied torques. In the absence of torques, there is no change in motion about the center of mass. To keep the imaging sensor pointed toward the Earth, the spacecraft must rotate about the orbit normal once per orbit. This ideal attitude is defined as the target attitude. A control torque must be applied to maintain the target attitude. Earth sensing spacecraft carry attitude control systems which generate the control torques from measurements of the error attitude (usually referred to simply as attitude) or difference between the actual attitude and target attitude.

In reality, the control torques also counteract other "disturbance" torques which act on the spacecraft. These torques are due to a combination of the Earth's gravitational and magnetic fields, the Earth's atmosphere, and solar radiation.

Control torques for different spacecraft are generated in a similar manner. Flywheels are used for momentum exchange and storage; they can be viewed as generators of cyclic torques (i.e., like those required to rotate the spacecraft once per orbit). Typical systems employ three wheels, one about each of the spacecraft axes (i.e., Landsat-1, -2, -3,\* and SPOT). Other spacecraft (i.e., Landsat-4) employ four wheels. This approach allows the operation of all wheels about some bias velocity, thus avoiding speed reversals inherent in the three wheel system.

External torquing mechanisms such as gas release and variable magnetic moments are used to "dump" momentum which accumulates in wheels because of the noncyclic disturbance torques. The gas release method was used for the earlier Landsat series. Magnetic torquing is planned for SPOT and is used in Landsat-4.

The one component that gives rise to the accuracy limitations of the different control systems is the attitude sensor. This sensor provides the input to the control system. The earlier Landsats used infrared Earth scanners (roll, pitch) and a rate integrating gyro (yaw). SPOT will use, in addition to the sensor carried by Landsat, a three-axis magnetometer to provide increased accuracy. Landsat-4 also uses a star sensor to remove biases in the gyros,

\* Actually, Landsat has four flywheels but two are along the roll axis and nominally rotate in opposite directions; thus, they can be modeled as one wheel. providing measurements an order of magnitude more precise.

The attitude model consists of a set of differential equations that describe the state of the spacecraft. These equations are integrated numerically to yield the attitude (and all other state variables) at any given time. The equations are (Wertz, 1980; Lefferts and Markley, 1976)

$$\frac{d\overline{L}}{dt} + \overline{w} \times \overline{L} = \overline{N} \tag{1}$$

$$\overline{w} = I^{-1} \left( \overline{L} - \overline{H} \right) \tag{2}$$

$$\frac{d\overline{q}}{dt} = \frac{1}{2} \Omega \overline{q} \tag{3}$$

$$\frac{dH}{dt} = \overline{T}_c \ (\overline{q}) \tag{4}$$

Equation 1 represents the dynamics which governs the evolution of the total spacecraft angular momentum  $(\overline{L})$  as a function of the applied torques  $(\overline{N})$ and the spacecraft angular velocity  $(\overline{w})$  (Lefferts and Markley, 1976). The angular velocity is relayed to the angular momentum through Equation 2 (Lefferts and Markley, 1976), where I is the moment of inertia and  $\overline{H}$  is the angular momentum of the flywheels. The spacecraft orientation is described by Equation 3 in terms of the evolution of the Euler parameters  $(\overline{q})$  (Wertz, 1980) (these are easily converted to roll, pitch, and yaw) where  $\Omega$  is a matrix that contains the spacecraft angular velocity  $(\overline{w})$ . Finally, the flywheel angular momentum is described in terms of Equation 4 (Lefferts and Markley, 1976).  $\overline{T}_c$  is the control torque, which is a function of the attitude (or euler parameters  $\overline{q}$ ) (General Electric, 1970).

## The Pseudo-Physical Orbit-Attitude Model and Its Estimation

The pseudo orbit-attitude model consists of the following time series for spacecraft attitude and height:

roll	=	$r_0$	+	$r_1 t$	+		
pitch	=	$p_0$	+	$p_1 t$	+		
yaw	=	$y_0$	$^{+}$	$y_1 t$	+		
height	=	$h_0$	$^{+}$	$h_1t$	+		

These parameters allow correction for any reasonably small (less than 10 km) deficiencies in the spacecraft orbit or attitude model as detailed below:

- roll and across track orbit errors with the roll time series,
- pitch and along track orbit errors with the pitch time series,
- yaw and orbit inclination errors with the yaw time series, and
- height and scale errors with the height time series.

The pseudo-model is estimated from GCPs using a recursive filter. The recursive filter (Caron and

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Simon, 1975; Gelb, 1979; Kalman, 1960; Ranch et al., 1965) is based on the following equations:

$$\begin{split} \boldsymbol{\Psi}_{k}\left(-\right) &= \boldsymbol{\Phi}\boldsymbol{\Psi}_{k-1}\left(+\right) & \text{State Prediction} \\ \boldsymbol{P}_{k}\left(-\right) &= \boldsymbol{\Phi}\boldsymbol{P}_{k-1}\left(+\right)\boldsymbol{\Phi}^{\mathrm{T}} + \boldsymbol{Q} & \text{Covariance} \\ & \text{Prediction} \\ \boldsymbol{B}_{k} &= \boldsymbol{P}_{k}\left(-\right) \mathbf{H}^{\mathrm{T}}\left[\mathbf{H}\boldsymbol{P}_{k}\left(-\right)\mathbf{H}^{\mathrm{T}} + \mathbf{W}_{k}\right]^{-1} & \text{Gain} \\ \boldsymbol{P}_{k}\left(+\right) &= \left[\mathbf{I} \cdot \mathbf{B}_{k}\mathbf{H}\right] \boldsymbol{P}_{k}\left(-\right) & \text{Covariance} \\ & \text{Covariance} \\ & \text{Correction} \\ \boldsymbol{\Psi}_{k}\left(+\right) &= \boldsymbol{\Psi}_{k}\left(-\right) + \mathbf{B}_{k} \mathbf{Y}_{k} & \text{State Correction} \\ \end{split}$$

 $\begin{aligned} \mathbf{\Psi}_k &= \text{ state vector } (r_0, r_1 \dots; p_0, p_1, \dots; y_0, y_1 \\ \dots; h_0, h_1) \\ \mathbf{Q} &= \text{ plant noise matrix,} \end{aligned}$ 

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- $\Phi$  = transition matrix,
- $\mathbf{P}_k = \text{covariance matrix},$
- $\mathbf{W}_k$  = measurement error covariance matrix, and  $\mathbf{B}_k$  = gain matrix.

The "+" in the equations refers to the value of the variable after a measurement, whereas the "-" refers to the value before a measurement. The measurement model used is

$$\mathbf{Y}_k = (\mathbf{R} - \mathbf{H} \boldsymbol{\Psi}_k) + \mathbf{E}_k$$

where  $\mathbf{H}\Psi_k$  is the vector of the measurement derived from the combination of state vector elements indicated in the measurement matrix, **H**.  $\mathbf{H}\Psi_k$  gives the predicted location of the input GCP. **R** is the



FIG. 3. Comparison of telemetry and model predicted variables for orbit 6120. A slow varying component has been subtracted from these plots to allow display with a finer scale. (a) Telemetry pitch flywheel speed (for 40 scenes) vs time. (b) Model predicted pitch flywheel speed vs time. (c) Telemetry roll fine error vs time. (d) Model predicted roll fine error vs time.

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true ECR (Earth-rotating coordinates) of the GCP, and  $\mathbf{E}_k$  is the uncertainty in the measurement.

## ATTITUDE MODEL RESULTS

Because the attitude model considers the spacecraft physics, it is capable of predicting not only the spacecraft attitude but also other state variables such as flywheel speeds. Some of these variables are included in the spacecraft telemetry. By comparing the model-predicted results with the telemetry, it is possible both to check the model and to refine model parameters (constants which are not well known) in order to achieve higher accuracy. Figure 3 contains examples of telemetry versus predicted results (for orbit 6120 of the Landsat-2 spacecraft). Figures 3a and 3b show the telemetry and modelpredicted pitch flywheel speeds. The difference between the telemetry and predicted speeds, if plotted, is less than the quantization error (3.0 RPM) on the telemetry except for the first 100 seconds where initial conditions are damping out (i.e., not all variables are known at t = 0 and, therefore, the model starts with some wrong initial conditions). Figures 3c and 3d show the telemetry and predicted roll fine error (this is an input to the attitude control system). Again, the difference (if plotted) is within the telemetry quantization error (0.045°). Similar results are obtained for other telemetry functions.

The main model output—roll, pitch, and yaw used for image correction—is shown in Figures 4a, 4b, and 4c.

In the case of Landsats-1, -2, and -3, the spacecraft roll and pitch are measured with a static horizon detector (the attitude measurement sensor, AMS). However, this telemetry has not been widely used because of its relatively poor accuracy. Plots of the AMS roll and pitch look very similar to Figures 4a and 4b, indicating that the AMS provides good measurement. However, it is not uncommon for the difference between the AMS and the model to be as large as 0.04 degrees (equivalent to about five pixels). The spacecraft yaw is not measured. Many (Tsuchiya and Yamarra, 1981) have been computing yaw from the AMS roll by the formula

## Yaw = -1.15 roll.

It is clear from comparing Figures 4a and 4c that this formula is incorrect (i.e., if one were to multiply the plot in Figure 4a by -1.15, one would not obtain Figure 4c).

Figure 4 also shows that there are some special instances when roll, pitch, and yaw are varying very slowly (for example at 250s < t < 275s). In these cases a scene (interval per scene  $\approx 25s$ ) over the area could be corrected (using present methods) with only three GCPs as has been reported (Caron and Simon, 1975; Savarde, 1981).

#### **RECTIFICATION RESULTS**

The correction process was tested on two Landsat-2 passes. Both passes cover track (path) 19



FIG. 4. Examples of model predicted roll (a), pitch (b), and yaw (c). It can be seen that yaw bears some relation to roll, but it is not a simple one. It can also be seen that in certain areas (e.g.,  $250 \sec < t < 275 \sec$ ) roll, pitch, and yaw vary slowly (over 25 seconds or one scene) permitting correction with few GCPs.



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FIG. 5. Vector error plot of Orbit 6120 showing the error at each GCP. Each vector is drawn from the measured to the model predicted location of each GCP. The GCPs that were used for model refinement are circled. The number beside each circle indicates the order in which these GCPs were used. All other GCPs (not circled) are used for calculation of the total imagery error. The scale for the vector (on a different scale) is shown separately on each figure. The whole plot shows 11 scenes, starting at scene 19 ( $\cong$  line 31000) and ending at scene 30. Seventeen GCPs were marked. The total RMS error is 66 m.

and frames (row) 17-30. One pass, orbit 6120, was imaged on 5 April 1976 and the other pass, orbit 33479, was imaged on 8 August 1981. The imagery data and orbital elements were obtained from the Prince Albert ground station. The telemetry data were obtained from the Canada Centre for Remote Sensing.

1:50,000 scale maps for the area were obtained from Energy, Mines and Resources, Canada-Canada Map Office. The accuracy of these maps ranged from 25 m\* (Class A) to 100 m (Class C). About 150 GCPs were identified in the imagery from each pass and on the maps. However, it was impossible to identify GCPs in many of the northern frames, especially those from the spring pass, due to ice and snow. For some of the other remote frames, it took a day to identify five to ten GCPs.

The pseudo-model used in the test was first-order in roll and pitch and zeroth order in yaw and height. Figures 5 and 6 show vector error plots of the two orbits after the input of 17 GCPs (circled and numbered in the figures). The scale for the vectors is shown separately within each figure. All GCPs used in the rectification process are circled. Each vector is drawn from the measured to the predicted loca-

\* 90 percent of points have less than 25-m error.

tion of the GCP. The total root-mean-square error (line, pixel) versus number of GCP plots for the two orbits are shown in Figures 7 and 8. It can be seen that an accuracy of 60 to 80 m can be attained (for about ten scenes) after four GCPs.

Figure 9 shows the interpolative ability of the method. Four GCPs were marked in scenes 19 and 30, with ten scenes separating the GCPs. The accuracy achieved after rectification was 67 m.

#### DISCUSSION OF RESULTS

There are contributions of measurement errors of the observation GCPs to the total measured RMS error. This marking error; (i.e., error due to location of GCPs on the map and on the imagery) is estimated at 42-m RMS. The actual error can be computed by

$$E_{ACTUAL} = (E_{measured}^2 - 42^2)^{1/2},$$

so it can be seen from the results that an accuracy of approximately 0.5 pixels can be achieved after 4 to 9 GCPS.

#### CONCLUSIONS

An auxiliary data processing method consisting of the currently used Earth shape and sensor models and a new physical spacecraft model has been pre-



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FIG. 6. Plot showing the error at each GCP for orbit 33479. The plot starts at scene 19 and finishes at scene 30. Seventeen GCPs (circled) were marked. RMS error is 60 m.

sented. Results show that ten scenes of Landsat-2 imagery can be precision rectified with only four GCPs. This compares with the present technology which requires an average of 15 GCPs per scene for the equivalent accuracy.

The new method reduces the number of GCPs by processing multiple scenes and by physically modeling the Landsat-2 spacecraft. The physical model is sufficiently general to apply to any three-axis stabilized polar orbiting satellite. In particular, it



FIG. 7. Measured RMS error vs number of GCPS—Orbit 6120. The plot represents the total RMS error calculated (having the observation GCPs) after the input of each GCP. (Error calculation does not include the GCPs marked to update the pseudo model—map and marking error  $\approx 42$  m).

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Fig. 8. Measured RMS error vs number of GCPs Orbit 33479 (map and marking error  $\approx 42$  m).

should apply, with some modification, to Landsat-4 (Iwens, 1977) and SPOT (CNES, 1978). The stability of Landsat-4 is such that the geometric error in the imagery due to spacecraft attitude should be under four pixels RMS. Although much smaller than the error in Landsat-2 imagery, it is still significant (Doyle, 1982). SPOT will also be more stable than

Landsat-2. However, because of its higher resolution (4 to 8 times more), the error, in terms of pixels, will be on the same order as the error in Landsat-2 imagery.

The order of magnitude reduction in the number of GCPs makes the production of precision geocoded products both feasable and affordable. In many





countries, good maps are available for the populated areas. GCPs obtained from these maps will allow precision correction of remote areas in the same pass of imagery (by extrapolation or interpolation). In other countries, maps may be outdated, be of drastically different scales, offer only partial coverage, or be entirely absent. In these cases, a few very accurate GCPs can be obtained by making use of the TRANSIT satellites and, in the future, of the global positioning system. These GCPs can then be used to rectify passes of imagery. The present cost of obtaining these GCPs varies from \$2000 in the Canadian Prairies to \$15,000 plus in Indonesia per GCP (Personal Communication). The actual cost depends on spacing between GCPs, terrain cover, transportation, latitude, etc. Considering that only a few GCPs are required per pass of imagery and that there is overlap (especially at higher latitudes) between the passes, economical production of precision geocoded products should become a reality, even in areas with no maps.

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