Forum

Orientation and Construction of Models

YOUR JOURNAL has recently carried three long articles on orientation and construction of models in close-range photogrammetry by Atsushi Okamoto (October, November, and December, 1981) in which he explores the relationship between geometric and algebraic methods of considering the problem. He states in his opening paragraph that little has been done in the way of investigating the problem when using non-metric cameras but then goes on to ignore the disturbing feature of non-metric cameras, that is, the lack of central perspective geometry, and so the article really relates to metric photography so that we can consider the collinearity condition between image and object vectors to be valid.

In this context I do not believe that the papers add anything useful to the body of knowledge on this subject and feel that there is a danger inherent in this kind of approach which confuses quite separate bases of photogrammetry. E. H. Thompson set out these confusions very clearly in a series of papers (Thompson, 1965, 1968, 1971, 1975), three of which are published in the volume, *Photogrammetry and Surveying*, recently reviewed in your journal (Dowman, 1981). I would be grateful for the opportunity of summarizing the main points for the benefit of your readers.

First let me state some axioms:

(a) Any two figures in three-dimensional space may be related by a projective transformation which may be expressed as

$$X' = \frac{a_{11}X + a_{12}Y + a_{13}Z + a_{14}}{a_{41}X + a_{42}Y + a_{43}Z + a_{44}}$$

$$Y' = \frac{a_{21}X + a_{22}Y + a_{23}Z + a_{24}}{a_{41}X + a_{42}Y + a_{43}Z + a_{44}} \qquad \text{(D1)}$$

$$Z' = \frac{a_{31}X + a_{32}Y + a_{33}Z + a_{34}}{a_{41}X + a_{42}Y + a_{43}Z + a_{44}}$$

where the elements $a_{11}, \ldots, a_{44}$, forming a matrix $\mathbf{A}$, are independent and XYC and $X'Y'Z'$ are three-dimensional coordinates representing the figures.

(b) Any projective transformation can be expressed as a perspective and a transformation. (This explains the theoretical background to rectification.)

(c) The collinearity equations which can be derived geometrically are

$$x - x_p = - c\,\frac{r_{11}(X - X_s) + r_{21}(Y - Y_s) + r_{31}(Z - Z_s)}{r_{13}(X - X_s) + r_{23}(Y - Y_s) + r_{33}(Z - Z_s)}$$

$$y - y_p = - c\,\frac{r_{12}(X - X_s) + r_{22}(Y - Y_s) + r_{32}(Z - Z_s)}{r_{13}(X - X_s) + r_{23}(Y - Y_s) + r_{33}(Z - Z_s)} \qquad \text{(D2)}$$

where the elements $r_{11}, \ldots, r_{33}$ are dependent elements of an orthogonal matrix $\mathbf{R}$, x, y are photograph coordinates, XYZ are coordinates of a point in object space, X_s, Y_s, Z_s are the coordinates of the perspective center in the same object system, x_p, y_p are the coordinates of the principal point in the photograph coordinate system, and c is the principal distance.

(d) Equations D2 are a special case of Equations D1 when $Z' = c$. That is, we are transforming three-dimensional space to a plane, a distance c from the perspective center, and the transformation is not reversible. This is seen mathematically in that the matrix of $\mathbf{A}$ in Equations D1 becomes singular in this case.

Relating these axioms to Okamoto's equations, we can see that his Equations 8 and 15 are of the same form as Equations D1 but he does not state that when relating these equations to the photographic case the matrix of coefficients is singular. Equations 1 refer to this case but there seems little advantage in relating the equations to the errors being introduced by the measuring system when we are discussing the geometry of the image forming process. It should also be noted that the collinearity equations can be written in the form of Equation 7 but that there are still only nine independent elements and indeed Thompson (1971) and Bopp and Kraus (1978) also use these equations and relate them to the general linear transformation (Equation 1) but point out the dependence of the 11 elements with the nine elements of the collinearity equations and show that these can be handled by the use of constraints. This seems a more satisfactory approach where the mathematical model of the image formation is not confused with errors in the measuring device. The relative simplicity of the direct solution of the equations is an attraction of using them in certain situations.

It is a great shame that photogrammetric textbooks do not generally treat this topic in any depth at all, especially because the basic relationships are straightforward. However, I do not think that the subject is advanced by taking the relationship too far.

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Author's Response

I do not wish to enter into a protracted discussion with Dowman on the orientation problem of a stereo pair of photographs taken with non-metric cameras, because I believe that almost all of his criticisms are based on his misinterpretation of my papers. Thus, I shall be as brief as possible.

(1) He states that my orientation theories are valid only for metric cameras, because I go on to ignore the disturbing features of non-metric cameras. This is a clear misunderstanding. A metric camera is defined as a camera whose interior orientation parameters (conventionally three elements) are known and also whose disturbing features such as lens distortion and film deformation are negligibly small. Film deformation and lens distortion are neglected in the discussion of my papers, because parameters describing the non-linear part of the disturbing features can be determined from the coplanarity condition of corresponding rays, and those describing the linear part are absorbed by the coefficients of the collinearity equations (Hallert, 1956; Abdel-Aziz and Karara, 1971; Karara, 1972; Koelbl, 1972; Karara and Abdel-Aziz, 1974; Okamoto, 1982).

(2) I disagree with Dowman's opinion that my papers do not add anything useful to the body of knowledge on this subject. The characteristics of the coplanarity condition of corresponding rays, those of the central projective one-to-one correspondence between the model and object spaces, and those of the model construction theory with multiple photographs taken with an overlap have not been fully clarified for pictures taken with non-metric cameras.

(3) He criticizes that there seems to be confusions and contradictions in the presentation of my papers. This criticism is clearly based on an assumption that the general case where a picture has 11 independent central perspective parameters should never occur in photogrammetry. However, many photogrammetrists, e.g., Abdel-Aziz and Karara (1971), Faig (1975), Moniwa (1976, 1981) and others have solved the general collinearity condition of photogrammetry and reported that excellent results can be obtained. In addition, the geometrical characteristics of the general orientation problem of a stereo pair of pictures must first be clarified so as to investigate those of the orientation problem for various special cases in close-range photogrammetry.

(4) Some axioms in projective geometry and photogrammetry cited by Dowman are written in many textbooks and are quite well known. His criticism that the axiom (d) is not considered in the paragraph for the characteristics of the general central projective one-to-one correspondence between two three-dimensional spaces,* is quite unreasonable, because I investigate in that paragraph the geometrical properties of the one-to-one correspondence between two three-dimensional spaces such as the model and object spaces. The axiom (d) is valid for the relationship between an object space and a plane (the picture taken).

(5) I also analyze geometrically a special case where a photograph has nine independent central projective parameters (the six exterior and three interior orientation elements in conventional photogrammetry). An algebraic approach to the special case is possible, as Dowman points out, by introducing necessary constraints between the coefficients of the collinearity equations in the form of Equation 7 in Part I.

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* The term "two three-dimensional spaces" does not mean a two-dimensional space and a three-dimensional space.

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Southeast Asian Regional Conference on Photogrammetry and Remote Sensing Education

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