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Generalized Adjustment by Least Squares (GALS)

GALS is a standardized least-squares software engine which can power virtually any network adjustment system.

Introduction

The impact of digital computers on the field of computational adjustment was recognized as early as the middle 1950's. Indications of this recognition are evident in the work of Brown (1955) and Schmid (1959). Their work resulted in a significantly effective formulation of the least-squares principle as applied to photogrammetric triangulation.

During the 1960's a large number of computer programs were written dealing with every aspect of adjustment in the mapping fields. Highly efficient algorithms were developed which made possible the practical adjustment of networks of observations containing tens of thousands of unknown parameters (Elassal, 1969). Experimentation with various perceptions of the mathematical model of the mapping process were prevalent during this period.

The 1970's saw computational adjustment programs put into daily production. Attention was therefore focused on the operational aspect of these programs. Overall productivity became the goal to be achieved. Data processing became more pertinent than mathematical or algorithmic questions in the evolution of computational adjustment. This trend is expected to continue in the foreseeable future.

Data management requirements, which must take into account the optimization of an involved least-squares application, can result in an unmanageable programming task. Modern ideas in software engineering provide the answer in the form of independently produced and verifiable software components. The need for techniques that promote reliability and simplify the maintenance of software is well established. It is time to explore the possibility for a "Least-Squares Engine" as the standard powering module for least-squares adjustment systems in the allied fields of surveying, geodesy, and photogrammetry.

GALS is a software system aimed at removing from the process of building adjustment programs all those mathematical and algorithmic aspects associated with application of least-squares principles to the adjustment of linear mathematical models. It is intended to be the general tool that might lead to a standardization of least-squares operations in adjustment software.

Abstract: *The least-squares principle is universally accepted as the basis for adjustment procedures in the allied fields of geodesy, photogrammetry, and surveying. A prototype software package for Generalized Adjustment by Least Squares (GALS) is described in this paper. The package is designed to perform all least-squares-related functions in a typical adjustment program. GALS is capable of supporting development of adjustment programs of any size or degree of complexity.*

Overview

GALS is a software subsystem which must be executed from a higher level program. It can be viewed as a subroutine package with somewhat involved interface requirements. Figure 1 is a functional diagram showing the major components

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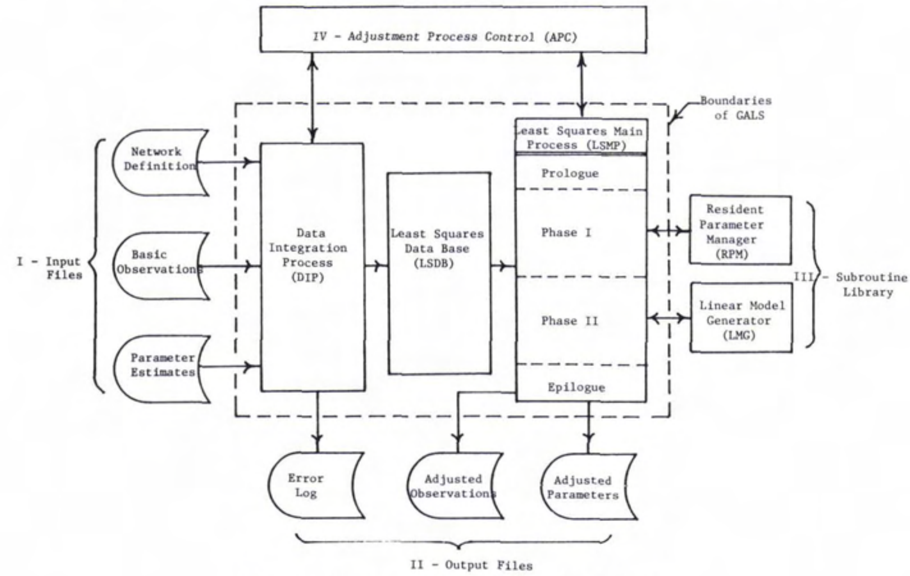

FIG. 1. Components and Interface of GALS.

of GALS and the various elements of its interface. Knowledge of the detailed internal operations of GALS is not essential for the utilization of its facilities. It is perfectly possible to deal with GALS as a "black box" and still create very involved adjustment systems. However, a user of GALS should be aware that the order of entering basic observations into GALS can have a significant effect on the overall efficiency of the least-squares adjustment process. This order determines the structure for the system of normal equations and subsequently influences optimization of its solution. There are several empirical methods for ordering the basic observations so as to arrive at a favorable structure for the normal equations (Cuthill and McKee, 1969; King, 1970; Snay, 1975). Feasibility for adjusting very large networks may very well depend upon arriving at an optimum order for the basic observations. The issue of observations order is outside the scope of this discussion of GALS and is briefly mentioned here to emphasize its importance. However, it should be realized that, for any given order of observations, GALS will automatically perform the most efficient least-squares adjustment possible. GALS is a basic building module for very-efficient and highly reliable adjustment software.

MATHEMATICAL MODEL

GALS is designed to deal with least-squares adjustment of linear models of observations. Although real world problems are generally nonlinear, they can be replaced by local linear approximations. The solution then is reduced to solving a sequence of linear least-squares problems. It is assumed that the mathematical model can be expressed as a set of N linear equations,

$$\mathrm{F}(\mathbf{X},\mathbf{Y},\mathbf{Z}) = 0, \tag{1}$$

where

$\mathbf{X}$ is a set of original observations

$$= \mathbf{X} + \mathbf{X}^0 \tag{2}$$

= observed values + residuals,

$\mathbf{Y}$ is a set of adjustable parameters

$$= \mathbf{Y} + \mathbf{Y}^{00} + \mathbf{Y}^0 \tag{3}$$

= initial approximations + offset + correction,

$\mathbf{Z}$ is a set of known parameters, and

F is linear in $\mathbf{X}^0$ and $\mathbf{Y}^0$.

It is further assumed that

$\bar{\mathbf{X}}$ can be divided into n statistically independent observed subsets,

$$= (\bar{\mathbf{X}}_1, \bar{\mathbf{X}}_2, \ldots, \bar{\mathbf{X}}_n) \tag{4}$$

$\bar{\mathbf{Y}}$ can be divided into m statistically independent adjustable parameter subsets,

$$= (\bar{\mathbf{Y}}_1, \bar{\mathbf{Y}}_2, \ldots, \bar{\mathbf{Y}}_m), \text{ and} \tag{5}$$

$\mathbf{Z}$ consists of l subset of known parameters,

$$= (\mathbf{Z}_1, \mathbf{Z}_2, \ldots, \mathbf{Z}_l). \tag{6}$$

Therefore, Equation 1 can be rewritten as a sequence of n subsets of equations,

$$\mathrm{f}_i(\bar{\mathbf{X}}_i, \bar{\mathbf{Y}}_{i2}, \ldots, \bar{\mathbf{Y}}_{im_i}, \mathbf{Z}_{i1}, \mathbf{Z}_{i2}, \ldots, \mathbf{Z}_{il_i}) = 0, \tag{7}$$

where

f_i is the i^{th} subset of N_i equations ($N = N_1 + N_2 + \ldots + N_n$),

$\bar{\mathbf{X}}_i$ is a subset of basic observations $\bar{\mathbf{X}}$ which appears only in the i^{th} subset of equations,

$\bar{\mathbf{Y}}_{i*}$ is a subset of $\bar{\mathbf{Y}}$ (Equation 5) and may appear anywhere in the mathematical model ($* = 1, 2, \ldots, m_i$), and

$\mathbf{Z}_{i}*$ is a subset of $\mathbf{Z}$ (Equation 6) and may appear anywhere in the mathematical model ($* = 1, 2, \ldots, l_i$).

A priori estimates of covariances for both basic observations and adjustable parameters are fully supported in GALS. Any practical problem of adjustment in the allied fields of mapping that admits a unique solution can be formulated to fit the GALS mathematical model.

GALS Internal Components

There are two major processes that take place within GALS. The following is a brief description of these processes and how they are accomplished.

DATA INTEGRATION PROCESS (DIP)

During this process, all input data are globally cross-correlated and verified for consistency and completeness. A file containing a log of all structural and logical errors for the network is produced. The data are reorganized and reordered for the most efficient application of least-squares principles. Control directories which precisely chart the sequence of operations for the least-squares process are generated. An exact evaluation of the computing resources required to accomplish a solution for the adjustment problem is done. A count of the degrees of freedom for the network is evaluated. DIP is performed only once during any adjustment process, including nonlinear problems which require iterative techniques. Results of DIP are stored in the Least Squares Data Base (LSDB). In reality, LSDB consists of four sequential data files. There is one entry point for DIP through which it can be activated by the Adjustment Process Control (APC) high-level program. Because all data for a network are retained in LSDB, an adjustment task may be interrupted following the completion of DIP and processing resumed at a later time. This arrangement allows for the opportunity to examine the results of DIP before proceeding with a committment to the Least Squares Main Process (LSMP). This early examination could be very important when dealing with very large adjustment problems where the LSMP represents considerable use of sizable computing resources. In some instances, an examination of the results of DIP might lead to a trial of a different ordering of the basic observations to reach a more favorable structure for the normal equations. Of course, a decision to proceed with the adjustment could always be made without requiring a reapplication of DIP.

LEAST SQUARES MAIN PROCESS (LSMP)

This is where the actual least-squares computation takes place. LSMP has an entry point to each one of its four subprocesses. The first entry into LSMP is always done through the Prologue subprocess. The Prologue section performs initialization for the LSMP and checks for the availability of needed computer resources. The Prologue section is executed only once during an adjustment task.

Phase I of the least-squares computations is the second section of LSMP. During this phase, the forward solution for the least-squares computation is carried out. This constitutes the simultaneous formation of the linear mathematical model and the components of the reduced normal equations. Residuals of basic observations and the weighted sum of squares are also computed. For nonlinear networks, convergence of the iterative solution can be assessed at this point, based on an examination of the weighted sum of squares.

Phase II of the least-squares computations is the third section of LSMP. Its execution must always be preceeded by an execution of Phase I. During this phase the back solution for the least-squares computation is accomplished. This entails the evaluation of corrections to the adjustable parameters. Optionally, estimates of the *a posteriori* covariance matrices for the adjustable parameters are also computed. The analytical background of Phase I and Phase II of LSMP can be found in Elassal (1969).

The last section to be executed in LSMP is always the Epilogue subprocess. This section performs necessary housecleaning tasks and creates data files for the final results of the adjustment. It also releases any computing resources that are no longer needed by GALS.

Interface Components

GALS communicates with its surrounding environment through a number of interface components. One can build a completely functioning adjustment system by only satisfying GALS minimum interface requirements. In research studies where user interface is not a critical requirement, this approach to building models of complicated adjustment systems is the most cost effective possible. GALS interface is divided into four groups: Input Files, Output Files, Subroutine Library, and Adjustment Process Control (APC).

INPUT FILES

The bulk of data enters GALS through three sequential files:

Network Definition File.

This file contains vital information concerning characteristics of parameters and observations for the class of networks to be addressed by an adjustment system.

GALS requires the classification of problem parameters (both adjustable and known) into "repeat-group" subsets which can then be assigned common characteristics. For example, known parameters are assigned to one or more subsets which are in turn assigned the characteristic of being unadjustable. Similarly, the nature of a network might make it favorable to group all angular parameters, such as camera station attitudes in photogrammetric aerotriangulation, into one subset. This allows for operations which are particular to camera attitudes, such as computations of corresponding rotation matrices, to be requested for all members of the group. The number of occurrences within each "repeat-group" subset is unlimited. GALS allows for each parameter "repeat-group" subset to be characterized by

- Adjustability (adjustable or known)
- Number of data elements for members of the group. For example, a camera attitude group subset will require three data elements for the three rotation angles which are required to define each member of the group.
- Minimum number of observation equations which are required to resolve each occurrence of the repeat group. This is used to detect and issue warnings for inadequately observed parameters.
- Storage requirements for special functions which need to be computed for each member of the group during each least-squares solution. Examples of such special functions are rotation matrices for attitude angles and matrices relating spatial coordinates in various reference systems.

GALS can service any number of types of observations. From GALS point of view, an observation type is characterized by the following:

- Number of data elements for that type of observation. For example, an observed distance constitutes one observed data element while measured image coordinates contain two observed data elements (x and y).
- Number of parameter subsets (both adjustable and nonadjustable) which are functionally related to the specified type of observation. A distance type observation will be related to the coordinates of its two terminal points (this is true whether or not the terminal points are classified as adjustable or nonadjustable parameters). In the classical aerotriangulation of frame photography, the mathematical model can be entirely based on the collinearity condition. Therefore, an implementation which does not simultaneously adjust for interior camera distortion, may specify four functionally related parameter sets for the collinearity condition (ground point object coordinates, camera station position, camera attitude, and camera interior orientation). An aerotriangulation system could be transformed to be self-calibrating by simply specifying a fifth parameter group subset to accommodate the corresponding distortion parameters. The effect will be that GALS will deliver to the Linear Model Generator (LMG) values of the appropriate self-calibration parameters, augmented to the other parameter subsets which are functionally related to the observation in question.
- Number of condition equations for the observation type. For example, a range observation type should specify one condition equation; a photogrammetric image coordinate observation type should indicate two condition equations.

Besides the general characteristics of the parameters and observations, the Network Definition File contains logical addresses of all external files required by GALS. This consists of three input files, three output files, four files for the LSDB, and five utility files for temporary storage of intermediate results.

Basic Observation File.

This is a sequential data file containing all the basic observations on a network. For each observation, a record containing the following information is included:

- Type of observation code (to identify the observation type)
- Observed values
- Covariance matrix for observed values
- Identification of occurrences for parameter subsets which are functionally related to the observation. Each identification contains the repeating group number for the parameter subset and the alphanumeric code of the parameter group occurrence.

Parameter Estimates File.

This is a sequential data file that contains initial estimates for adjustable parameters and known values for nonadjustable parameters. The following information is included for each occurrence of a parameter subset:

- Parameter subset occurrence identification (both group number and alphanumeric identification)
- A set of computational flags to indicate whether or not initial parameter values are to be updated after each least-squares iteration. These flags are needed to protect selected parameter subset occurrences, whose values are closely known, from contamination by other poorly estimated parameters during the early stages of a solution.
- Estimated or known values of the parameter subset occurrence and their covariance matrix.

OUTPUT FILES

This is the second component of the GALS interface. GALS reports the results of its operations through a set of three sequential output files.

Error Log File.

This file contains one record for each error discovered during the Data Integration Process (DIP). Messages are logged into this file, accompanied by all necessary identifying parameters. An error message is included in the file whenever one of the following five conditions is encountered:

- Estimates or known values for a parameter subset occurrence are reported to GALS but that occurrence is not referenced by any of the basic observations.
- Estimates or known values for a parameter subset occurrence are reported to GALS more than once.
- A parameter subset occurrence is referenced in one or more basic observations but no information is reported about its values.
- A parameter subset occurrence is involved in fewer condition equations than the required

minimum as defined in the Network Definition File.
- A combination of the above errors occurred that caused the network to be logically unsolvable.

Adjusted Observation File.
This is a sequential file which contains one record for each observation. A typical record in this file reports the following data:

- Observation type code
- Original values of the observation
- Least-squares residuals for the original values
- Covariance matrix of the observation's original values

Adjusted Parameter File.
This file contains the adjusted values for all parameter subsets. The file is sequential and contains one record for each parameter occurrence. Each record reports the following information:

- Identification of the parameter subset occurrence
- Original values of the parameter occurrence (this is true for both adjustable and nonadjustable parameters)
- Corrections to the occurrence's original values as estimated by the least-squares solution
- *A priori* estimate of covariance matrix for the original values of the parameter subset occurrence
- *A posteriori* estimate of covariance matrix for the adjusted values of the parameter subset occurrence.

SUBROUTINE LIBRARY

This is the third component of the GALS interface. In the course of its operation, GALS calls on two subroutines which must be supplied by the user. The first subroutine receives from GALS, during each call, the value for an observation accompanied by the values of all parameter subset occurrences which are functionally related to that observation. This subroutine, called Linear Model Generator (LMG), computes all coefficients for the condition equations resulting from the subject observation. The subroutine performs a strictly algebraic task and is therefore very straightforward to write.

The second subroutine, called the Resident Parameter Manager (RPM) subroutine, handles temporary storage and retrieval of data for a limited and predefined number of parameter subset occurrences. It involves the management of a resident two-dimensional table and therefore represents a trivial programming task. GALS assists the management process by supplying the subroutine with internal addresses and values for the data to be managed.

ADJUSTMENT PROCESS CONTROL (APC)

The APC is the last of the four components of the GALS interface. It is essentially a high-level program from which GALS can be called in a coordinated fashion. APC would normally perform the following functions:

- Initiate DIP component of GALS
- Perform tests on the errors reported by DIP through the Error Log File and make a go/no-go decision to proceed to the LSMP component of GALS.
- Initiate LSMP component and monitor its progress by controlling calls to its four entry points.

Again, APC is a very simple program with uncomplicated logic requirements.

Operating Environment

A prototype of GALS has been built to operate on IBM-compatible hardware running under either the MVT or MVS Operating System. The prototype was written exclusively in FORTRAN, with the exception of two small subroutines for bit manipulation, which are written in assembly language. GALS requires the services of a sort/merge software facility which is assumed to exist within the operating environment. The development of GALS is considered to be largely a software engineering project. As such, a prototype was needed to satisfy the important functions of:

- System design verification for correctness and completeness,
- Experimentation with the many possible design compromises, under realistic conditions
- Actual simulation of GALS in connection with various types of adjustment problems

Because of the inability to access computer resources from within a FORTRAN environment, the prototype is configured as sequence of six independent program tasks. This situation will be cured in the second phase of GALS development by using PL/I programming language. This will make possible a single-step "black-box" type of service. It will also provide for the automatic allocation of all necessary computing resources by GALS, depending on an assessment of actual needs.

GALS overall system was verified by using it as the basis for two experimental adjustment programs. The first program dealt with adjustment of a triangulation/trilateration net resulting from strictly planimetric observations. A simple mathematical model involving more than a single type of observation was selected to numerically verify the operations of GALS. Various networks of different sizes and configurations were successfully tested.

The second experimental adjustment program dealt with the general bundle adjustment of standard photogrammetric aerotriangulation of frame photography. The mathematical model selected for the program was based on the collinearity condition with allowances made for the self-calibration parameters. A significant number of cases were used to verify the operations of GALS. These cases dealt with numerous combinations of adjustable and known parameters.

Conclusions

GALS is intended to be a cost-effective and comprehensive tool to assist in building least-squares adjustment programs of considerable complexity. A GALS-like facility is indispensable for at least one class of adjustment programs. This class is characterized by a complicated mathematical model involving numerous conditions and a very large number of unknowns.

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Analytical Plotter Workshop (1983 ACSM-ASP Annual Convention)

Washington, D.C. 13-18 March 1983

The Instrumentation Committee of the Photogrammetric Applications Division, American Society of Photogrammetry, is sponsoring a mini-workshop on Analytical Plotters during the ACSM-ASP Annual Convention at the Washington Hilton Hotel, 13-18 March 1983. In April 1980 this committee, under the leadership of Larry Fritz, sponsored an Analytical Plotter Workshop/Symposium in Reston, Virginia. This present effort, at a much smaller scale, is designed as a follow-up to provide updated information on these and any recent instruments.

The Workshop will consist of three events. The first will be held Tuesday evening from 7 to 10 p.m. in the Exhibit Hall, and will consist of hands-on operation of all the Analytical Plotters displayed. Approximately a dozen instruments will be available for all participants to inspect and operate. A special badge will be required for all participants in this event. These will be available at a special registration booth. A small charge will be assessed for this event.

The second event is a session of invited papers scheduled for Thursday morning. These papers will describe new instruments and experiences by users. The third event will be held Thursday afternoon and consists of a panel discussion. Representatives of the manufacturers of those instruments will be on the panel. A question and answer session will be included. Both events are listed in the ACSM-ASP program and are open to all registrants of the Convention.

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