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Accuracy of Earthwork Calculations from Digital Elevation Data

Elevation data generated from the orthophoto scanning process, although lower in accuracy than that which is normally needed for earthwork volume computation, can potentially be used to provide accurate volume measurements.

Abstract: *Results from an investigation have clearly demonstrated that elevation data generated from the orthophoto scanning process can potentially be used to provide accurate volume determination. General prediction formulas were developed to relate the reliability of volume determination with elevation accuracy, data spacing along the cross-section profile, distance between cross-sections, and the type of topography.*

Introduction

Recent developments in photogrammetric methods in general, and orthophoto mapping in particular, have resulted in the increasing availability of digital elevation data. There are basically two approaches being used to produce orthophotomaps. The off-line approach requires the generation of elevation profiles of the terrain, which are then used to drive the orthophoto machine. In the on-line approach, the orthophoto is created strip by strip while the operator scans the stereo model along profiles. In both approaches, digital elevation data of the terrain being mapped can be generated as a direct by-product of the orthophoto mapping process.

The developments in remote sensing, digital cartography, and automation of land information systems have provided the impetus for the increasing use of digital terrain models in engineering planning and design. Usefulness of the massive amount of digital elevation data in route location and design is obvious. Optimization methods have been developed for route location using digital terrain models (Turner, 1978). Computer programs have also been developed for the automatic selection of vertical alignment based on cut-and-fill volumes (Organization for Economic Cooperation and Development, 1973).

However, the ultimate usefulness of digital elevation data depends not only on availability, but also on accuracy. The accuracy of earthwork calculation using digital terrain elevation data depends on accuracy of the elevation data, point density, and distribution of data points. This subject has yet to be fully investigated. Marckwardt (1978) and Blachut and van Wijk (1976) reported on the accuracy of elevation data generated from the orthophoto scanning process. Chandra (1979) and Ternryd (1966) attempted to relate the accuracy of volume determination with accuracy of elevation data but made no definitive conclusion.

It has long been proven in practical applications that earthwork volumes can be accurately calculated from elevation measurements made by stereophotogrammetric methods. However, in the orthophoto scanning process, the accuracy of the elevation data is dictated by the requirements of that process. The elevation profiles need only to be sufficiently accurate to avoid visible mis-

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matches between the edges of adjacent scan lines. Consequently, the accuracy of the digital elevation data obtained from the orthophoto scanning process is usually below the level normally needed for earthwork calculations. On the other hand, the density and quantity of the elevation data collected from orthophoto mapping exceeds by many folds the coverage usually used in the conventional method of photogrammetric profiling for earthwork calculation. The critical question is, Can the increase in density and quantity of the data base compensate for the decrease in data accuracy? This paper reports on an investigation aimed at resolving this question and at finding a quantitative measure of the accuracy of earthwork volumes as functions of data accuracy and density.

Error Propagation

The most commonly used formula for computing earthwork volumes in route location is the average-end-area formula

$$v = \frac{L}{2}(A_1 + A_2) \tag{1}$$

where A_1 and A_2 are the cross-sectional areas of two sections, L is the distance along the center line between the two sections, and v is the volume of earth between the two sections (see Figure 1). The total volume of earthwork along a proposed highway is then computed as the algebraic sum of the individual volume sections v_i; i.e.,

$$V = \sum_{i=1}^{n} v_i \tag{2}$$

where n is the total number of volume sections. Applying the law of propagation of random error to Equation 1 yields the expression,

$$\sigma_v = \pm \sqrt{\frac{L^2}{4}\sigma_{A_1}^2 + \frac{L^2}{4}\sigma_{A_2}^2} \tag{3}$$

where σ_v is the standard error of the computed volume v, and σ_{A_1} and σ_{A_2} are the standard error of the cross-sectional areas A_1 and A_2, respectively. Assuming $\sigma_{A_1} = \sigma_{A_2} = \sigma_A$, Equation 3 can be simplified to the following:

$$\sigma_v = \pm \frac{1}{\sqrt{2}} L\, \sigma_A \tag{4}$$

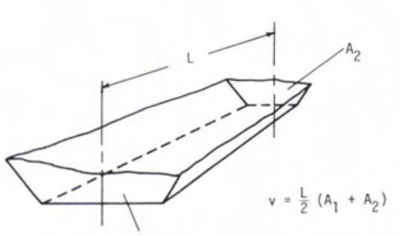

Fig. 1. Average-end-area formula.

where σ_A is the standard error of the cross-sectional areas. Equation 4 does not take into consideration the error effects due to inexactness of the basic average-end-area formula and nonuniformity of the terrain between the two end sections. Let σ_T represent the standard error of these two combined effects on the volume. Then Equation 4 may be modified to the form,

$$\sigma_v = \pm \sqrt{\frac{L^2}{2}\sigma_A^2 + \sigma_T^2} \tag{5}$$

Similarly, applying the law of propagation of random errors to Equation 2 yields the expression,

$$\sigma_v = \pm \sum_{i=1}^{n} \sigma_{v_i}^2 \tag{6}$$

where σ_{v_i} is the standard error of the volume of section i and σ_V is the standard error of the total volume of earthwork along a proposed route. For the purpose of simplicity, it can be assumed that the error characteristics of all the sections are the same. Then

$$\sigma_V = \pm \sqrt{n}\, \sigma_v \tag{7}$$

where σ_v is as expressed in Equation 5.

In Equation 5, the magnitude of σ_A is a function ofthe error in the elevation data, the data density along the ground profile of the cross-section, and the nature of the topography. A mathematical expression relating σ_A to the standard error of elevation measurements, σ_h, can be derived. Figure 2 shows a cross-section which is defined by m data points. The position of each point is defined by its horizontal distance from the center line (x_i) and its elevation (h_i). The general equation for computing the area of the cross-section by the coordinate method is

$$A = \tfrac{1}{2}[h_1(x_2 - x_m) + h_2(x_3 - x_1) + h_3(x_4 - x_2) + \ldots + h_m(x_1 - x_{m-1})] \tag{8}$$

Assuming that all of the elevations (h_i) are measured with the same order of precision as represented by the standard error, σ_h, the following expression can be derived by application of the law of propagation of random errors:

$$\sigma_A = \pm \frac{1}{2}\left[(x_2 - x_m)^2 + (x_3 - x_1)^2 + \ldots + (x_1 - x_{m-1})^2\right]^{1/2} \sigma_h \tag{9}$$

which may be simply stated as

$$\sigma_A = \pm C\sigma_h. \tag{10}$$

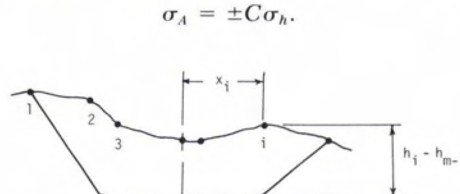

Fig. 2. Cross-section geometry.

Thus, for a given combination of road cross-section and topography, σ_A is linearly proportional to σ_h. Equation 8 assumes that the terrain has a uniform slope (i.e., follows a straight line) between adjacent data points on the cross-section profile. The larger the number of data points, the more accurate will be the computed area, A. Conversely, the fewer the number of data points and the more irregular the terrain, the larger will be the error in A. Let σ_t denote the standard error in A caused by this source. Then, the combined effects of σ_t and σ_h on σ_A can be expressed as

$$\sigma_A = \pm\sqrt{C^2\sigma_h^2 + \sigma_t^2} \quad (11)$$

Equations 5, 7, and 11 constitute a prediction model which may be used to estimate the uncertainties in volume computation. The magnitude of σ_h depends on the methods and types of instruments used for making the elevation measurements and can be easily estimated for a given project. As can be observed from Equation 9, the magnitude of the coefficient, C, depends on the geometry of the cross-section and the number of elevation points being used to define the cross-section profile. The magnitudes of σ_t and σ_T depend on the density of the data points and on how well the data points represent the topography, as well as on the geometry of the cross-section. A simulation study was conducted to provide some insights into the orders of magnitude of C, σ_t, and σ_T.

Simulation Study

THREE TYPES OF TOPOGRAPHY

Digital elevation data was generated for a road corridor situated in each of three different types of topography: flat, hilly, and mountainous. One corridor was situated in a relatively flat area, which had a relief of 20 m within the corridor. The corridor was 34 m wide and 1,504 m long. Within the first 300 m of this corridor, the average ground slope was about 10 to 15 percent in a direction transverse to the centerline of the road and about 2 percent along the centerline. Within the next 800 m, the average ground slope was between 1 and 4 percent in both directions. Within the last 400 m, the average ground slope ranged between 3 and 7 percent in both directions.

The second corridor was situated in a hilly area which had a total relief of 40 m. The corridor measured 34-m wide and 1,392-m long. The ground slope ranged between 8 and 40 percent but mostly around 12 percent.

The third corridor was situated in a mountainous area which had a total relief of 100 m within the corridor. The corridor was 34-m wide and 1,504-m long. Within the first 500 m of the corridor, the ground slope ranged from 10 to 40 percent. In the second 500 m, the slope ranged from 8 to 20 percent. Within the last 500 m of the corridor, the ground slope was irregular and ranged between 10 and 100 percent. In few locations, the slope exceeded 100 percent.

Table 1. Geometric Parameters of thf Roads Included in the Simulation Study

Geometric Parameters	Road Situated In		
	Flat Area	Hilly Area	Mountainous Area
Length of road, m	1,504	1,392	1,504
Base width of road, m	10	10	10
Side slope	2:1	2:1	2:1
Minimum road gradient, percent	1.5	3.4	6
Maximum road gradient, percent	4	6	11
Total cut volume, m^3	5,497	14,730	38,905
Total fill volume, m^3	5,765	27,223	34,019

ROAD GEOMETRY

Table 1 lists the geometric parameters of the road situated in each of the three corridors.

DIGITAL ELEVATION DATA

For each of the three corridors, digital elevation data were initially generated on a grid pattern, with a grid spacing of 2m by 2m. Thus, there were 13,554 elevation points in the corridors situated in flat and mountainous areas, and 12,546 elevation points in the corridor situated in the hilly area. These data sets constituted the basic data sets of this study. From these basic data sets, additional data sets were generated for various combinations of elevation accuracy (σ_h), distance between data points along the cross-sectional profile, and distance between cross-sections (L) along the centerline. A random number generator was used to generate fictitious elevation errors of specified standard errors (σ_h). The study included five levels of spacing between data points along a cross-sectional profile: 2 m, 4 m, 6 m, 8 m, and irregular spacing with a data point situated at any point showing a large change in ground slope. Four levels of spacing between cross-sections were included: L = 4 m, 12 m, 24 m, and 36 m.

SIMULATION APPROACH

The cut-and-fill volumes for each of the three roads were first computed using a data spacing of 2 m along the cross-section profile, 2 m between cross-sections, and no error in the point elevations. The cross-sectional areas and sectional volumes thus computed were assumed to be error free and were used as standards to determine the errors in other cases.

Figure 3 is a graph derived from the simulation

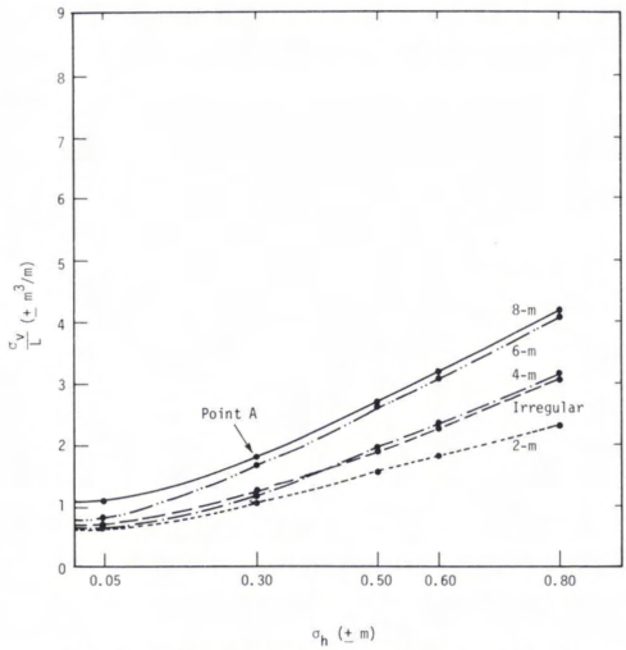

FIG. 3. σ_v/L versus σ_h for hilly terrain (L = 12 m).

study for the road located in the hilly area. The graph plots the standard error per unit length (σ_v/L) of the computed volume of a section with a length of L = 12 m versus the different levels of standard errors in the point elevations. For example, point A in Figure 3 shows that σ_v/L = 1.81 m^3/m for $\sigma_h = \pm 0.3$ m, L = 12 m, and a data spacing of 8 m between data points along the cross-section profile. To obtain this point on the graph, a data set was generated for the road with L = 12 m and a spacing of 8 m between points along the profile. Fictitious random errors with a standard error of ± 0.3 m were generated and introduced into the original point elevations. The cut-and-fill volume was then computed for each 12-m section and compared with the correct volume. Since the road

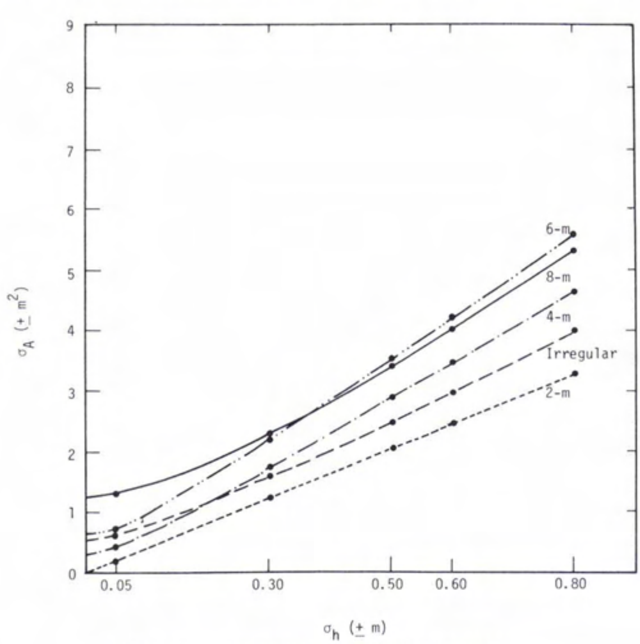

FIG. 4. σ_A versus σ_h for hilly terrain.

was 1,504-m long and $L = 12$ m, there were 125 sectional volumes. Let v_i denote the correct volume of section i, and v_i^o denote the computed volume of the same section. Then the error (ϵ_i) in the computed sectional volume was computed as

$$\epsilon_i = v_i^o - v_i \tag{12}$$

The mean ($\bar{\epsilon}$), root-mean-square error (σ_v), and root-mean-square error per unit length (σ_v/L) of the 125 sectional volumes were then computed as follows:

$$\bar{\epsilon} = \frac{\sum_{i=1}^{125} \epsilon_i}{125} \tag{13}$$

$$\sigma_v = \pm\sqrt{\frac{\sum_{i=1}^{125} (\epsilon_i - \bar{\epsilon})^2}{124}} \tag{14}$$

and

$$\frac{\sigma_v}{L} = \frac{\sigma_v}{12} \tag{15}$$

In the case of point A in Figure 3, $\sigma_v = 12 \times 1.81 = 21.7$ m³.

The standard error (σ_A) in the computed cross-sectional areas for each combination of σ_h and data spacing may also be determined using the same approach. For example, consider again the case of Point A in Figure 3. Let A_j denote the correct area of the j^{th} cross-section, and A_j^o be the computed area of the same cross-section. Then, the error (d_j) in the computed area was computed as

$$d_j = A_j^o - A_j. \tag{16}$$

Because there were $125 + 1 = 126$ cross-sections in the road, the mean ($\bar{d}$) and root-mean-square error (σ_A) of the areas were computed as

$$\bar{d} = \frac{1}{126}\sum_{i=1}^{126} d_j \tag{17}$$

$$\sigma_A = \pm\sqrt{\frac{\sum_{j=1}^{126} (d_j - \bar{d})^2}{125}} \tag{18}$$

Figure 4 shows the relationship between σ_A and σ_h for the hilly area. This graph was derived using $L = 12$ m. However, the quantity σ_A is independent of the value of L.

DETERMINATION OF C AND σ_t

Figure 4 shows in graphical form the same relationship that was expressed in Equation 11. Thus, by using the simulation results illustrated in the figure, the quantities C and σ_t in Equation 11 could be determined for each level of data density along the cross-section profiles. The results are tabulated in Table 2.

TABLE 2. VALUES OF C AND σ_t FOR VARIOUS POINT DENSITY

Point Density Along Profile	Flat σ_t	Flat C	Hilly σ_t	Hilly C	Mountainous σ_t	Mountainous C
2 m	0.00	3.95	0.00	4.15	0.00	4.19
4 m	0.34	5.21	0.35	5.77	1.10	6.78
6 m	0.78	6.30	0.64	7.00	1.71	7.23
8 m	0.93	6.39	1.29	6.34	3.10	8.69
Irregular	0.28	4.89	0.57	4.91	0.67	5.83

DETERMINATION OF σ_T

The quantity σ_T in Equation 5 was determined for each of the three different types of terrain at four different values of L. Simulation cases were conducted using error-free elevations, 2-m point spacing along the cross-sectional profiles, and four different values of L. The cross-sectional areas were thus free of errors in these cases. Any error in the computed volumes would be attributed to the topography and the imperfection of the average-end-area formula. That is, for these cases, $\sigma_T = \sigma_v$. The results from these cases are summarized in Figure 5, in which the values of σ_T/L are plotted against L.

VERIFICATION

Equations 5, 7, and 11 can now be used with the values of C and σ_t in Table 2 and σ_T/L in Figure 5 to predict the accuracy of volume determination for the three roads. In fact, the value of σ_v/L for each of about 141 simulation cases was computed by this approach and compared with the value obtained by simulation. The maximum difference in the values of σ_v/L was 25 percent of σ_v/L. Thus, the simulation results and the prediction formulas verified the validity of each other.

HYPOTHETICAL CASES

The prediction model described above was used to determine the accuracy of volume computation for several hypothetical cases. The following procedure was used:

- Define the values of the following parameters for each hypothetical case: σ_h, L, spacing between data points along a cross-sectional profile, and type of topography.
- Look up the values of C and σ_t from Table 2.
- Compute σ_A using Equation 11.
- Obtain value for σ_T from Figure 5.
- Compute σ_v using Equation 5.
- Compute σ_V using Equation 7.

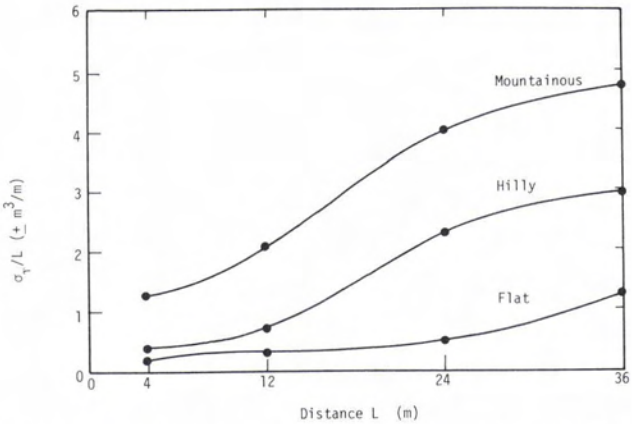

FIG. 5. σ_T/L versus L for flat, hilly, and mountainous terrain using error-free elevations and 2-m interval between data points along profile.

The results are tabulated in Table 3. In all cases, the total length of the road was 100 km and the distance between elevation points along the cross-section profile was assumed to be 4 m. In order to make use of the values of C and σ_t in Table 2 and the values of σ_T/L in Figure 5, the road was assumed to have the same geometry as that used in the simulation study, i.e., base width = 10 m and side slope is 2:1.

The following observations can be made from the results in Table 3:

- The parameter L, i.e., the distance between cross-sections, has the largest influence on the accuracy of volume computation. The accuracy decreases with increasing value of L.
- Decrease in elevation accuracy (i.e., increase in σ_h) can be compensated by decrease in the distance L. For example, in case 1, where σ_h = $\pm$ 0.01 m and L = 10m, σ_V amounted to $\pm$ 380, 650, and 2,010 m^3 for flat, hilly, and mountainous terrain, respectively. In case 3, when σ_h was increased to $\pm$ 0.1 m and L was decreased to 4 m, σ_V amounted to $\pm$ 304, 392, and 980 m^3 for flat, hilly, and mountainous terrain, respectively. Thus, even though there was a ten-fold increase in σ_h, case 3 actually yields more accurate volumes than case 1.
- Even low-accuracy elevation data (with σ_h amounting to as much as $\pm$ 0.3 m) may be used to yield accurate volume measurements as long as such data are available in high density. See cases 5 and 6 in Table 3.
- The accuracy of volume determination is largely independent of the magnitude of the earthwork volumes.

TABLE 3. ACCURACY OF VOLUME COMPUTATION FOR SEVERAL HYPOTHETICAL CASES*

Case No.	σ_h ($\pm$ m)	L (m)	σ_V($\pm$ m^3) Flat Terrain	Hilly Terrain	Mountainous Terrain
1	0.01	10	380	650	2,010
2	0.01	35	2,282	5,444	8,905
3	0.1	4	304	392	980
4	0.1	10	530	760	2,060
5	0.2	4	506	595	1,113
6	0.3	4	727	829	1,303
7	0.3	10	1,170	1,390	2,470

* All cases: Total length of road = 100 km
Base width of road = 10 m
Side slope = 2:1
Distance between elevation points along cross-section profile = 4 m.

CONCLUSION

This study has shown that elevation data generated from the orthophoto scanning process, although lower in accuracy than that which is normally needed for earthwork volume computation, can potentially be used to provide accurate measurements. General prediction formulas were developed to relate the reliability of volume determination with elevation accuracy, data spacing along the cross-section profile, distance between cross-sections, and the type of topography. The values of C, σ_t, and σ_T that have been derived from the simulation study are applicable only to a road which has a base width of 10 m and a side slope of 2:1. However, by using the approach described in this paper, these parameters can be determined for any cross-section geometry.

The investigation reported in this paper is part of the dissertation research conducted by Dr. Y. M. Siyam. More details on this study can be found in his dissertation (Siyam, 1981).

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