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Multi-Segments Approximation to a Three-Dimensional Curved Line Using the Least-Squares Locating Method

In the multi-segments approximation of a spiral wire, the RMS value of errors was less than 1 mm when using three photographs taken with a camera located about 2 metres from the object.

ABSTRACT: *The least-squares locating method (LSLM), which has been proposed for the location of linear objects lacking discernible points in a three-dimensional space by using multiple projections, is revised based on the direct linear transformation (DLT) method in order to apply it to close-range photogrammetry using control points. The multi-segments approximation method for locating a three-dimensional curved line is developed on the basis of the revised LSLM in the case of the central projection. An experiment to locate a three-dimensional spiral wire is performed by applying the approximation method, and the results of the experiment show that the method is useful for the accurate location of curved linear objects.*

INTRODUCTION

CLOSE-RANGE PHOTOGRAMMETRY has been widely applied in various scientific fields for measurements of three-dimensional objects, for example, the analyses of bubble chamber photographs (Garfield, 1964) and electron microscopic photographs and in consulting for radiation therapy (Rener *et al.*, 1980). It has, however, some defects for certain uses. Conventional analysis methods used in close-range stereo photogrammetry are based on triangulation, and they can only determine the position of a point which is clearly discernible on the object. A common defect of those methods then, is, that it is difficult to determine the position of a line on which discrete points are not discernible.

Another defect of those methods is that it is not always possible to measure the projective parameters—for example, the projective orientation, the position of the projection center and of the principal point, the magnification, and so on—for every photograph with the desirable accuracy.

In order to counteract the above mentioned first defect, we have already proposed a new method, the least-squares locating method (LSLM) (Kondo *et al.*, 1982a). It has been shown to be effective for locating linear objects without discernible marks. It was applied to the location of crystal lattice dislocations in ice, and the results proved the usefulness of the method (Kondo *et al.*, 1982b). The second defect, generally, can be eliminated by using a sufficient number of control points whose positions are definitely known because the projective parameters are substituted for by the control points (Rener *et al.*, 1980; Wong, 1975; Abdel-Aziz and Karara, 1971; Abdel-Aziz and Karara, 1974). Recently, we revised the early version of the LSLM in this defect by using control points, and experimentally investigated the accuracy of the revised LSLM.

In this paper, first, the principle of the LSLM

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utilizing control points is introduced. Second, the multi-segments approximation algorithm (Kondo *et al.*, 1982b) for a curved line in a three-dimensional space is presented for the central projection, applying the revised LSLM. Finally, the algorithm is applied to the location of a complicated curve, and the experimental results are discussed.

Least-Squares Locating Method (LSLM)

The LSLM is a least-squares method used to determine the position of a linear object lacking discernible marks from its multi-projection. If the projective parameters (exterior- and interior-orientation parameters) and the coordinates of a sampled point (data point) on each projection are given, the ray (data line) passing through both the data point and the center of projection can be defined in a three-dimensional space. If the object is a point, then the location of the point could be determined in principle as the intersection of two data lines corresponding to their respective projections. In practice, however, various measurement errors involved in available data often make the data lines fail to cross. As is often the case with linear objects without discernible points, there are many instances in which we can neither determine such intersections nor obtain data points which are well correspondent to each other on each projection. Therefore, it is necessary to locate the object based on other than the intersections of data lines. As every data line, ideally, should intersect the object, one possible method is to locate the object so that the average distance between data lines and the object is minimized. The LSLM has been developed on the basis of this idea in order to determine the position of such an object. The principle of the LSLM is as follows:

As shown in Figure 1, let a vector **V** denote the parameters describing the position of an object, Q. Then the error $d_j^{(i)}$ between Q and the data line $\mathbf{L}_j^{(i)}$, which is defined by both the center $\mathbf{S}^{(i)}$ and the j^{th} data point $\mathbf{p}_j^{(i)}$ of the i^{th} projection, is a function of **V**. Based on the least-squares method, the position of Q is given by $\mathbf{V}^*$, which minimizes the cost function,

$$f(\mathbf{V}) = \sum_i \sum_j \{d_j^{(i)}(\mathbf{V})\}^2. \tag{1}$$

The solution $\mathbf{V}^*$ is determined by using all data points and is optimal in the sense of least squares. The minimization of Equation 1 can be performed by the use of the simplex method (Nelder and Mead, 1965), which is well known to be effective for the optimization problem of the unimodal function.

When the errors $d_j^{(i)}$ in the cost function (Equation 1) are calculated, the linear projective transformation from an image space into an object space is required in order to define the data line corresponding to a data point in the object. A method which is similar to one developed by Wong (1975) is applicable. Wong's method is the modification of the direct linear transformation (DLT) method of Abdel-Aziz and Karara (1971, 1974) by using control points.

If lens distortion is negligible, the basic equations of the DLT are

$$\begin{aligned} x_n &= \frac{C_1X_n + C_2Y_n + C_3Z_n + C_4}{C_9X_n + C_{10}Y_n + C_{11}Z_n + 1}, \\ y_n &= \frac{C_5X_n + C_6Y_n + C_7Z_n + C_8}{C_9X_n + C_{10}Y_n + C_{11}Z_n + 1}, \end{aligned} \tag{2}$$

where X_n, Y_n, Z_n are the object space coordinates of the n^{th} point, x_n, y_n are the corresponding measured image coordinates, and $\{C_1, C_2, \ldots, C_{11}\}$ are 11 transformation parameters. Because Equation 2 is linear with respect to $\{C_1, C_2, \ldots, C_{11}\}$, they can be determined for a projection by using more than six control points, and by solving the following simultaneous equations:

$$\begin{bmatrix} x_1 \\ x_2 \\ \cdot \\ \cdot \\ \cdot \\ x_6 \\ y_1 \\ y_2 \\ \cdot \\ \cdot \\ \cdot \\ y_5 \end{bmatrix} = \begin{bmatrix} X_1 & Y_1 & Z_1 & 1 & 0 & 0 & 0 & 0 & -x_1X_1 & -x_1Y_1 & -x_1Z_1 \\ X_2 & Y_2 & Z_2 & 1 & 0 & 0 & 0 & 0 & -x_2X_2 & -x_2Y_2 & -x_2Z_2 \\ & & \cdot & & & \cdot & & & & \cdot & \\ & & & & & & & & & & \\ & & & & & & & & & & \\ X_6 & Y_6 & Z_6 & 1 & 0 & 0 & 0 & 0 & -x_6X_6 & -x_6Y_6 & -x_6Z_6 \\ 0 & 0 & 0 & 0 & \mathrm{X}_1 & \mathrm{Y}_1 & \mathrm{Z}_1 & 1 & -y_1X_1 & -y_1Y_1 & -y_1Z_1 \\ 0 & 0 & 0 & 0 & \mathrm{X}_2 & \mathrm{Y}_2 & \mathrm{X}_2 & 1 & -y_2X_2 & -y_2Y_2 & -y_2Z_2 \\ & & & & & & & & & & \\ & & \cdot & & & \cdot & & & & \cdot & \\ & & & & & & & & & & \\ 0 & 0 & 0 & 0 & \mathrm{X}_5 & \mathrm{Y}_5 & \mathrm{Z}_5 & 1 & -y_5X_5 & -y_5Y_5 & -y_5Z_5 \end{bmatrix} \begin{bmatrix} C_1 \\ C_2 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ C_{11} \end{bmatrix} \tag{3}$$

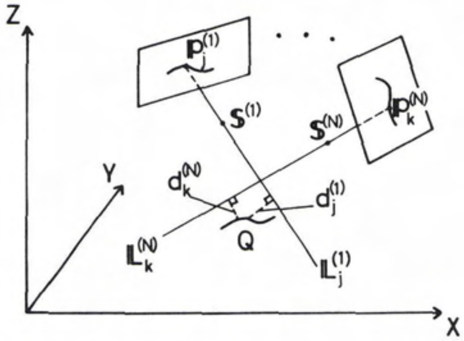

FIG. 1. The distance $d_j^{(i)}$ between the object Q and data line $\mathbf{L}_j^{(i)}$.

MULTI-SEGMENTS APPROXIMATION METHOD FOR A CURVED LINE

It is difficult to describe and to locate an object of a continuous curved line in a three-dimensional space by an analytical function of parameter, t. Thus, the position of such a curve is, usually, determined only approximately. The multi-segments approximation method (Kondo *et al.*, 1982b), which we have already developed for the parallel projection, is one of such location methods and can be extended to the central projection such as a close-range photograph.

Let a curve be located by approximating with a series of K segments. The K segments are defined by $K + 1$ nodes; $Q_1(X_1, Y_1, Z_1)$, $Q_2(X_2, Y_2, Z_2), \ldots,$ $Q_{K+1}(X_{K+1}, Y_{K+1}, Z_{K+1})$, and then the total number of the parameters necessary for locating the curve is $3(K + 1)$. The r^{th} segment is expressed by

$$\frac{X - X_r}{X_{r+1} - X_r} = \frac{Y - Y_r}{Y_{r+1} - Y_r} = \frac{Z - Z_r}{Z_{r+1} - Z_r}, \tag{4}$$

where $Q_r(X_r, Y_r, Z_r)$ and $Q_{r+1}(X_{r+1}, Y_{r+1}, Z_{r+1})$ are its nodes.

The distance between the j^{th} data point and the image of the r^{th} segment on the i^{th} projection plane is utilized for the distance measure $d_{jr}^{(i)}$ by multiplying it by the reciprocal of the magnification factor of the projection. Figure 2 shows the relation between data points and the image of the r^{th} segment $l_r^{(i)}$. The least value of K distances between the data point $\mathbf{p}_j^{(i)}$ and K segments is available for the distance measure $d_j^{(i)}$ of $\mathbf{p}_j^{(i)}$. That is,

$$d_j^{(i)}(\mathbf{V}) = \min_{r=1\sim K} \{d_{jr}^{(i)}(\mathbf{V})\}, \tag{5}$$

where $\mathbf{V}$ is the vector $(X_1, Y_1, Z_1, X_2, Y_2, X_2, \ldots, X_{K+1}, Y_{K+1}, Z_{K+1})^{\mathrm{T}}$.

The distance measure $d_{jr}^{(i)}$ can be evaluated in two cases shown in Figure 3 and is given as follows:

Case (I); if $\mathbf{h}_{jr}^{(i)}$ is on the segment $l_r^{(i)} = \overline{\mathbf{q}_r^{(i)}\mathbf{q}_{r+1}^{(i)}}$,

$$d_{jr}^{(i)} = \frac{1}{m^{(i)}} \left|(\mathbf{p}_j^{(i)} - \mathbf{q}_{r+1}^{(i)}) \times (\mathbf{q}_r^{(i)} - \mathbf{q}_{r+1}^{(i)})\right| / \left|\mathbf{q}_r^{(i)} - \mathbf{q}_{r+1}^{(i)}\right|, \tag{6a}$$

Case (II); otherwise

$$d_{jr}^{(i)} = \frac{1}{m^{(i)}} \min\{|\mathbf{p}_j^{(i)} - \mathbf{q}_r^{(i)}|, |\mathbf{p}_j^{(i)} - \mathbf{q}_{r+1}^{(i)}|\}, \tag{6b}$$

where $\mathbf{q}_r^{(i)}$ and $\mathbf{q}_{r+1}^{(i)}$ are the projected image of the nodes Q_r and Q_{r+1}, respectively, $\mathbf{h}_{jr}^{(i)}$ is the foot of the perpendicular from $\mathbf{p}_j^{(i)}$ to the segment $l_r^{(i)}$, and $m^{(i)}$ is the magnification factor of the i^{th} projection plane.

$\mathbf{V}^{*}$ which minimizes the cost function $f(\mathbf{V})$ in Equation 1 substituted $d_j^{(i)}(\mathbf{V})$ by Equation 5 and Equation 6a or 6b, is the least-squares position of the curve. The method deciding the initial multi-segments for the minimization of $f(\mathbf{V})$ is the same as one used in the previous paper (Kondo *et al.*, 1982b).

LOCATION OF A THREE-DIMENSIONAL CURVED WIRE

The multi-segments approximation method was applied to the location of a spiral wire, as shown in Figure 4. The wire was approximated with K segments, for $K = 5 \sim 9$, by employing three photographs, (a), (b), and (c) in Figure 4. Seven vertices of the H-shaped iron block were used for control points and taken in the photographs simultaneously. The origin of the reference coordinates was set on one of these vertices of the block and the X-, Y-, and Z-axis were set along either edge of the top plane and the height of the block, respectively.

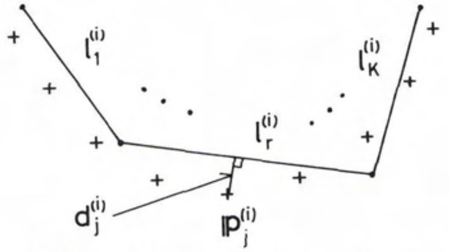

FIG. 2. Data points and projected multi-segments.

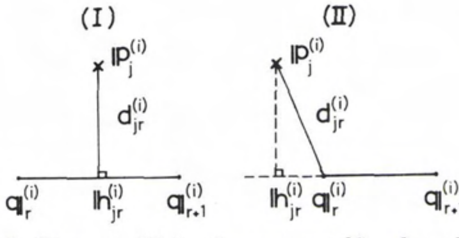

FIG. 3. Distance $d_{jr}^{(i)}$ for the segment $l_r^{(i)}$, where $\mathbf{h}_{jr}^{(i)}$ is the foot of the perpendicular from $\mathbf{p}_j^{(i)}$ to the segment.

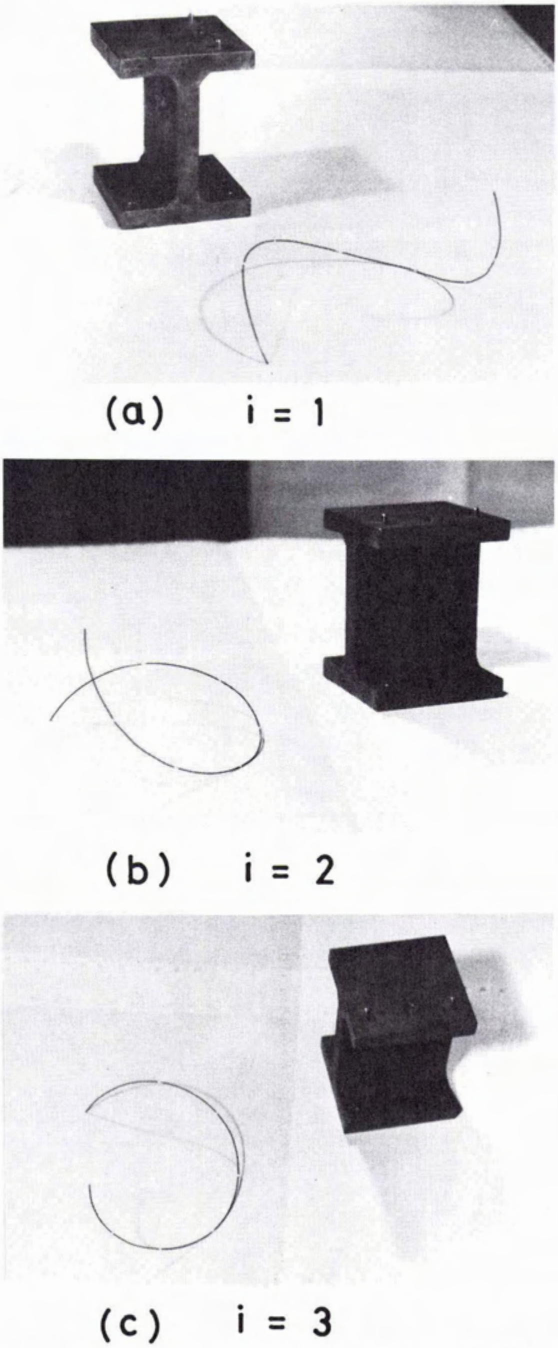

FIG. 4. Photographs of a wire to be located and a block as control points.

TABLE 1. RMS VALUES D OF $d_j^{(i)}$ FOR K = 5, 6, 7, 8, AND 9 (IN mm)

K	5	6	7	8	9
D	1.4	1.1	1.0	0.8	0.7

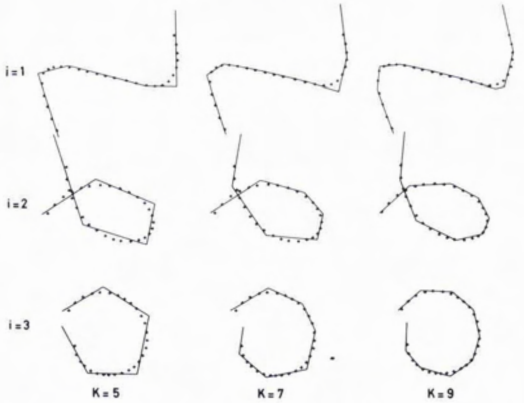

FIG. 5. Projections of least-squares segments on each projection plane.

Eighty-seven total data points on the images of the curved line, (a); 27, (b); 31, and (c); 29 points, were randomly sampled from three photographs and used for the location.

The more accurate result corresponds to the smaller root-mean-square (RMS) value D is defined by,

$$D = \sqrt{\frac{1}{N}\sum_{i}\sum_{j}\{d_j^{(i)}\}^2}, \qquad (7)$$

where N is the total number of data points. Table 1 shows the RMS values D for $K = 5 \sim 9$. It can be seen from the table that D decreases as K increases, that is, the approximation is improved by increasing K. This fact is ascertained by Figure 5 which shows the projections of the finally approximated multi-segments, for K = 5, 7, and 9, on each projection plane overlapped onto data points. These figures also show that a more accurate approximation is achieved as K increases.

The accuracy of the location is investigated by

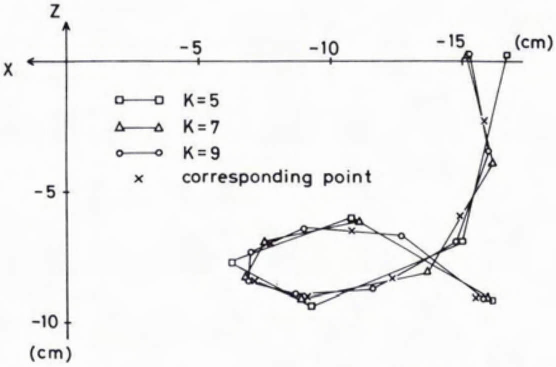

FIG. 6. Projections of the optimally estimated wire on the Z-X plane.

comparing the resultant position of each multi-segment with the least-squares positions of the eight points which had been marked on the wire for the discernment and could be located by the triangulation. Figure 6 shows the projections of each multi-segments onto the Z-X plane and the cross marks in the figures show projections of the eight points. The distances between those points and the multi-segments indicated the accuracy of the location. If the probability density function of locating error is assumed to be normal, then it can be stated that, in the case of $K = 9$, 68 percent of the whole curve has been located with an error less than ±0.7 mm.

CONCLUSION

A revised LSLM using control points based on the DLT was applied to close-range photogrammetry, and the result of multi-segments approximation for a complicated curve was discussed.

In the multi-segments approximation of a spiral wire, the RMS value of errors was less than 1 mm when using three photographs taken with a camera about 2-m distant from the object. The accuracy of the location obtained by this experiment is satisfactory for the measurement error of the projection coordinates, ±0.2 mm.

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