G. E. Bormann
Wild Heerbrugg Ltd.
CH-9435 Heerbrugg, Switzerland
E. Vozikis
Technical University Vienna
A-1040 Vienna, Austria

Map Projection Transformation with Digitally Controlled Differential Rectifiers

An unconventional application of modern orthophoto equipment.

Abstract: *The geometrically correct transformation of the entire map content from one cartographic projection to another is an old problem. It can be solved economically with digitally-controlled differential rectifiers such as the Wild AVIOPLAN OR1, using additional software modules. Examples illustrate the potential of this method and show also the great advantages of the color capability of the OR1 system.*

Introduction

The content of existing maps is very often the basis for new cartographic products. In many cases the extracted information will be used for various different purposes. It happens quite frequently, however, that a geometric projection other than the original one is desired or needed.

Cartographers are then faced with a problem which could in the past only be solved in either a crude and not very accurate procedure or in a very costly way. Meshwise re-projection with a rectifier and/or re-drawing was necessary.

Today digitally controlled instruments and systems, which were basically developed for the production of orthophotographs, offer a new economic solution of the problem. The Wild AVIOPLAN OR1, a differential rectifier, belongs to this category of equipment.

First Experiment

A practical case in the field of aviation navigation, approximately two years ago, led to a closer investigation of the new possibilities. In connection with some of the data supplied by an inertial navigation system, a special navigation chart was needed very quickly. The specifications asked for a rectangular grid formed by the geographic meridians and parallels (with additional constraints).

The navigation charts of central Europe and elsewhere are, however, on the Lambert conformal (angle-true) conical projections of the International (Hayford) or Bessel Ellipsoids with either one or two length-true parallels at scales of 1 : 1 million or 1 : 500 000. The meridians are straight lines converging at the (North) Pole and parallels are imaged as concentric circles about the (North) Pole. In this particular case the 1 : 500 000 scale Aeronautical Chart ICAO 2253-B of Switzerland (one length-true parallel, Bessel Ellipsoid) had consequently to be transformed into a (special) rectangular Cassini projection of 1 : 1 million scale.

The work was successfully carried out on the AVIOPLAN OR1 in Heerbrugg using additional and modified software originally developed at the Institute of Photogrammetry of the Technical University in Vienna, Austria. The converging meridians became parallel and the latitude circles were rectified to straight lines with high accuracy. The entire map content was transformed simultaneously into the new projection in its original colors.

Although the differences of the two cartographic projections are obvious and considerable at the map scale, even over such a small area as Switzerland, another more spectacular example was selected for presentation in a publication where the scale of reproduction is limited by the page size.

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The choice fell on the physical map of Europe taken from the *Swiss Atlas for High Schools*, a creation of the well known Swiss cartographer Prof. Ed. Imhof.

SOME REMARKS ON ORTHOPHOTOGRAPHY AND ON THE AVIOPLAN OR1

The principle of orthophoto production is well known and the operation of the AVIOPLAN OR1 has been described in detail in several publications (Kraus, 1976; Stewardson; 1976). We will therefore restrict the discussion of these items to some background remarks which seem to be necessary for the appreciation of the map transformation method.

In general, orthophotos cannot be obtained directly with a camera. A subsequent process is necessary to remove the perspective distortions from, for instance, an aerial photograph. The perspective effects are caused by central projection when the photographed terrain surface is not flat and/or the camera axis was not perpendicular to the datum plane.

For the elimination of these effects it is necessary to resolve the content of the original photograph into small area or line elements. They have to be changed in scale and rotated as well as shifted into the correct geometric position before recomposition to a new photograph, the orthophoto, can take place.

Figure 1 illustrates the principle of taking an (aerial) photograph. A regular, rectangular grid in the (horizontal) datum plane (x,y) is assumed to undergo vertical parallel projection onto an undulating terrain surface (or a model of that surface). There it forms a spatial network of irregularly shaped quadrangles. By central projection this grid net is consequently deformed by the shape of the terrain and by the spatial orientation of the camera axis.

The production of an orthophoto corresponds to the inverse process. The irregular grid (u,v) has to be transformed into the regular rectangular grid (x,y) or, multiplied with a scale factor, into (X,Y) as is shown in Figure 2. This transformation is, however, only possible when the shape of the terrain, the orientation of the camera axis, the spatial position of the projection center, and the principal distance of the camera are known in some digital form. Normally, this is not the case *à priori*. The necessary information has to be obtained either from measurements in stereoplotters, or by computation, for instance, from an existing digital terrain model.

The Wild AVIOPLAN OR1 performs the required transformation. It is an instrument with optical projection. The optical devices such as a double zoom system for continuous change of scale, a dove-prism for image rotation, and the (u,v) cross-slide of the photo-carrier for positioning are servo-driven by a process computer.

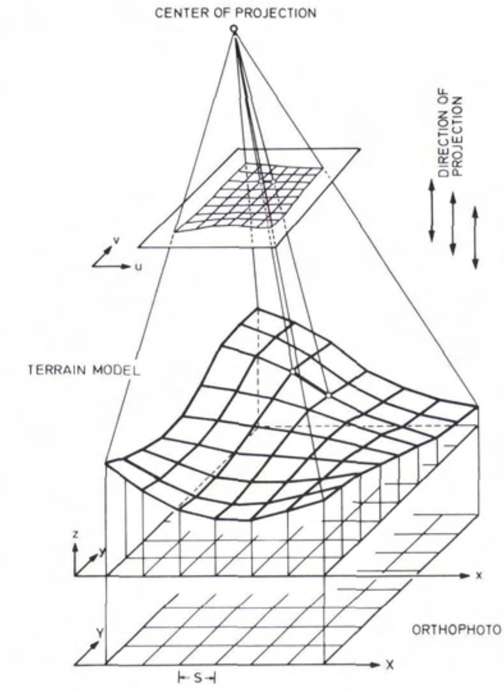

FIG. 1. Basic relations between central and orthogonal projections.

The OR1 applies the principle of differential rectification of an infinite number of line elements of finite (selectable) length which are arranged in adjacent profiles. The orthophotograph or rectified image is composed on a rotating film drum which is shifted in the direction of its axis at the end of each profile scan. The amount of shift corresponds to the length of the line element, i.e., the length of the selected slit mask through which the photographic image has to pass.

The input data to the OR1 process-computer are image coordinates (u,v) supplied from magnetic tape. With reference to Figure 1, this means that the projection scan width S can be chosen between 3 and 15 mm. The Y-interval (ΔY) in the rectified image is always 1 mm; however, it is lin-

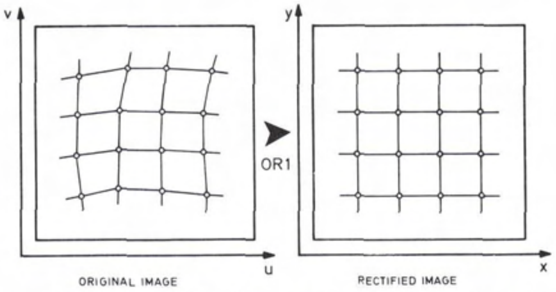

FIG. 2. Grid transfer from aerial photograph to orthophoto.

FIG. 3. Wild AVIOPLAN OR1 with projection unit, process computer, magnetic tape station, and printer.

early interpolated to 0.01 mm by the process computer. In mathematical terms the OR1 performs a bilinear transformation for each grid mesh.

As was mentioned above these image coordinates have to be established first either by measurement or by computation in an external computer. For the latter case a FORTRAN computer program SORA was developed at the Technical University at Vienna/Austria some years ago. (SORA stands for *S*oftware for *O*ffline *R*ectification, *A*violan). (Otepka and Loitsch, 1976).

GENERAL METHOD OF MAP TRANSFORMATION WITH THE OR1

From the explanation above, the following presentation should very easily be understood. What can be rectified by means of differential transformation can obviously also be distorted. The "distortion" may be a desired change of an original if the appropriate mathematical model can be formulated and is then imposed. The transformation of a map with its whole content from one projection to another type of map projection is such a case. The mathematical model can be formulated, usually in rigorous terms.

Reverting to Figure 2, the desired new map projection is directly comparable with the "rectified" image (x,y). The map projection to be transformed stands in place of the "original" (u,v). Usually it cannot be the original map directly but a reduced transparent reproduction on film. This is in most cases necessary because the picture carrier of the OR1 is designed for aerial photography, while the format of the transformed map may be up to 750 mm by 900 mm maximum.

It is now furthermore assumed that the mathematical conversion formulas between both map projections are known. In particular, the coordinates of the "old" projection (u,v) must be expressible in explicit form by the coordinates of the "new" (transformed) projection (x,y).

The principle is very simple. The "new" (transformed) projection shall be given in cartesian coordinates (x,y) as functions of longitude λ and latitude φ: i.e.,

$$x = f_1\,(\lambda,\varphi), \quad y = f_2\,(\lambda,\varphi). \tag{1}$$

The same shall be the case for the "old" (to be transformed) projection with the cartesian coordinates (u,v): i.e.,

$$u = h_1\,(\lambda,\varphi), \quad v = h_2\,(\lambda,\varphi). \tag{2}$$

Next a cartesian grid (x,y) is defined in the "new" projection (Figure 4). With the *inverse* formulas of Equation 1, corresponding geographic coordinates (λ,φ) are obtained: i.e.,

$$\lambda = g_1\,(x,y), \quad \varphi = g_2\,(x,y). \tag{3}$$

By substitution of Equation 3 into Equation 2, it follows that

$$u = h_1[g_1(x,y)] \text{ and } v = h_2[g_1(x,y),\, g_2(x,y)]. \tag{4}$$

The result of this numerical transformation (Equation 4) is illustrated in Figure 5.

The regular rectangular grid of the "new" (transformed) projection appears deformed and is superimposed on the "old" (to be transformed) map.

Consequently, a "rectification" of the grid of Figure 5 according to the orthophoto method will lead to Figure 4 and will at the same time transform the entire map content into the new projection.

In analogy to the orthophotographic process, the "old" projection corresponds to the aerial photograph and the "new" one to the orthophoto.

There are some additional points to be observed, however. A reproduction of the "old" map must be made on film (either negative or positive) not larger than 9 by 9 inches. Even if this is done in a good reproduction camera, small projective displacements cannot be excluded. It is therefore advisable to measure the rectangular coordinates (u',v') of 10 to 15 well distributed points in the

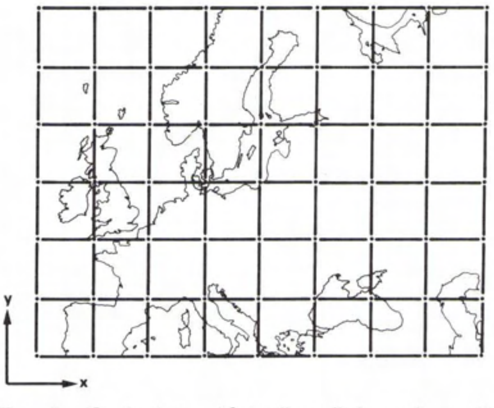

FIG. 4. Cartesian grid in "new" (transformed) projection.

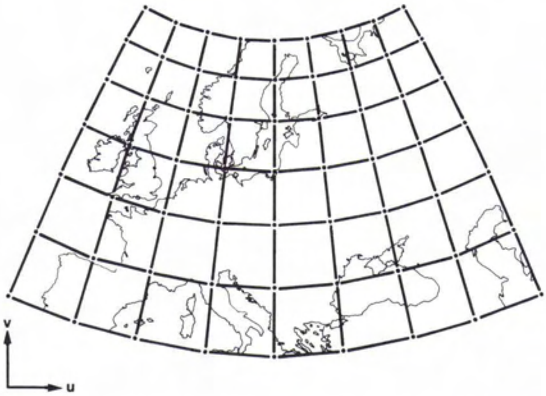

FIG. 5. Transformed grid in "old" (to be transformed) projection.

repro film with a comparator and assign them a principal distance. It can be an arbitrary value or, better, the principal distance (not focal length) of the repro-camera if the results are to be interpreted.

Following this, the length of the OR1 slit mask must be chosen. It determines the projection scan width and therefore also the number of points to be computed for a specified format of the "new" map.

A special version of the SORA program first computes a spatial resection using the measured (u',v') and their corresponding (u,v) and transforms the (x,y) grid into the (u',v') plane. As a result the input data for the OR1 process-computer are obtained.

The actual projection procedure in the OR1 is then identical with that for orthophoto production, and the "new" (transformed) map is generated on the film drum either in black and white or in color, positive or negative, direct or mirror-reversed.

THE TEST EXAMPLE

The Swiss atlas map, a ten-color print, is an equal-area azimuthal projection with its central point at $\lambda_o = 20°\text{E}$ and $\lambda_o = 20°\text{E}$ and $\varphi_o = 50°\text{N}$ (near Cracow, Poland). The scale is 1 : 15 million.

It was decided to transform this map into a Mercator projection (conformal) with its length-true parallel at $\varphi_o = 50°\text{N}$. The curved meridians and parallels of the azimuthal projection were to become straight lines intersecting at right angles. Large displacements in the north were to be expected.

The reproduction scales for both projections were chosen as 1 : 30 million.

For reasons of optimum equal-color reproduction, both projections were made in the OR1 although the azimuthal map required only a reduction to half of the original scale.

The following formulas were applied:

Inverse Mercator, corresponding to Equation 3:

$$\lambda = \lambda_o + \frac{180\ m\ x}{\pi\ R \cos \varphi_o}$$

$$\varphi = 2 \operatorname{arc\,tan} [\tan (\frac{\varphi_o}{2} + 45) \cdot \exp \frac{m\ y}{R \cos \varphi_o}] - 90$$

Azimuthal (equal area), corresponding to Equation 2:

$$u = \frac{\sqrt{2} \cos \varphi \sin (\lambda - \lambda_o)}{\sqrt{1 + \sin \varphi \sin \varphi_o + \cos \varphi \cos \varphi_o (\lambda - \lambda_o)}} \cdot \frac{R}{m}$$

$$v = \frac{\sqrt{2} [\sin \varphi \cos \varphi_o - \cos \varphi \sin \varphi_o (\lambda - \lambda_o)]}{\sqrt{1 + \sin \varphi \sin \varphi_o + \cos \varphi \cos \varphi_o \cos (\lambda - \lambda_o)}} \cdot \frac{R}{m}$$

with the Earth taken as a sphere (angular values in degrees).

λ	Longitude
φ	Latitude
λ_o	Longitude of central point of azimuthal projection
φ_o	Latitude of central point of azimuthal projection
R	Earth radius
m	Scale factor (30 million)

A projection scan width (length of slit mask in the OR1) of 3 mm was chosen. In the case of the Mercator projection, this resulted in 73 projection profiles each with a length of 408 mm. Consequently, nearly 30 000) (x,y) transformations were computed with the special SORA program in an external computer. The number has to be multipled by a factor of 100 when the interpolation calculations of the OR1 process computer are also taken into account. In other words, close to 3 million pairs of (x,y) coordinates were computed to perform this transformation. For the spatial resection 20 (u',v') coordinates were measured in the reproduction.

The net time per projection amounted to approximately 49 minutes as the drum rotation speed at the periphery (film location) was purposely limited to 10 mm/sec for correct color exposure.

All projections were made twice, first on Kodak Vericolor IIL Color Negative Film, Process C41, and second on Kodak Ektachrome Duplicating Film 6121 (color postive), Process E6.

For accuracy investigation 20 well distributed points were measured on the original 1 : 15 million scale azimuthal map (paper sheet), on the 1 : 30 million reduced scale azimuthal projection (film), and on the transformed 1 : 30 million Mercator projection (film). Three modes of transformations were then carried out: affine (six parameters), similarity (four parameters), and congruent (three parameters). The measurements were in all

PLATE 1. Azimuthal equal-area projection. Original scale 1 : 15 million. OR1 reduction 1 : 30 million. (Courtesy of Atlas Commission, Swiss Federal Directorate for Education.)

Plate 2. Mercator projection (conformal). Scale of OR1 transformation 1 : 30 million.

cases transformed against the theoretical values at the nominal scale.

The results showed in all three cases that there was practically no difference between the six-parameter and four-parameter transformations. This indicates the absence of any affinity. The largest errors were found for the original 1 : 15 million scale paper map. The positive scale factor of 0.3 percent (original too small) indicates paper shrinkage or perhaps the assumption of a slightly different Earth radius. After reduction by this scale factor, the mean square error of a coordinate amounted to 0.19 mm at map scale.

Surprisingly good were the results of the 1 : 30 million reduced azimuthal projection (Plate 1) as well as of the transformed Mercator projection (Plate 2). With a fixed scale factor of 1, the mean square error of a coordinate at map scale was found to be 0.09 mm and 0.11 mm, respectively.

These excellent results speak not only for the OR1 transformation process but also for the internal geometric quality of the original map, particularly if it is taken into account that the map was designed and drafted many years ago when automatic drawing tables were not available.

CONCLUSIONS

Although the map transformations carried out with the OR1 SORA system so far were of a test nature, they have proved that the method can be applied very successfully and has a potential.

The main applications may perhaps not be the transformation of complete maps but of certain features which cartographers may wish to transfer.

When entire maps are transformed, obviously also the nomenclature will be changed in scale and orientation. The principal applications will therefore most likely be in the revision of unannotated maps. Geometrically, the possibilities seem to be nearly unlimited.

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