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Impact of Geometry on Height Measurements from MLA Digital Image Data

A favorable combination of B/H ratio, attitude recovery, and correlation accuracy is required to ensure reliable height measurements from satellite data.

Abstract: *The possibilities for obtaining reliable height measurements from digital stereo data recorded with fore and aft pointing* MLA *sensors are heavily influenced by geometric principles involving* B/H *ratio, sensor attitude, pixel size, and correlation accuracy. As an example, to achieve 20-m contours meeting* NMAS, *10-m (pixel) or higher resolution stereo data acquired at a* B/H *ratio of 1.0 from an altitude of 700-900 km must be correlated to within approximately 0.25 pixel. Sensor attitude must be known to within 1 arc second. Graphs showing the interrelationships between height error, contour interval,* B/H *ratio, correlation accuracy/precision, and sensor attitude are presented.*

Interest in the potential for obtaining accurate height measurements from multispectral linear (or multilinear) array (MLA) image data has increased as SPOT, Mapsat, and other programs are discussed (Colvocoresses, 1979, 1981; Ducher, 1980; Welch, 1981). Two categories of sensor systems are being considered: (1) those which record stereo coverage in a cross-track pointing mode with pointable solid state cameras such as the High Resolution Visible (HRV) sensors of SPOT (Chevrel *et al.*, 1981); and (2) those which record stereo coverage in the along track mode with fixed fore and aft pointing solid state cameras such as those discussed for Mapsat (Itek, 1981; Colvocoresses, 1982).

A previous paper dealt with the possibilities for making planimetric and height measurements from film images produced with fore and aft pointing cameras (Welch, 1980). It is now appropriate to consider a more generalized situation in which either digital or analog data are available and accuracies (or errors) are expressed in fractions of a pixel. As in the previous paper, this discussion is limited to the fore and aft pointing camera system.

The parallel ray geometry associated with a fore and aft pointing system is illustrated in Figure 1, and the equation for determining relative heights or *height differences* (Δh) is (Welch, 1980)

$$\Delta h = \frac{\Delta p}{2 \tan \alpha} \cdot SF \qquad (1)$$

Δp = difference in x-parallax measured in the image plane ($\Delta p = (x_1 - x_2)$),
α = camera orientation angle referenced to local vertical, and
SF = scale factor.

This equation can be used to derive heights of geologic formations, tall tree stands, large buildings, etc. However, it also may be used to establish the relationship between correlation or measurement error and the threshold height difference which can be determined at some measure of statistical reliability. Because measurements are recorded at the top (t) and bottom (b) of objects, the cumulative effect of two measurement errors (σ_t, σ_b) must be considered.*

Noting that heights are a function of x-parallax, the measurement error ($\sigma\Delta p$) is computed according to:

$$\sigma\Delta p = \sqrt{\sigma_t^2 + \sigma_b^2} \qquad (2)$$

* In this example, measurement error is attributed to the precision (σ) to which the x-correlation or measurement can be made.

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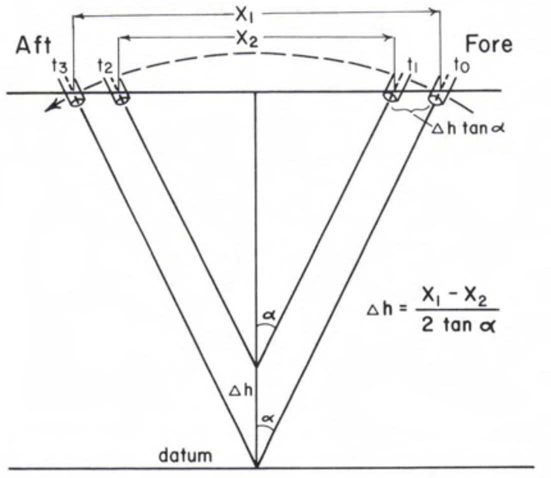

FIG. 1. Basic geometry for stereo MLA data recorded with fore and aft pointing cameras.

Then, $\sigma\Delta p$ is entered into Equation 1 (replacing Δp) to determine the minimum height difference error ($\sigma\Delta h$) at the 68 percent level of confidence. If required, $\sigma\Delta h$ at the 90 percent level of confidence can be calculated by multiplying the value obtained from equation 1 by 1.65.

The relationships between $\sigma\Delta h$ (in pixel units) and base-to-height (B/H) ratio for different $\sigma\Delta p$ values are presented in Figure 2. It is evident from this figure that $\sigma\Delta h$ increases exponentially as the B/H ratio is reduced. In order to minimize $\sigma\Delta h$ it will be necessary to record stereo satellite data at B/H ratios in excess of 0.6, with 1.0 or even 1.2 preferred. However, the larger B/H ratios (e.g., 1.2) may result in dead areas, intolerable vertical exaggeration, and/or significant variations in irradiance for rugged terrain, thus creating correlation or measurement problems.

It is also appropriate to consider the possibilities for compiling *relative* contours to some statistical measure of accuracy or reliability. The term "relative contour" implies that the contours are referenced to an arbitrary datum such as the bottom of a valley, a lake surface, etc. The reliability of relative contours and the appropriate closest contour interval (C.I.) may be readily computed by accepting a statistical level of confidence such as 90 percent (normally associated with United States National Map Accuracy Standards), which indicates that 90 percent of the *relative* spot heights measured from the image data or derived from the contours will be correct to within ½ the C.I. However, it must be recognized that the relative spot heights and contours will reflect any uncorrected distortions in the data set. Thus, the term "accuracy" is avoided.

The relationship between the closest relative contour interval meeting the 90 percent criteria and the precision of measurement (σ) for relative spot heights is (Figure 3)

$$\text{C.I.}_{90} = 3.3\ \sigma. \tag{3}$$

This equation is similar to that relating the closest topographic contour interval meeting NMAS speci-

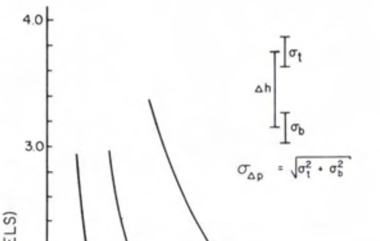

FIG. 2. Height difference error ($\sigma\Delta h$) in pixels as a function of B/H ratio for σ values of 1, 0.5, 0.25, and 0.1 pixel, corresponding to $\sigma\Delta p$ values of 1.4, 0.7, 0.35, and 0.14 pixel, respectively. Note: The values for $\sigma\Delta p$ are derived from $\sqrt{\sigma_t^2 + \sigma_b^2}$.

FIG. 3. Closest relative C.I. in pixel units as a function of B/H ratio based on 3.3 times the correlation/measurement precision (σ). Curves are shown for σ values of 1, 0.5, 0.25, and 0.1 pixel.

fications to photogrammetric spot height *accuracy* ($RMSE_Z$) (Thompson, 1979; Welch, 1982):i.e.,

$$C.I._{NMAS} = 3.3\ RMSE_Z. \quad (4)$$

Thus, the concepts are similar, but for relative contours the precision value σ based on the correlation or measurement capabilities of the operator/instrument may be used to define the C.I. relative to an arbitrary point or datum, whereas, for topographic contours the $RMSE_Z$ is referenced to a recognized datum (e.g., sea level) and includes the correlation or measurement capabilities of the operator/instrument *plus* any errors resulting from residual irregularities in the data after systematic adjustment to the datum.† For this reason relative contours can be constructed at much closer intervals than topographic contours designed to meet NMAS. It is of interest to note that the 20-m C.I. mentioned in conjunction with Mapsat is based on conformance to NMAS (Colvocoresses, 1982).

The accuracy of spot heights referenced to a recognized datum is largely determined by factors which create geometric displacements in the sensor data and produce correlation or measurement errors in the along-track direction. These displacements or errors (ΔX) may be expressed in pixels (or fractions of a pixel) at the ground and/or in arc seconds. In this example, an along track error (ΔX) is assumed to result from the deflection (ϕ) of one of the two corresponding rays as shown in Figure 4. The Z-error may be calculated as follows:

$$\text{Z-error} = \Delta X \cdot \cot \alpha$$
$$= \Delta X \cdot 2H/B \quad (5)$$

† Errors may be due to the map projection, ground control, random variations in sensor attitude during the recording period, coordinate adjustment procedures, etc.

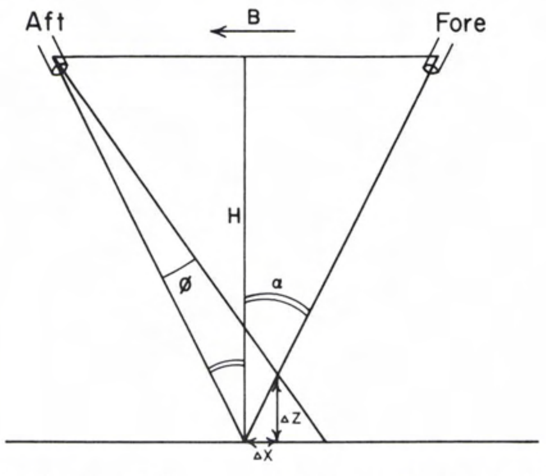

FIG. 4. An angular error (ϕ) of less than 5 arc sec in the along track direction is required to limit Z-errors to less than ±20 m.

where α is the camera orientation angle with respect to the local vertical, and ΔX is the displacement or error in the X-direction.

Thus, the Z-error in pixel-sized units may be determined for α values which are representative of the corresponding B/H ratios (Figure 5). From Figure 5 it is evident that, even for the favorable B/H ratio of 1.0, the ratio of ΔZ to ΔX (X-correlation error) is 2:1. When the B/H ratio is reduced to 0.5, the error ratio is increased to 4:1. This simple example indicates the critical geometric relationships governing elevation determinations from along-track MLA data.

Further insights to this problem are gained from an examination of a plot of the Z-error in pixel units as a function of B/H ratio for corresponding ΔX values of 0.1 to 1 pixel (Figure 6). From this figure it is again evident that B/H ratios of 0.6 or larger are necessary to minimize the Z-errors. It also clearly indicates the importance of reducing X-measurement or correlation errors to less than one pixel. For example, if it is assumed that ΔX values of 0.25 to 1.0 pixel are RMSE values which occur for 10-m image data recorded at a B/H ratio of 0.5, spot height accuracies will range from ±10 to ±40 m, and the closest contour intervals meeting NMAS (as defined by Equation 4) range from 35 to 150 m. When errors are related to pixel size, it becomes

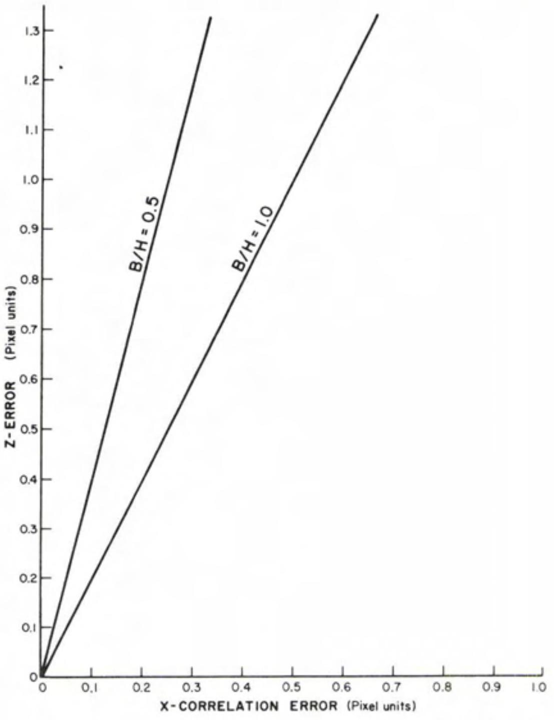

FIG. 5. Z-error in pixels as a function of X-correlation errors (ΔX), also in pixels, for B/H ratios of 0.5 and 1.0.

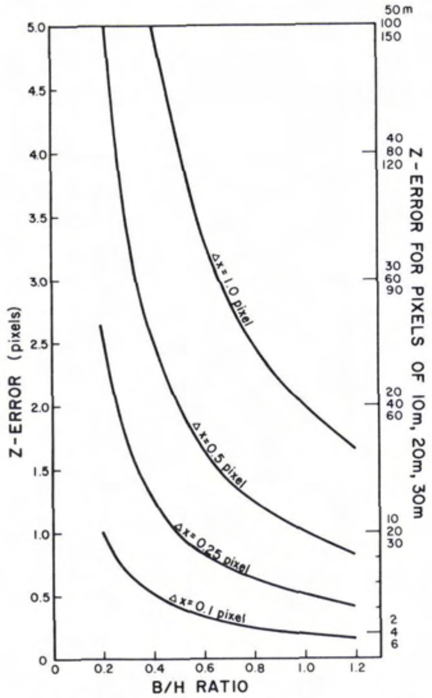

FIG. 6. Z-error in pixels as a function of B/H ratio for X-correlation errors (ΔX) of 1, 0.5, 0.25, and 0.1 pixel. The vertical axis on the right indicates the Z-error in metres for pixels of 10, 20, and 30 m.

evident that systems of high spatial resolution are required to accurately represent topography.

If the internal image geometry is disrupted, the magnitude of Z-errors will increase and further limit the use of MLA data for mapping tasks. For example, the geometric positions of pixels in an MLA data set are influenced by the attitude stability of the satellite sensor during the recording period (~100 sec for a B/H ratio = 1.0). Consequently, it is appropriate to examine the errors resulting from variations in sensor attitude (or from errors in the recovery of sensor position and attitude by means of ground control, the Global Positioning System (GPS), and star tracker data) (Welch and Marko, 1981). With fore and aft pointing systems at altitudes of 700 to 720 km and 910 to 930 km, the slant range distances are about 795 and 1025 km, respectively, for a B/H ratio of 1.0. Over these distances an angular displacement of 1 arc sec is equivalent to a planimetric error of about 5 m. In Figure 7, the Z-errors for image displacements equivalent to 1, 3, and 5 arc sec in the along track direction are plotted as a function of B/H ratio for a nominal altitude of 713 km. Although angular errors of 1 to 3 arc sec are approximately an order of magnitude less than those often obtained with aerial photographs subjected to analytical adjustments, it is obvious that contour intervals of 20 m meeting NMAS only can be

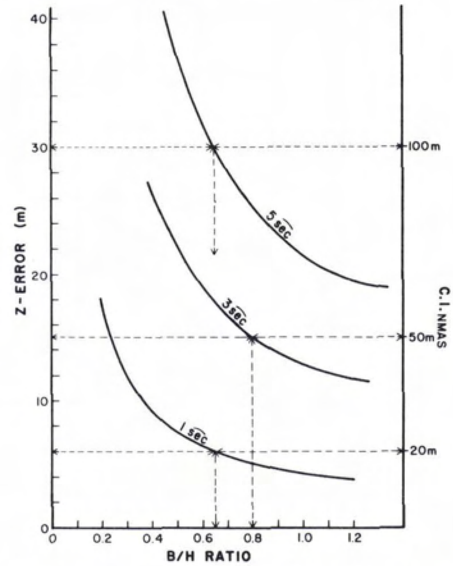

FIG. 7. Z-error in metres (considered to be $RMSE_Z$) and the closest C.I. meeting NMAS for along track angular errors of 5, 3, and 1 arc second with stereo data of different B/H ratios. A nominal altitude of 713 km is assumed.

realized at B/H ratios of 0.65 or better for angular errors of less than 1 arc sec. Similarly, to achieve 100-m contours, angular displacements must be limited to less than 5 arc sec. Rotation about the Y-axis (pitch) in excess of 5 arc sec will cause serious ele-

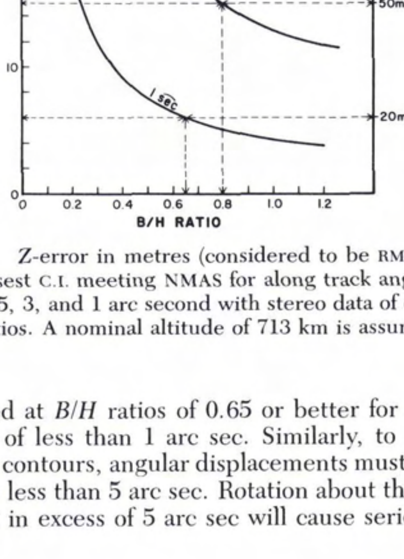

FIG. 8. Along track angular error for different attitude stability rates as a function of B/H ratio. Only with attitude stability rates of 10^{-5} deg/sec, or better, can these errors be constrained to less than 5 arc seconds over the time interval required to record stereo image data.

vation errors. In order to limit errors to less than 5 arc sec over the time interval required to record the stereo image, sensor stability must be controlled to 10^{-5} deg/sec or better as shown in Figure 8.

In conclusion, this brief analysis indicates that for stereo MLA data of high geometric fidelity the reliability of relative spot heights and contour intervals will be governed by the precision to which X-correlation can be accomplished, and that B/H ratios in excess of 0.6 are required to minimize the effect of measurement errors. The determination of elevations and contours to accepted accuracy standards will be greatly influenced by the internal geometric fidelity of image data. Sensor attitudes must be maintained or established to better than 5 arc sec to achieve $RMSE_Z$ values of ± 30 m (C.I. of 100 m) and to about 1 arc sec for an $RMSE_Z$ of approximately ± 6 m (C.I. of 20 m). Thus, the principal geometric requirements for good digital stereo MLA data are (1) B/H ratio ≥ 0.6, (2) angular errors of less than 5 arc sec, and (3) measurement or correlation to a fraction of a pixel. These requirements imply that attitude stability can be controlled to better than 10^{-5} deg/sec and that the sensor systems will provide data of high spatial resolution (e.g., pixels ≤ 10 m).

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