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Spatio-Temporal Position from Mirror Images

The use of mirrors allows the instantaneous coverage of all sides of three-dimensional objects.

Abstract: *The practical use of mirrors as an image generating medium is discussed. The detailed mathematics for a calibration process is presented. The achievable accuracy is about 1 to 2 mm when a 2-m photographic distance is used in connection with 60-mm focal length fixed base cameras. A variety of geometrics are shown with various forms of mirror utilization.*

Introduction

The photogrammetric reconstruction of spherical or round-shaped objects requires more than one stereopair of photographs. Three pairs of photographs provide adequate coverage of the object with some desirable overlap between models. Three pairs of stereo cameras must be operated simultaneously to obtain the photographs if the spatial position of the object is not repeatable in time (i.e., if the object is moving). The economy of such a close-range photogrammetric system may be unjustified considering the use of cameras and the time requirement for digital evaluation of the stereo pair of pictures. An alternate solution is the use of mirror photography, where two mirrors yield additional images of an object. Thus, one stereo pair of photographs taken with fixed based cameras can be used for the mathematical reconstruction of the spatio-temporal position of a model.

This geometrical arrangement is not without problems. The mirrors inserted in the object space must be front surfaced mirrors and free of distortion. These criteria cannot be satisfied completely in practice. Therefore, in this paper an attempt will be made to correct this problem.

Geometrical Concept

The use of plane surface mirrors has been reported upon in the literature by a number of authors. Among them, the most recent is Kratky (1975) who solved the problem analytically, employing transformations to eliminate the effect of mirror reflection. Veress *et al.* (1978) solved the problem stereoscopically by constructing a plotter to accommodate the mirror images.

To eliminate the rigid geometry required for the transformation method, a simplified mathematical solution was investigated.

The general geometry of the problem is shown by Figure 1, in which two front surfaced mirrors are placed in the object space beyond the subject at a 53° convergency angle to the camera axes. The photographic cameras are mounted on a fixed base, providing a stereo pair of photographs with about 60 percent overlap of the object as well as of its left and right mirror images. Therefore, complete pictorial information is obtainable from just a single pair of stereophotographs.

Mathematical Concept

The surface of a mirror, regarded as a plane, can be defined by the following equation:

$$AX + BY + CZ + D = 0 \qquad (1)$$

Assuming that the plane does not pass through the origin of the arbitrary coordinate system, then the equation simplifies to

$$aX + bY + cZ + 1 = 0 \qquad (2)$$

where

$$a = \frac{A}{D},\ b = \frac{B}{D},\text{ and } c = \frac{C}{D}$$

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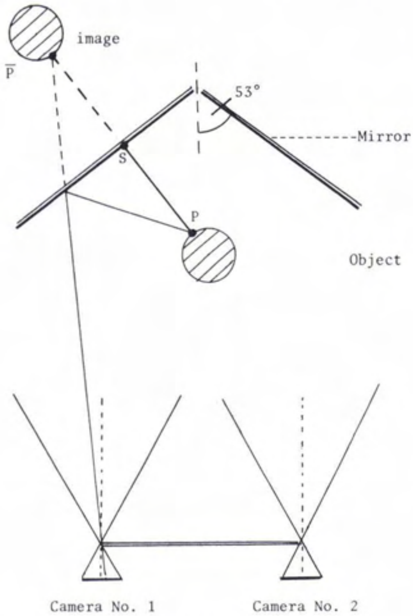

FIG. 1. General geometry.

The a, b, and c coefficients can be determined if a minimum of three known points are located on the surface of the mirror. If more than three points are available, then a least-squares adjustment provides a unique solution. According to Veress (1974), the normal equation of such a solution is

$$\begin{bmatrix} a \\ b \\ c \end{bmatrix} = -\,(\mathbf{A}^{\mathrm{T}}\mathbf{A})^{-1}\,\mathbf{A}^{\mathrm{T}} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \tag{3}$$

where

$$\mathbf{A} = \begin{bmatrix} X_1\, Y_1\, Z_1 \\ X_2\, Y_2\, Z_2 \\ \cdot \quad \cdot \quad \cdot \\ \cdot \quad \cdot \quad \cdot \\ \cdot \quad \cdot \quad \cdot \\ X_n\, Y_n\, Z_n \end{bmatrix}$$

Using this mathematical arrangement, the equation of the mirror surface can be computed. The required three or more points on the mirror surface can be determined by spatial intersection from the two photographs, or by a conventional control survey as part of calibration process.

Using the same photogrammetric intersection, the three-dimensional coordinates of any point on the surface of the subject directly facing the cameras can also be determined. To determine the coordinates of points from the mirror images needs further mathematical treatment.

If one measures the photo coordinates of a mirror-image, then the spatial coordinates ($\bar{X}$, $\bar{Y}$, $\bar{Z}$) of point $\bar{P}$ can be determined (Figure 1). These coordinates must be transformed into the X, Y, Z coordinates of the corresponding point P on the object.

The line $P\bar{P}$ of Figure 1 is perpendicular to the surface of the mirror and the points $\bar{P}$ and P are equidistant from the plane of the mirror. Therefore, the corresponding mirror surface point S has the coordinates

$$\begin{aligned} X_s &= 0.5\,(X + \bar{X}) \\ Y_s &= 0.5\,(Y + \bar{Y}) \\ Z_s &= 0.5\,(Z + \bar{Z}) \end{aligned} \tag{4}$$

These coordinates should satisfy the equation of the mirror; that is,

$$aX_s + bY_s + cZ_s + 1 = 0 \tag{5}$$

or

$$\frac{a}{2}(X + \bar{X}) + \frac{b}{2}(Y + \bar{Y}) + \frac{c}{2}(Z + \bar{Z}) + 1 = 0$$

or, after modification,

$$aX + bY + cZ + (a\bar{X} + b\bar{Y} + c\bar{Y} + 2) = 0. \tag{6}$$

This equation is the equation of the plane passing through point P and lying parallel to the surface of the mirror.

The equation of the line P$\bar{\text{P}}$ line, which is perpendicular to the mirror surface, is

$$\frac{X - \bar{X}}{a} = \frac{Y - \bar{Y}}{b} = \frac{Z - \bar{Z}}{c}\ . \tag{7}$$

Let the common ratio of this equation be t, which is constant. Then,

$$\begin{aligned} X &= at + \bar{X} \\ Y &= bt + \bar{Y} \\ Z &= ct + \bar{Z} \end{aligned} \tag{8}$$

The t can be determined by substituting Equation 8 into Equation 6. That is,

$$t = -\frac{2\,(a\bar{X} + b\bar{Y} + c\bar{Z} + 1)}{a^2 + b^2 + c^2}\,. \tag{9}$$

The coordinates of object point P from Equation 9 and 8 are

$$\begin{aligned} X &= \frac{-2a}{a^2 + b^2 + c^2}(a\bar{X} + b\bar{Y} + c\bar{Z} + 1) + \bar{X} \\ Y &= \frac{-2b}{a^2 + b^2 + c^2}(a\bar{X} + b\bar{Y} + c\bar{Z} + 1) + \bar{Y} \\ Z &= \frac{-2c}{a^2 + b^2 + c^2}(a\bar{X} + b\bar{Y} + c\bar{Z} + 1) + \bar{Z} \end{aligned} \tag{10}$$

Using this equation, the coordinates of any object point can be obtained.

These equations provide the correct coordinates only if the mirrors are without distortion. The formulation of these equations, however, permits a relatively simple solution to calibrate the mirrors and, thus, correct the distortions. The above equations can be modified into the following form for this purpose:

$$X = \frac{-2a}{a^2 + b^2 + c^2}(a\bar{X} + b\bar{Y} + c\bar{Z} + 1) + \bar{X} + f(d, X)$$

$$Y = \frac{-2b}{a^2 + b^2 + c^2}(a\bar{X} + b\bar{Y} + c\bar{Z} + 1) + \bar{Y} + g(d, Y)$$

$$Z = \frac{-2c}{a^2 + b^2 + c^2}(a\bar{X} + b\bar{Y} + c\bar{Z} + 1) + \bar{Z} + h(d, Z) \quad (11)$$

where d is the perpendicular distance from point P to the surface of the mirror, and f, g, and h are error functions which should be determined by calibrating the mirror surface. It should be noted here that the purpose of this article is only to show the feasibility of this geometry and not to establish a theoretically correct error function. Therefore, this correction will only be approximate in the following practical example.

System Calibration

Two mirrors were arranged in a geometry shown by Figure 1. The mirrors are ¼-inch thick front-surfaced mirrors but are not made of optical quality glass. The calibration process consisted of two phases, one of which was to obtain the equation of the mirror surface in order to determine the correction functions (Equation 11) of the object coordinates.

Two cameras (Figure 1) were arranged such that their positions were known in a local coordinate system. Thus, coordinates of points in the object space could be computed by space intersection. Targets were distributed on the surface of the mirror in a grid pattern and their coordinates were computed (Figure 2). The errors due to the distortion of the mirror were somewhat minimized by computing different equations for each grid element using the coordinates of four corner targets.

The determination of error functions (f, g, and h, Equation 11) required the knowledge of coordinates of points located in the object space. For this purpose, two piano wires for each mirror were suspended from the ceiling as plumb lines, each weighted with a heavy plumb bob immersed in a can of oil. Each of the lines were placed different distances from the mirror surface. Small beads with a diameter of about 3 mm were mounted on the wires to serve as coordinated target points. The arrangement is shown in Figure 2. The coordinates of the beads can be determined by conventional survey methods or, as in this case, by photogrammetric means. A second set of coordinates can be obtained from the mirror images by utilizing Equation 10). The error functions (f, g, and h) can then be determined from Equation 11.

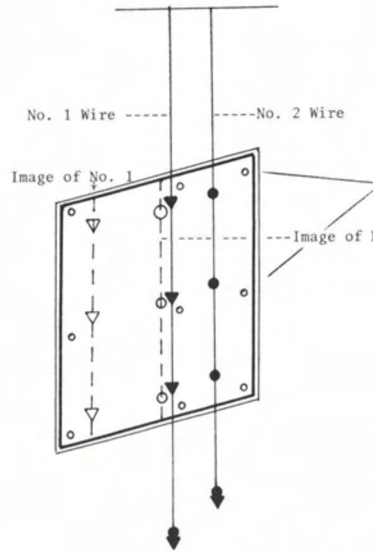

FIG. 2. Target distribution.

The error function may be expressed as polynomials. The order of the polynomials can be determined from the available number of control points and the desirable degree of refinement.

Practical Results

A parallelepiped object was placed in front of the mirrors. Target points were placed on each side of the object. A stereopair of photographs were taken with MK-70 Hasselblad metric cameras with 60-mm focal length, f/5.6 Biogon lenses. The cameras are factory matched pairs, having the same focal lengths, and they are nearly distortion free (maximum radial distortion, 5 μm).

A 620-mm horizontal baseline was used with an average photographic distance of 2 m. Such a photograph, and the target arrangements, is shown in Figure 3.

The exterior orientation of the cameras was determined from previous experiments (Fraser and Veress, 1980). The photo coordinates were measured on a Kern MK-2 monocomparator with a least reading of 1 μm.

The error functions were determined according to Equation 11 and found to be

$$\begin{aligned} f &= 0.52099 + 0.08265d \\ g &= -0.33 \\ h &= -0.24665 - 0.05347d. \end{aligned} \quad (12)$$

where d is the perpendicular distance from the object point to the surface of the mirror in cm.

The distances between the targets located on the surface of the parallelepiped were measured with a vernier micrometer with a least reading of 25 μm (0.001 inch). This least reading does not reflect the accuracy of the distance measurement,

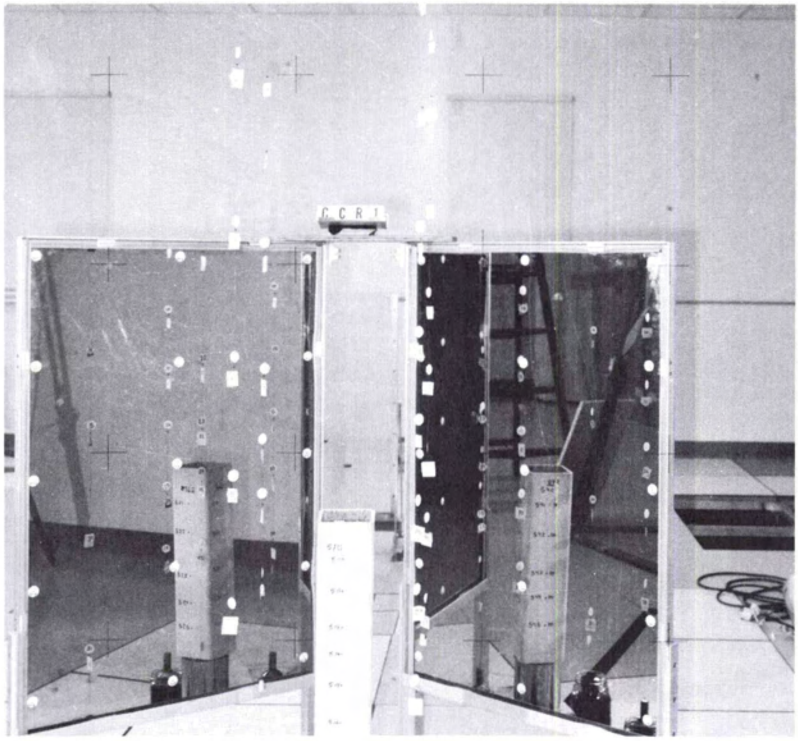

FIG. 3. Left photograph of a stereopair.

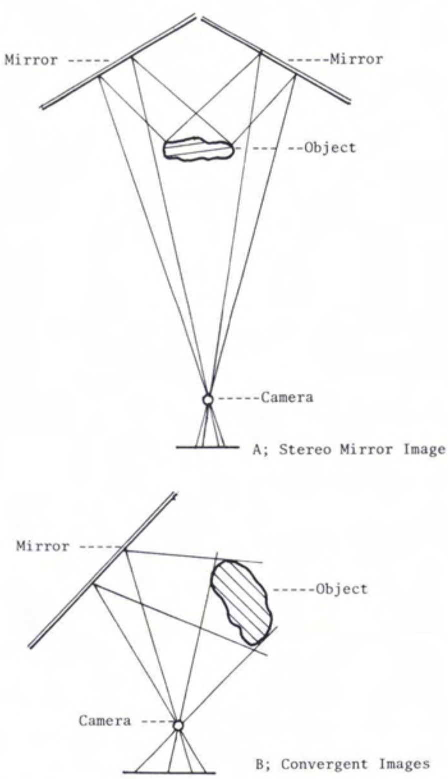

FIG. 4. Geometrical arrangements.

due to the difficulty in centering the micrometer on the targets.

Comparisons, however, were made between the measured distances and the distances computed from the photogrammetrically determined object space coordinates. The average standard error, computed from distance differences, was found to be ± 2.3 cm. using uncorrected coordinates and ± 1.4 mm using corrected coordinates according to the correction functions in Equation 12.

Discussion

This experiment showed that mirror images can be utilized numerically and can be corrected to provide spatial coordinates with acceptable accuracy. It must be pointed out, however, that the correction functions (f, h, g) are unable to correct image distortions due to small local undulations of the mirror. Therefore, it is necessary to use optical flat glass with practically no undulation, which was not the case in this experiment.

This and other experimentations reported in the literature indicate that mirrors can be used effectively as an image generating medium. Once the mirror is regarded as such, then interesting geometries can be developed which will lead to different analytics to obtain the digital model of the object.

For example, the geometry presented in this paper in Figure 1 used a horizontal baseline for the cameras. This resulted in an asymmetrical image geometry in that the left and right mirror images have different scales such as is shown in the photograph of Figure 3.

A vertical camera base would have resulted in a complete symmetrical picture where the left and right mirror images of the photograph exhibit nearly the same scale. Thus, the images could have been observed stereoscopically with a stereoscope without the disturbing effect of a different scale.

Another variety of this geometry is shown in Figure 4-A where a single camera is used to generate stereo images using two mirrors. This geometry is advantageous in connection with relatively long focal length cameras.

If a wide angle camera is available and stereoscopic images are not essential for a project, a convergent geometry as shown in Figure 4-B may be desirable. In this geometry the corresponding rays intersect each other at nearly a 90 degree angle. Thus, this arrangement will give a very precise point definition.

The above mentioned cases include only one or two mirrors, but interesting solutions can be obtained by using multiple mirrors such as three or four for a suspended object.

The mathematics shown in this paper are applicable in principle for most of those cases; however, the detailed solutions will be different.

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