C. S. Fraser
Division of Surveying Engineering
The University of Calgary
Calgary, Alberta T2N 1N4, Canada

Photogrammetric Monitoring of Turtle Mountain: A Feasibility Study*

Free-network adjustment and sensitivity analysis are employed in the design of a deformation monitoring scheme.

Abstract: *Analytical photogrammetry is nowadays finding wide application as a precise non-contact measuring tool for deformation monitoring. This paper reports on an investigation which was carried out to ascertain the feasibility of photogrammetrically monitoring the south peak area of Turtle Mountain, Alberta, site of the famous Frank Slide. Simulations employing free-network adjustment, and sensitivity analysis using formulated deformation models, indicate that low-level aerial photography, analytically restituted, provides a monitoring network which is sensitive enough to detect subcentimetre object target point movements. Aspects of the monitoring network design are discussed, the free-network adjustment approach is outlined, and the application of sensitivity analysis is detailed.*

Introduction

Turtle Mountain and the Frank Slide

In the morning hours of 29 April 1903, above the town of Frank in the Crowsnest Pass of southern Alberta, the eastern side of the summit of Turtle Mountain broke free and plummetted into the valley below. This major rockslide avalanche, which was named the Frank Slide, contained some 90 million tons of rock and is one of the largest known to have occurred in the past few thousand years. Parts of the south side of the town of Frank were buried by up to 30 m of debris and 70 people were lost. As a result of the 100-second slide, about three square kilometers of the valley floor were covered by rubble to an average depth of 14 m. The volume of broken limestone which moved down Turtle Mountain is estimated to have been about 30 million cubic metres (Cruden and Krahn, 1978).

The appearance of the slide scar and rock debris, which are shown in Figure 1, gives a stark reminder of the mammoth dimensions of the Frank Slide. It is worth noting in the figure that the height difference between the Crowsnest River and the south peak of Turtle Mountain is close to 1000 m, and also that the eastern extremeties of the slide debris are at elevations of 50 m or so above the elevation of the river.

The fundamental structural characteristic of Turtle Mountain which contributed to the slide appears to be that the top of the mountain consists of a tightly folded anticline, with steeply sloping bedding planes on the eastern limb. The slide mass, which was in places over 100 m thick, initially moved downward along these bedding planes. For further details on the geological and geotechnical aspects of the Frank Slide, the reader is referred to Cruden & Krahn (1978) and Beaty (1975).

Today, 80 years after the landslide, there are still concerns about the stability of Turtle Mountain. Most attention is centered on a five million cubic metre rock wedge comprising the south peak, which may fall eastwards. A map of this area is illustrated in Figure 2, and a photograph of portions of the potentially unstable pyramid block is shown in Figure 3. Outlining the wedge on the west side is a major fissure, Crack 1 in Figures 2 and 4, one of many gaping cracks in the broken rock of the mountain top.

* Presented paper, 49th Annual Meeting of the American Society of Photogrammetry, Washington, D.C., 13-18 March 1983.

0099-1112/83/4911-1551$02.25/0

FIG. 1. Oblique aerial view of the Frank Slide, Crowsnest Pass, Alberta (photo courtesy of C. Beaty, University of Lethbridge).

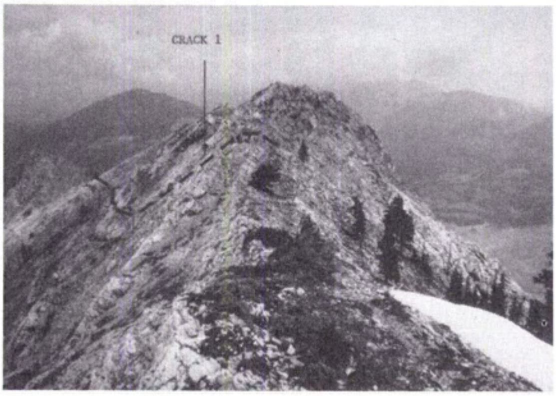

FIG. 3. The south peak of Turtle Mountain, as viewed from the south.

IN-SITU MONITORING AT THE SOUTH PEAK

The history of displacement monitoring on Turtle Mountain dates back to the early 1930s when crack surveys of the crown of the Frank Slide were carried out. Since then, a number of further surveys have been undertaken, with the most recent being an *in-situ* monitoring program designed to detect crack movements at the margins of the rock wedge on the south peak. Two TM 71 Crack Motion Detectors are employed to monitor dilation and shear effects in one gaping crack. These measurements are supplemented by extensometer and leveling data between bolts driven into the rock on opposite sides of Crack 1 on the west side of the south peak. The steep sides of the peak area are certainly less than ideal for spirit leveling, as is indicated by the position of the rodman in Figure 4. Complete details of this program are given in Cruden *et al.* (1982).

Whereas the current monitoring scheme is capable of detecting crack motion to a high accuracy—0.1 mm/year for the TM 71 gauges and about 1 mm for the leveling and extensometer observations—it does have two main drawbacks. The first is that the area being monitored is fairly small in relation to the area covered by the five million cubic metre rock wedge. Thus, any movement that does not take place across Crack 1 is unlikely to be detected; i.e., the measuring scheme effectively examines only the hypothesis that Crack 1 is the boundary of the unstable zone. The second drawback is that, if hazardous movements occur to the extent where failure is an-

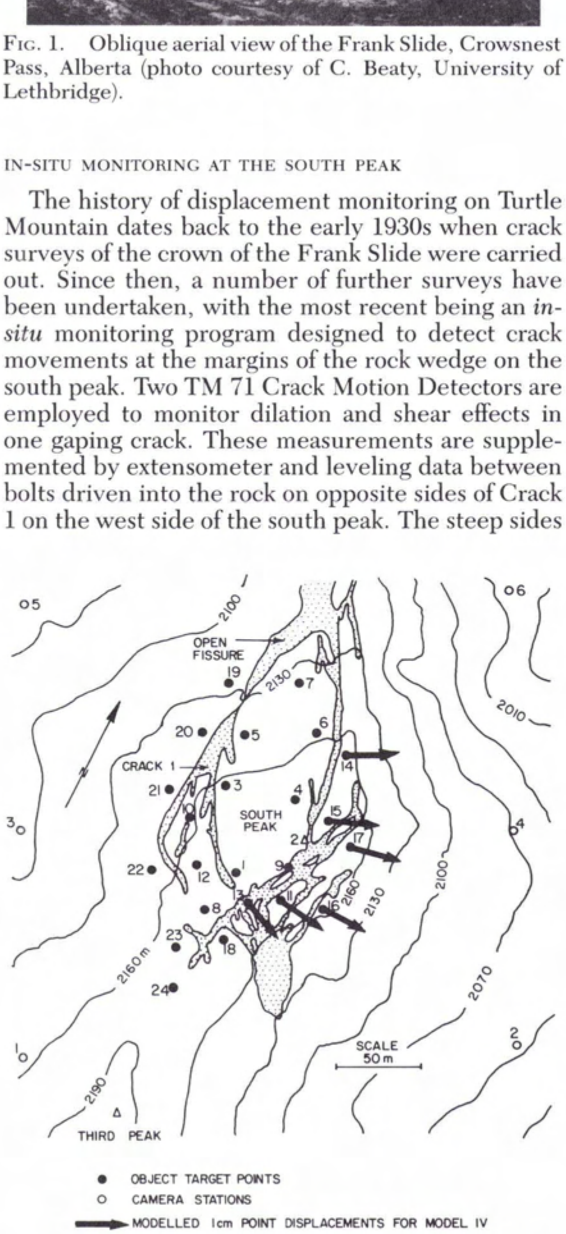

FIG. 2. Map of the south peak area of Turtle Mountain showing object target point and camera station locations.

FIG. 4. A portion of Crack 1 on the west face of the south peak.

ticipated, it will no longer be safe to occupy the south peak area and carry out measurements.

THE NEED FOR A PHOTOGRAMMETRIC MONITORING SCHEME

As a precise non-contact measuring tool, analytical photogrammetry is ideally suited to the task of deformation monitoring. Over the past decade or so, reported applications of close-range photogrammetry have included the monitoring of rock faces and earth walls (Allam, 1975; Torlegård and Dauphin, 1975; Veress *et al.*, 1980; Robertson *et al.*, 1982), mining subsidence (Armenakis and Faig, 1982), and landslide areas (Erlandson and Veress, 1975; Negut, 1980).

In comparison to other deformation monitoring systems, photogrammetry offers a few unique advantages. In cases where structures are loaded to failure, or where hazardous movements are expected, the non-contact feature of photogrammetric measurements is particularly beneficial. Further, there are few limitations on the number of object points employed. One extensometer reading yields a single distance (or change of distance) observation, and a point in a surveying network needs to be "tied in" by physically setting up an instrument or target over that point. However, it is immaterial as far as the photogrammetric data acquisition system is concerned whether there are 10 or 100 targetted points at which movement is to be measured, so long as all points are imaged on a sufficient number of photographs. Thus, a comprehensive picture of the entire deformation pattern can be built up simply by setting out a sufficient number of targets, and no extra field measurements are required. Another feature of photogrammetric networks for deformation monitoring—about which a number of misconceptions remain in the surveying community—is that there is no need for independently surveyed control to absolutely orient the photogrammetric block. Analytical photogrammetry is essentially a stand-alone monitoring technique, which can rigorously incorporate additional surveying observations should they be available.

At the invitation of the provincial department of the environment, Alberta Environment, a preliminary investigation was undertaken to ascertain the practicability of using analytical photogrammetry to monitor the South Peak of Turtle Mountain. This study, the results of which were reported by Fraser (1982b), was conducted without the benefit of a site visit to the mountain top. Basically, two imaging geometries where considered: a convergent configuration using helicopter borne photography (of 200-mm focal length), and a "normal" low-level aerial geometry comprising 100 percent overlap of the target array and 152-mm focal length photography. Simulations employing free-network adjustment and sensitivity analysis indicated that in the case of both geometries subcentimetre point displacements could be detected between two measuring epochs. On visiting Turtle Mountain, however, it was found that the steepness of the sides of the south peak (see Figure 3) effectively ruled out the more expensive convergent imaging geometry as a practical proposition. Thus, attention was directed to the "normal" geometry, the object target array was redesigned, and a number of deformation models were formulated for use in a sensitivity analysis. The purpose of this paper is to demonstrate, through rigorous analysis techniques, that low-lewel aerial photography, analytically restituted, provides a photogrammetric network which is sensitive enough to detect subcentimetre deformation movements on Turtle Mountain. In addition, an explanation is given as to why a free-network approach is warranted, and the theory of network sensitivity analysis is briefly explained.

THE PHOTOGRAMMETRIC MONITORING SCHEME

NETWORK DESIGN AND DIAGNOSIS

A photogrammetric network shares many common features with three-dimensional surveying networks in Euclidean space. It is, therefore, not surprising that techniques employed in the design, diagnosis, and optimization of monitoring networks in surveying are in many instances applicable to photogrammetry. For example, the problems of zero- and first-order design are directly applicable to photogrammetric networks, although the second- and third-order design problems are simplified to a considerable extent. Moreover, aspects of statistical quality and accuracy can be evaluated according to standard methods by reference to the precision, reliability, and sensitivity of the network. (Recent reviews of these aspects, as they relate to monitoring networks in engineering surveying, are given by Alberda (1980) and Niemeier (1982).) Prior to examining the precision and sensitivity of the proposed photogrammetric monitoring network, it is useful first to examine aspects of the network geometry.

NETWORK GEOMETRY

In designing a photogrammetric monitoring network for Turtle Mountain, a few physical constraints apply. Low-level, slow-speed flying in the front ranges of the Rocky Mountains can be—and frequently is—a hazardous operation. This fact, coupled with the need to minimize image motion to an acceptable level, led to the selection of a minimum design flying height of 320 m above the south peak, i.e., an altitude of about 2480 m. A standard mapping camera of 152 mm used at this height yields an image scale of about 1:2100. The network was designed around this photographic scale, such that there would be a six-fold coverage of each object point. Accordingly, the camera stations would be situated as shown in Figure 2.

Also shown in Figure 2 are the locations of 24 object target points. As well as affording the detection of outward motions at Crack 1, in accordance with the principal geotechnical hypothesis, the object points are positioned so that relative movements between different broken rock segments of the south peak can be detected. The actual positioning of the object target points is a fairly arbitrary procedure at the design stage, and considerable point location leeway can be tolerated before the geometric strength and sensitivity of the photogrammetric network are noticeably influenced.

Until such time that a full diagnosis based on simulated data is carried out, there can be no assurance that the photogrammetric monitoring network meets the accuracy criteria for which it was designed. However, having the preliminary geometric design does allow the photogrammetrist to ascertain a coarse measure of the attainable object point positioning accuracy. Such an overall accuracy indicator is provided by the formula

$$\bar{\sigma}_c \simeq q\ S\ \sigma \tag{1}$$

where $\bar{\sigma}_c$ can be termed the mean standard error of the object point coordinates, S is the average scale number, σ is a global estimate of the standard error of image coordinate measurements, and q is a factor whose magnitude is primarily dependent on the network geometry. For a "normal" six-photo imaging configuration, with 100 percent overlap of the target array, a value of q of about 0.5 to 0.7 can be anticipated if the network is adjusted by the bundle method with inner constraints (Fraser, 1983). Application of Equation 1 to the network described, with q = 0.5 and σ = ± 2.5 μm, yields a rough standard error estimate of about 2.6 mm. Given that the major design criteria is the detection of movements of 1 cm or more, this level of precision would indicate that the network design should be subjected to a full diagnosis, prior to any changes being made to the preliminary imaging geometry.

PHOTOGRAMMETRIC ADJUSTMENT APPROACH

Although it is more a problem of network design than diagnosis, the fixation of the zero-variance computational base, or datum, has a considerable impact on the precision of a minimally constrained photogrammetric adjustment (Fraser, 1982a). The dynamic nature of the network, coupled with the geology of the south peak area, precludes the establishment of a minimal object space control field (either arbitrarily assigned or independently surveyed) which can be considered stable over all measuring epochs. Thus, the deformation problem becomes one of "relative" motions and a free-network adjustment procedure needs to be followed.

A commonly adopted approach in the analysis of deformation measurements obtained by geodetic methods involves the use of the pseudo-inverse (Pelzer, 1979; Niemeier, 1981). This so-called "main" solution can be written for the self-calibrating bundle adjustment as

$$\begin{aligned}\hat{\mathbf{x}} &= (\mathbf{A}^T\mathbf{P}\mathbf{A} + \mathbf{P}^o)^+ (\mathbf{A}^T\mathbf{P}l - \mathbf{P}^o l^o)\\ &= \mathbf{Q}_x (\mathbf{A}^T\mathbf{P}l - \mathbf{P}^o l^o)\end{aligned} \tag{2}$$

where

$\hat{\mathbf{x}}$	is the vector of unknown parameters (object point coordinates, exterior orientation parameters, and additional parameters),
$\mathbf{A}$	is the design matrix,
$\mathbf{P}$	is the image coordinate weight matrix,
$\mathbf{P}^o$	is the weight matrix of the prior means of the parameters,
l and l^o	are discrepancy vectors, and
$\mathbf{Q}_x$	is the cofactor matrix of the parameters.

In applying Equation 2 in the context of deformation monitoring, the basic restriction on the weight matrix $\mathbf{P}^o$ is that it should be insufficient to yield zero variance for the seven datum parameters. From a practical point of view, $\mathbf{P}^o$ can generally be omitted from Equation 2 if it serves only to impose a loose prior constraint on the parameter estimates $\hat{\mathbf{x}}$. Indeed, for a large range of variance values in $(\mathbf{P}^o)^{-1}$—the condition being that individual variances are not "too small"—the estimates $\hat{\mathbf{x}}$ and $\mathbf{Q}_x$ are invariant (Fraser, 1980). The pseudo-inverse yields a solution whereby $\|\hat{\mathbf{x}}\|$ and the trace tr $\mathbf{Q}_x$ are both a minimum. Geometrically, this condition can be interpreted as fixing the datum coordinate system origin at the "center of gravity" of the photogrammetric network (as defined by both exterior orientation parameters and object target point coordinates), maintaining average network orientation, and holding the mean scale as fixed. One important feature of this free-network approach is that, so long as the same provisional exterior orientation parameter values and the same provisional object space coordinates are adopted at each measuring epoch, it is possible to apply tests for departures from network congruency (Pelzer, 1971, 1980; Niemeier, 1981).

Unfortunately, whereas the adoption of the same provisional parameter values is generally practicable in a photogrammetric network incorporating terrestrial camera stations which can be relocated, it is not necessarily possible if aerial camera platforms and different cameras at each measuring epoch are employed. In the Turtle Mountain investigation, which utilizes photography from an aircraft, the provisional exterior orientation parameters of the first measuring epoch may be inappropriate for subsequent epochs because of changes in camera station geometry, and a possible change in the camera selection. For this reason it is desirable that the network datum be defined only in terms of the provisional object target point coordinates.

In addition to requiring the zero-variance computational base to be defined only in terms of the target point coordinates, an optimization of the variance of the parameter estimates is generally also sought. These two criteria are met by obtaining the solution for the vector of object point coordinates $\hat{\mathbf{x}}_2$, whereby

$$\|\hat{\mathbf{x}}_2\| = \text{minimum}$$

and (3)

$$\text{tr } \mathbf{Q}_x^{(2)} = \text{minimum.}$$

The matrix $\sigma_0^2 \mathbf{Q}_x^{(2)}$ is the covariance matrix corresponding to the object space XYZ coordinates, with σ_0^2 being the variance factor. If there are p target points in the network, $\hat{\mathbf{x}}_2$ is a $3p \times 1$ vector and $\mathbf{Q}_x^{(2)}$ will have a rank R of 3p − 7. The conditions stated above can be satisfied by subjecting the target point array to three rotations, a scale change, and three translations, all differentially small. This approach is referred to as the method of inner constraints (Meissl, 1969; Blaha, 1971), and it has been applied in close-range photogrammetry by Fraser (1982a) and Papo and Perelmuter (1980, 1982). The most straightforward technique for implementing inner constraints (relating only to $\hat{\mathbf{x}}_2$ involves "bordering" the normal matrix ($\mathbf{A}^T\mathbf{P}\mathbf{A} + \mathbf{P}^o$) with a Helmert transformation matrix $\mathbf{G}$, where $\mathbf{AG} = 0$. Note that the same restrictions apply to $\mathbf{P}^o$ as were outlined previously. For an object point j, the appropriate 3×7 matrix $\mathbf{G}_j$ is given as

$$\mathbf{G}_j = \begin{pmatrix} 1 & 0 & 0 & 0 & Z_j & -Y_j & X_j \\ 0 & 1 & 0 & -Z_j & 0 & X_j & Y_j \\ 0 & 0 & 1 & Y_j & -X_j & 0 & Z_j \end{pmatrix} \quad (4)$$

The transformation matrix $\mathbf{G}$ is then incorporated into the photogrammetric normal equation system in the following manner:

$$\begin{pmatrix} \mathbf{A}_1^T\mathbf{P}\mathbf{A}_1 + \mathbf{P}_1^0 & \mathbf{A}_1^T\mathbf{P}\mathbf{A}_3 & \mathbf{A}_1^T\mathbf{P}\mathbf{A}_2 & 0 \\ & \mathbf{A}_3^T\mathbf{P}\mathbf{A}_3 + \mathbf{P}_3^0 & \mathbf{A}_3^T\mathbf{P}\mathbf{A}_2 & 0 \\ & & \mathbf{A}_2^T\mathbf{P}\mathbf{A}_2 + \mathbf{P}_2^0 & \mathbf{G} \\ \text{symmetric} & & & 0 \end{pmatrix} \begin{pmatrix} \hat{\mathbf{x}}_1 \\ \hat{\mathbf{x}}_3 \\ \hat{\mathbf{x}}_2 \\ \mathbf{k} \end{pmatrix} = \begin{pmatrix} \mathbf{A}_1^T\mathbf{P}\boldsymbol{l} - \mathbf{P}_1^0\boldsymbol{l}_1^0 \\ \mathbf{A}_3^T\mathbf{P}\boldsymbol{l} - \mathbf{P}_3^0\boldsymbol{l}_3^0 \\ \mathbf{A}_2^T\mathbf{P}\boldsymbol{l} - \mathbf{P}_2^0\boldsymbol{l}_2^0 \\ 0 \end{pmatrix} \quad (5)$$

where subscripts 1, 2, and 3 refer to the exterior orientation parameters, the object point coordinates, and the additional parameters, respectively; and $\mathbf{k}$ is a 7×1 vector of Lagrangian multipliers. The "bordered" normal matrix of Equation 5 is nonsingular and thus a solution for the parameters, along with the minimum-trace covariance matrix $\sigma_0^2\mathbf{Q}_x^{(2)}$, can be obtained via a standard Cayley inverse.

The simulated photogrammetric monitoring network for Turtle Mountain was adjusted according to Equation 5 and the overall accuracy attained is indicated by the results listed in Table 1. In the table the mean standard error $\overline{\sigma}_c$ is simply computed from

$$\overline{\sigma}_c = \left(\frac{\sigma_0^2}{3p} \text{ tr } \mathbf{Q}_x^{(2)}\right)^{1/2} \quad (6)$$

where p is the number of object target points. The RMS value s_c is obtained in the usual manner from the square root of the meaned sum of squared differences between the true and adjusted coordinate values. As mentioned, for the simulation conducted an *a priori* value of $\sigma = \pm 2.5$ µm was set for the normal random deviate generator used in the perturbation of the true image coordinates.

From Table 1 it can be seen that the accuracy of the photogrammetric point positioning, as expressed by the estimators $\overline{\sigma}_c$ and s_c, is well within the subcentimetre range. Further, there is a basic correspondence between the value of $\overline{\sigma}_c$ obtained by means of Equation 6 and the coarse estimate computed by Equation 1. In addition to yielding XYZ coordinate standard errors of a few millimetres, the network displays very homogeneous precision from point to point, as expressed by the ratio λ_1/λ_{65} of the largest, λ_1, over the smallest eigenvalue, λ_{65}, of the cofactor matrix $\mathbf{Q}_x^{(2)}$. Such homogeniety of object point precision tends to be a feature of photogrammetric adjustments which employ partial inner constraints. This characteristic impacts on the variances of quantities which are derived as functions of the adjusted object space coordinates. For example, regardless of the separation of any two target points in the network, a distance standard error of about 2.5 to 3.5 mm is obtained. In terms of proportional accuracy, the average distance standard error is about 1 part in 60,000 of the target array diameter.

ADDITION OF SURVEYING OBSERVATIONS

Whereas point-to-point precision in the simulated network is very homogeneous, the anticipated discrepancies between accuracies in planimetry and height were evident. The mean standard error in

TABLE 1. BUNDLE ADJUSTMENT RESULTS FOR THE SIMULATED TURTLE MOUNTAIN PHOTOGRAMMETRIC MONITORING NETWORK

• Mean standard error $\overline{\sigma}_c$	±0.0022 m
• RMS value s_c of XYZ object point coordinates	±0.0023 m
• Square root of the ratio λ_1/λ_{65} of the largest over the smallest eigenvalue of $\mathbf{Q}_x^{(2)}$	2.7
• RMS value $s_{x,y}$ of image coordinate "measurements"	±2.2 µm

planimetric coordinates was computed to be $\bar{\sigma}_{X,Y} = \pm 1.9$ mm, while a value $\bar{\sigma}_Z = \pm 2.8$ mm was obtained for the Z-coordinate precision. In an effort to improve the heighting accuracy, it was decided to incorporate leveling data and redundant scale control in the network to supplement the photogrammetric point positioning. These data are routinely obtained as part of the existing monitoring program, though extensometer and spirit level measurements would of course cease if movements accelerated to hazardous levels.

Current crack motion measurements (Cruden *et al.*, 1982) center on Crack 1, and the surveying data from this program which have been incorporated in the photogrammetric network comprise a level traverse between the points 19 to 24, shown in Figure 2, and the distances 19-20, 20-21, and 23-24. Height difference observations between two adjacent target stations were assigned standard error values of $\pm$ 1 mm, as were distance measurements. These eight surveying observations on the "stable" side of the main fissure were then adjusted simultaneously with the photogrammetric data, the normal equations taking the general form

$$\begin{pmatrix} \hat{\mathbf{x}} \\ \mathbf{k} \end{pmatrix} = \begin{pmatrix} \mathbf{A}^T\mathbf{P}\mathbf{A} + \mathbf{A}_g^T\mathbf{P}_g\mathbf{A}_g + \mathbf{P}^0 & \mathbf{G} \\ \mathbf{G}^T & 0 \end{pmatrix}^{-1} \begin{pmatrix} \mathbf{A}^T\mathbf{P}l - \mathbf{P}^0 l^0 + \mathbf{A}_g^T\mathbf{P}_g l_g \\ 0 \end{pmatrix} \tag{7}$$

where the subscript g refers to the terrestrial surveying observations. For details of the algorithmic approach adopted for this combined adjustment, the reader is referred to Fraser (1982c).

Perhaps the most significant feature of the addition of distance and leveling data in the monitoring network was that to a large degree there was an insignificant overall improvement in object point precision. The standard errors of the Z coordinates of target points 19 to 24 improved by up to 1.5 mm, but the mean value $\bar{\sigma}_Z$ was reduced by only 0.2 mm. In planimetry there was a half-millimetre improvement in the Y-coordinate standard errors of points 19, 20, 21, 23, and 24 (measured distances ran roughly in the Y direction). In view of the minimal influence of the leveling and distance observations on the accuracy of the designed photogrammetric network, it was decided not to supplement the aerial photographic data with any terrestrial measurements.

SENSITIVITY ANALYSIS

Of crucial importance in deformation analysis is the sensitivity of the photogrammetric network to critical deformations. Under the assumption that there is prior information regarding the magnitude, direction, and extent of expected deformations, the network design must be such that these deformations can be detected at a certain confidence level. Measures of deformation are relative in nature, i.e., they are usually functions of positional differences which have occurred between two or more measuring epochs. It is, therefore, not surprising that the covariance matrix $\sigma_0^2\mathbf{Q}_x^{(2)}$ plays a key role in assessing how sensitive a network is to the detection of systematic point displacements which are likely to occur under formulated deformation models.

The concept of sensitivity analysis can be outlined in terms of a deformation analysis which employs tests for departures from congruency (Pelzer, 1971; 1972; Leonhard and Niemeier, 1980). Alternatively, because they indicate the quality of conformance of observations to a network design, external reliability measures can be employed in a sensitivity analysis (Niemeier *et al.*, 1982). Here, the former, computationally more straightforward approach is adopted, and the discussion is limited to a two-epoch analysis. As a result of two photogrammetric network adjustments, one at each measuring epoch, the following object point coordinates and covariance matrices are obtained: $\hat{x}_2^{(a)}$, $\sigma_0^2\mathbf{Q}_x^{(2a)}$ and $\hat{x}_2^{(b)}$, $\sigma_0^2\mathbf{Q}_x^{(2b)}$, with a common variance factor σ_0^2. According to the global congruency test of Pelzer (1971) the null hypothesis

$$H_o : \mathrm{E}\{\hat{x}_2^{(a)}\} = \mathrm{E}\{\hat{x}_2^{(b)}\} \tag{8}$$

is tested using the test statistic

$$\bar{F} = \frac{\mathbf{d}^T\mathbf{Q}_d^+\mathbf{d}}{R(\mathbf{Q}_d)\,\sigma_0^2} \tag{9}$$

where

$$\mathbf{d} = \hat{\mathbf{x}}_2^{(b)} - \hat{\mathbf{x}}_2^{(a)}$$

and

$$\mathbf{Q}_d = \mathbf{Q}_x^{(2a)} + \mathbf{Q}_x^{(2b)}.$$

The parameter $\bar{F}$ is referred to as the mean gap, and in the photogrammetric context $R(\mathbf{Q}_d) = 3p - 7$. If the null hypothesis is correct, i.e., no point displacements occur, $\bar{F}$ follows the central Fisher distribution. Essentially, the test of the null hypothesis asks the question: Are the two geometric networks congruent within the tolerance expressed by their respective precisions? If the null hypothesis is rejected, then point movements are implied.

Of more importance for sensitivity analysis is the alternative hypothesis

$$H_a : \mathrm{E}\{\mathbf{d}\} = \tilde{\mathbf{d}} \neq 0 \tag{10}$$

where $\bar{F}$ is distributed according to the non-central F-distribution with non-centrality parameter

$$\omega = \frac{\tilde{\mathbf{d}}^T\mathbf{Q}_d^+\tilde{\mathbf{d}}}{\sigma_0^2} \tag{11}$$

The probability $\gamma = 1 - \beta$ that $\tilde{d}$ will lead to the rejection of the null hypothesis H_o at significance level α is termed the power of the test with respect to the alternative hypothesis H_a. For a specified probability α that H_o will be rejected when it is true (Type I error), and a chosen probability β that H_o will be accepted when not true (Type II error), a critical value of the non-centrality parameter ω_o can be computed as a function of α, γ, and $R(\mathbf{Q}_d)$. Nomograms for ω_o, for three power values, are given in Baarda (1968). For example, in the Turtle Mountain investigation values of $\alpha = 0.05$ and $\gamma = 0.8$ were selected, the appropriate critical non-centrality parameter value being $\omega_o = 33.9$ for $R(\mathbf{Q}_d) = 65$.

For a given $\tilde{d}$, if $\omega > \omega_o$ the null hypothesis is rejected and thus it can be concluded that $\tilde{d}$ represents a detectable deformation at the probability levels α and β. The importance of the test of ω in terms of sensitivity analysis is that any so-called form vector $\tilde{d}$ of modeled point displacements can be assessed in order to ascertain a just-detectable deformation. Let c be a scalar value; then if

$$c^2\omega = \frac{(c\tilde{\mathbf{d}})^{\mathrm{T}}\,\mathbf{Q}_d^{+}\,(c\tilde{\mathbf{d}})}{\sigma_0^2} = \omega_0 \tag{12}$$

the form vector $c\tilde{d}$ represents a critical amount of movement, i.e., a just-detectable deformation. In designing a photogrammetric network of sufficient sensitivity to detect movements described by more than one likely deformation model, the photogrammetrist may need to consider a number of form vectors. In general, a network is least sensitive to a single point displacement and most sensitive to multiple point movements.

For the Turtle Mountain investigation, five deformation models describing anticipated patterns of point displacements were formulated and the corresponding form vectors were constructed. Table 2 lists the points subjected to movement according to each of these deformation models. Of the models, number V corresponds to the principal geotechnical hypothesis that the whole south peak block will move outwards, thus causing a dilation at Crack 1. The remaining models serve in an examination of the sensitivity of the proposed monitoring network to single and multiple point movements. The 10 mm point displacement vectors shown in Figure 2 correspond to model IV, and also include the anticipated movements under models I, II and III. As an example of a form vector, consider Model I:

$$\tilde{\mathbf{d}}_I^{\mathrm{T}} = (0, \ldots, 0 \quad 0.009 \\ 0.0 \quad -0.005 \quad 0, \ldots, 0) \tag{13}$$

where the displacements $dX_{14} = 0.009$ m, $dY_{14} = 0.0$, and $dZ_{14} = -0.005$ represent the movements in the directions of the three coordinate axes. Having formulated $\tilde{\mathbf{d}}_I$, the scalar $c = (\omega_o/\omega)^{1/2}$ is determined according to Equation 12, and $c\tilde{d}_I$ becomes the vector of critical (i.e., just-detectable) movements.

Table 2. Magnitudes of Critical Point Movements Corresponding to the Five Deformation Models (at Probability Levels of $\alpha = 0.05$ and $\gamma = 0.8$)

Models	Number of Points Subjected to Movement	Magnitude of Critical Movement (mm)
I	1 (pt. #14)	17
II	1 (pt. #13)	16
III	2 (pts. 14 and 15)	13
IV	6 (pts. 11 and 13–17)	9
V	18 (pts. 1–18)	9

Table 2 lists the magnitudes of critical movements for each of the deformation models formulated. As anticipated, the network is least sensitive to single point movements. A just-detectable movement of either points 13 or 14 corresponds to about eight times the mean positional standard error $\bar{\sigma}_c$. For models IV and V, the latter of which essentially examines the deformation of six points (numbers 19 to 24) with respect to a "stable" rock wedge, critical movement is at the subcentimetre level. This degree of sensitivity is considered to be sufficient for the monitoring of hazardous, accelerating movements of or within the south peak block.

The critical magnitudes listed in Table 2 are dependent upon, among other factors, the image coordinate measuring precision and the selected probability levels α and β. If in practice the photo coordinate standard error was reduced to 2 μm from the 2.5 μm simulated value, the values of just-detectable movement listed in the table would be scaled down by a factor of 0.8. On the other hand, if the commonly employed type II error probability level, β, of 0.1 rather than 0.2 is adopted, the movement values would increase by a factor of about 1.2.

Although the five assumed deformation models considered in Table 2 are rather simplified formulations, they do serve to illustrate the high degree of sensitivity of the proposed photogrammetric monitoring network. The results of this sensitivity analysis, coupled with the findings of a previous study (Fraser, 1982b), indicate that a multi-station photogrammetric monitoring network is sensitive enough to detect multiple point deformations between two or more measuring epochs as small as four times the mean positional standard error value.

Conclusions

The basic criterion which governed the design of the photogrammetric network for the south peak of Turtle Mountain was that it should be sufficiently accurate for the monitoring of deformation movements of one centimetre or greater. The sensitivity analysis conducted for the proposed network indicated that multiple-point, subcentimetre movements between two measuring epochs could be detected at the assigned confidence level, thus meeting the main design monitoring accuracy criterion. As a result of this investigation, it can be concluded that analytical restitution of low-level aerial photography would provide a feasible means of detecting hazardous deformations on the south peak of Turtle Mountain.

As a final remark on sensitivity analysis, this technique affords the photogrammetrist a tool by which he can rigorously ascertain, at a given confidence level, the amount of movement which will be just detectable according to the deformation model or models formulated. Thus, at the planning stages a photogrammetric network can be diagnosed, and if the sensitivity does not meet the required design criteria, appropriate redesign or optimization steps can be initiated.

In the summer of 1982 an object target array of 24 points was established on the summit ridge of Turtle Mountain, and aerial photographic coverage was obtained at heights of between 320 and 350 m above the south peak area. The results of the network adjustment for this first epoch of photogrammetric measurement corroborated the findings of the feasibility study. These results, along with an account of the deformation analysis procedure adopted, are summarized in Fraser & Stoliker (1983).

Acknowledgments

For their useful discussions on various aspects of this investigation, the author wishes to thank Drs. D. M. Cruden and W. Niemeier, and Messrs. D. W. Prosser and T. Sneddon. The work was partially supported by Alberta Environment, and the Natural Sciences and Engineering Research Council of Canada.

References

Alberda, J. E., 1980. A Review of Analysis Techniques for Engineering Survey Control Schemes. *Proc. Industrial and Engineering Survey Conference*, London, September.

Allam, M. M., 1975. Spatial Rock Face Deformations and Orientations of Geological Discontinuities in Open-Pit Mines Using Analytical Photogrammetry. *Proc. of the ASP Symposium on Close-Range Photogrammetric Systems*, University of Illinois, Urbana, pp. 5-23.

Armenakis, C., and W. Faig, 1982. Photogrammetric Monitoring of Mining Subsidence. *Paper presented to the Centennial Convention of the CIS*, Ottawa, April.

Baarda, W., 1968. *A Testing Procedure for Use in Geodetic Networks*. Netherlands Geodetic Commission, New Series, Vol. 2, No. 5, 97 p.

Beaty, C. B., 1975. *Landscapes of Southern Alberta*. University of Lethbridge Press, 95 p.

Blaha, G., 1971. *Inner Adjustment Constraints with Emphasis on Range Observations*. Dept. of Geodetic Science Report No. 148, Ohio State University, Columbus, 85 p.

Cruden, D. M., and J. Krahn, 1978. Frank Rockslide, Alberta, Canada. In: Voight, B. (Ed.) *Rockslides and Avalanches*, Vol. 1, Elsevier, Amsterdam.

Cruden, D. M., D. W. Prosser, and T. Sneddon, 1982. Monitoring the South Peak of Turtle Mountain. *Presented Paper, Fourth Canadian Symposium on Mining Surveying and Deformation Measurements*, Banff, June 7-9.

Erlandson, J. P., and S. A. Veress, 1975. Monitoring Deformations in Structures. *Photogrammetric Engineering & Remote Sensing*, Vol. 41, No. 11, pp. 1375-1384.

Fraser, C. S., 1980. On Variance Analysis of Minimally Constrained Photogrammetric Adjustments. *Aust. J. Geod. Photo. Surv.*, No. 33, pp. 39-56.

——— 1982a. Optimization of Precision in Close-Range Photogrammetry. *Photogrammetric Engineering & Remote Sensing*, Vol. 48, No. 4, pp. 561-570.

——— 1982b. The Potential of Analytical Close-Range Photogrammetry for Deformation Monitoring. *Presented Paper, Fourth Canadian Symposium on Mining Surveying and Deformation Measurements*, Banff, Alberta, June 7-9.

——— 1982c. On the Use of Non-Metric Cameras in Analytical Close-Range Photogrammetry. *The Canadian Surveyor* Vol. 36, No. 3, pp. 259-279.

——— 1983. Network Design Considerations for Non-Topographic Photogrammetry. *Submitted for publication in Photogrammetric Engineering Remote Sensing*.

Fraser, C. S., and P. C. Stoliker, 1983. Deformation Monitoring of a Landslide Area by High-Precision Photogrammetry. Presented Paper, FIG XVII International Congress, Sofia, Bulgaria, June 19-28.

Leonhard, T., and W. Niemeier, 1980. Diagnosis of Trans-Canada Levelling Nets by Principal Component Analysis. *Presented Paper, Second International Symposium on Problems Related to the Redefinition of the North American Vertical Datum*, Ottawa, May.

Meissl, P., 1969. Zusammenfassung und Ausbau der inneren Fehlertheorie eines Punkthaufens. *Deutsche Geodätische Kommission*, Series A, 61, pp. 8-21.

Negut, N., 1980. Technical-Geological Interpretation of the Phenomena Having Occurred in the Case of a Land Sliding. *Int. Archives of Photogrammetry*, Vol. 23 (B5), Hamburg.

Niemeier, W., 1981. Statistical Tests for Detecting Movements in Repeatedly Measured Geodetic Networks. *Tectonophysics*, Vol. 71, pp. 335-351.

——— 1982. Design, Diagnosis and Optimization of Monitoring Networks in Engineering Surveying. *Presented Paper, The Centennial Convention of CIS*, Ottawa, April.

Niemeier, W., W. F. Teskey, and R. G. Lyall, 1982. Precision, Reliability and Sensitivity Aspects of an Open-Pit Monitoring Network. *Presented Paper, Fourth Canadian Symposium on Mining Surveying and Deformation Measurements*, Banff, Alberta, June 7-9.

Papo, H., and A. Perelmuter, 1980. Free Net Analysis of Storage Tank Calibration. *Presented Paper, XIV Congress of ISP, Comm. V*, Hamburg.

——— 1982. Free Net Analysis in Close-Range Photogrammetry. *Photogrammetric Engineering & Remote Sensing*, Vol. 48, No. 4, pp. 571-576.

Pelzer, H., 1971. Zur Analyse geodätischer Deformationsmessungen. *Deutsche Geod. Komm., Series C*, No. 164, Munich.

———1972. Nachweis von Staumauerdeformationen unter Anwendung statistischer Verfahren. *Berichte Arbeitsgruppe B, Deutscher Geodätentag, Braunschweig.*

———1979. *Some Criteria for the Accuracy and Reliability of Networks. Presented Paper, XVII General Assembly of IUGG*, Canberra.

———, editor, 1980. *Geodatische Netze in Landes- und Ingenieurvermessung.* Konrad Wittwer Verlag, Stuttgart.

Robertson, G. R., A. M. R. Macrae, J. Tribe, D. W. Sibley, and D. H. Smith, 1982. Use of Photogrammetric Methods for Mine Slope Deformation Surveys. *Presented Paper, Fourth Canadian Symposium on Mining Surveying and Deformation Measurements*, Banff, Alberta, June 7-9.

Torlegård, A. K. I., and E. L. Dauphin, 1975. Deformation Measurement by Photogrammetry in Cut and Fill Mining. *Proc. of the ASP Symp. on Close-Range Photogrammetric Systems*, University of Illinois, Urbana, pp. 24-39.

Veress, S. A., N. C. Jackson, and J. N. Hatzopoulos, 1980. Monitoring a Gabion Wall by Inclinometer and Photogrammetry. *Photogrammetric Engineering & Remote Sensing*, Vol. 46, No. 6, pp. 771-778.

(Received 7 August 1982; accepted 16 May 1983; revised 22 June 1983)

CALL FOR PAPERS

International Conference
Inventorying Forest and other Vegetation of the High Latitude and High Altitude Regions

Fairbanks, Alaska
23-26 July 1984

This conference is sponsored by the Society of American Foresters and the International Union of Forestry Research Organizations. Contributed papers on the major workshop topics are solicited. Concurrent sessions are planned to permit in-depth discussion of these topics:

- Planning high latitude and high altitude vegetation inventories.
- Sampling designs and statistical analyses used in inventorying high latitude and high altitude vegetation.
- Vegetation measurement methods used for inventories of high latitude and high altitude vegetation.
- Special considerations related to inventorying and reporting results of inventories for high latitude and high altitude vegetation.
- General topics related to inventorying high latitude and high altitude vegetation.

Those interested in presenting papers should submit a title, 200-word abstract (maximum), and author's name, address, affiliation, and phone number by *1 December 1983* to

Dr. H. H. Gabrial
Bureau of Land Management
701 C Street, Box 13
Anchorage, AK 99513