J. D. SIEGWARTH
J. F. LABRECQUE
C. L. CARROLL
Thermophysical Properties Division
National Bureau of Standards
Boulder, CO 80303

Volume Uncertainty of a Large Tank Calibrated by Photogrammetry

A volume uncertainty of 0.03 percent was obtained.

INTRODUCTION

BECAUSE OF THE HIGH COST of liquefied natural gas (LNG), contractual agreements often require custody transfer measurement uncertainties no greater than a few tenths of a percent. The volume of liquid changing ownership is currently determined by level measurements in calibrated cargo tanks. The calibration method must provide volume of the tank as a function of height. For ship cargo tanks, calibration tables are provided for the ship at even keel. List and trim correction tables are provided for deviations from even keel.

Both storage and ship cargo tanks are large, and the shape of the latter often deviates from a simple geometry. The large size of these tanks and their inability to support full loads of water preclude application of flow meter calibration techniques, and their irregular shapes often render conventional taping techniques for calibration unsatisfactory.

The 37-m diameter spherical tanks of the first U.S. built LNG cargo ships were some of the first tanks to be calibrated by photogrammetry (Brown, 1980, 1981). The photogrammetric method used to calibrate these tanks was an extension of a method used to establish dimensions and shapes of calibrated antenna dishes (Brown, 1958, 1980, 1981; Kenefick, 1971). The technique was new to the petroleum industry so the claim of dimensional accuracies to 1 part in 10^5 needed verification.

The National Bureau of Standards (NBS) was asked to examine the accuracy of the photogrammetric methd as applied to one of the spherical tanks. A limited study of the photogrammetry was carried out on this one tank (Jackson *et al.*, 1979) with some $4^1/_2$-m long calibrated rods. This study did not disclose any reason to believe that the calibration uncertainties were larger than claimed by the photogrammetrist. The rods, however, were too short to confirm the estimated uncertainties.

Later, NBS was asked to examine the accuracy of the photogrammetric calibrations of free standing prismatic cargo tanks. A brief description of the calibration measurements of these tanks has been reported (Brown, 1981). Extensive tests of the calibrations of these tanks were made by NBS personnel and the results have been published (Siegwarth and LaBrecque, 1981). In this paper, the testing of the photogrammetric method and the results of the tests are presented. Other sources of calibration uncertainty are discussed briefly and overall estimates of the tank calibration uncertainties are given.

ABSTRACT: *The volume calibration uncertainty of large (30,000m³) liquefied natural gas tanks calibrated by photogrammetry has been independently estimated by the National Bureau of Standards. The independent estimates were obtained using surveying tapes.*

CALIBRATION OF TANKS BY PHOTOGRAMMETRY

The photogrammetric calibration examined in the present paper was carried out on 15 free standing prismatic cargo tanks of three LNG ships; the *El Paso Savannah*, the *El Paso Cove Point*, and the *El Paso Columbia*. A schematic diagram of one of the tanks is shown in Figure 1. The largest of these tanks are about 36-m square by 25-m high. The calibration accuracy required was 0.2 percent maximum uncertainty.

As many as 1600 photogrammetric targets were applied to the exterior side walls of the tanks. The internal structure of the tank prohibited an internal measurement. The 19-mm diameter white targets were first painted on the tank walls. Later, printed targets were glued on. The targets were placed in

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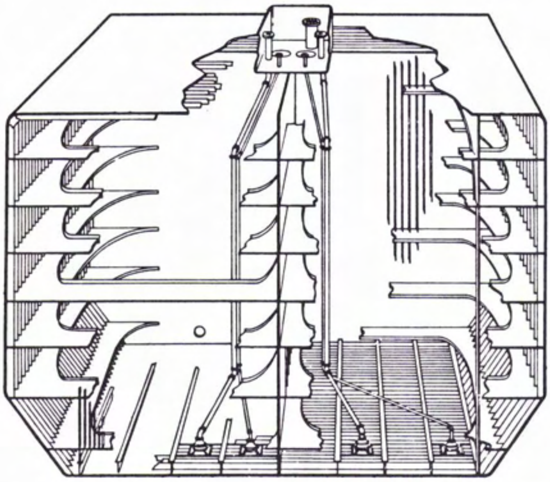

FIG. 1. Cutaway drawing of two quadrants of a cargo tank showing the internal bracing configuration.

approximate rows and columns with spacings of about 1.5 m and 3 m respectively.

The photography was done outside at night after the tank thermal gradients had minimized and the temperature stabilized. A specially constructed 23 by 23 cm (9 by 9 in.) format camera designed for 6-mm thick microflat glass plates was used for the photography. A 240-mm focal length lens gave a 55° square field of view. Eight ground-level camera stations were employed. Four were located on each of the perpendicular bisectors of the sides. The other four were located at each vertical corner on the extended diagonals. One photograph was taken at each position. Often, one side-camera position was obstructed by an adjacent tank. The adjacent corner-camera positions were moved to permit a complete view of the tank side and two photographs were taken at each of those stations to compensate for the missing side photograph. The obstructed side was targeted with 19-mm balls to increase the image size.

The photogrammetric scale factor for the *Savannah* tanks was provided by measuring the vertical separations of some targets on the tank with calibrated surveyors tapes. For these tanks six vertical distances approximately 15¼ m (actually 50 ft) in length provided the scale calibration. The scaling was provided by targets mounted on tapes for the last two ship sets. Again, six lengths were used.

The x, y coordinates of the target images were measured on a 1-μm resolution digital comparator to an uncertainty of about ±2.5 μm. The X, Y, Z coordinates of the tank targets were calculated by the bundle method. The camera location, orientation, and lens corrections are simultaneously calculated. Details of the photogrammetric reduction method are proprietary but the method is generally explained in Brown (1958, 1980, 1981) and Kenefick (1971).

The coordinate system was then rotated and translated so that Z is vertical and along the level gage axis. The tank volume tables are then generated by a numerical integration procedure.

THE INDEPENDENT TEST OF PHOTOGRAMMETRIC ACCURACY

Photogrammetric dimensioning is estimated by the photogrammetrist to yield inaccuracies of less than 1 part in 10^5 of the major dimension of an object. This means that a tank 36-m long can be dimensioned to 0.36 mm. The method used to test the photogrammetric accuracy should have at least that resolution. Because these tanks were to be calibrated with an uncertainty of no more than 0.2 percent, the highest photogrammetric accuracy was not required.

The large plane surfaces of these tanks offered the possibility of measuring target separations with a surveyors tape independently and comparing the tape determined separation with the separation calculated from the photogrammetrically derived target coordinates. Rather than trying to obtain the required accuracy by taping the distance between existing targets on the tank, special targets were made to clamp directly to tapes hung vertically or stretched horizontally on the surfaces of the tank. Two to a dozen targets were placed on each tape. These targets were included in the photogrammetric reduction.

The vertical tapes were hung from the top of the tank, allowed to slide through aluminum foil guides at the bottom of the tank, and tensioned by 5-kg weights hung on the bottom end. The targets, consisting of a 19-mm diameter white spot on an 87-mm square black aluminum plate, could only be placed near the top and bottom ends because of the frequent wind in the area. The tapes were then immune to all but the strongest winds.

On the first four tanks, a horizontal tape was placed near ground level in a catenary suspension. This was discontinued because of the suspicion that wind and moisture condensation were reducing the accuracy of this tape. Horizontal tapes placed on subsequent tanks were supported at about 3-m intervals by aligned brackets. The brackets were first aligned by visual sighting, but in later tests a laser beam was used. The tape was free to slide over the supports but was fixed at one end and tensioned by a 5-kg weight attached to the other. Either the flat targets described above or 19-mm diameter white balls attached to black anodized aluminum plates were clamped to the horizontal tapes. Horizontal tapes were passed under three of the tanks, supported at 3-m intervals, with ball targets at each end. These tape installations differed from the other tapes in that no one photographic plate contained images of the targets on both ends of the tape.

The tapes used for these measurements were made of iron-36 percent nickel alloy and unruled. The target positions were referenced to scribe marks placed on the tapes. The distance of the reference edge of a target from the target position relative to the nearby scribe mark was measured to 0.1 mm with a steel scale before and after the photography. This distance was never more than a few centimetres. The distance from the reference edge to the center of the target circle was measured in the laboratory.

The scribe mark separations on the tapes were determined at NBS-Gaithersburg using a laser interferometer measuring device. Selected lengths were calibrated three times during the course of the tests as a control for the tape lengths. The pooled standard deviation of the difference between repeated laser calibrations of the same scribe mark spacings was 0.0129 millimetres with 28 degrees of freedom, the maximum difference being 0.042 millimetres.

Because a half kilogram change in tension weight produces about 0.05-mm/m change in length of the tape, the calibrated scribe mark separations are corrected for the tension at the time of photography, taking into account the target and the tape weights. A thermal expansion correction is applied even though the assumed thermal expansion coefficient, 0.4×10^{-6} m/m°C (Siegwarth and LaBrecque, 1981) and the maximum temperature difference between the time of calibration and the photography of 7° only gives a 3-ppm correction. The uncertainty in the distance between the test targets on the tanks is estimated to be ± 0.32 mm, which includes ± 0.28-mm uncertainty in the positioning of the targets and three times the standard deviation of the scribe mark spacings.

RESULTS OF THE TAPE TESTS

The n targets on a tape represent only n-1 independent distances. An end target on each tape was chosen as target 1, and the distances to the other targets were determined from this target. If targets 1 and i on a tape had photogrammetric coordinates (x_1, y_1, z_1) and (x_i, y_i, z_i), then the distances between them, d_p, assuming an orthogonal coordinate system, is

$$d_p = \{(x_i - x_1)^2 + (y_i - y_1)^2 + (z_i - z_1)^2\}^{1/2} \tag{1}$$

Table 1 shows a comparison of the photogrammetrically determined target separation, d_p, and the NBS value for the same separation, d_{NBS}, for the tapes placed on the tanks of the *El Paso Cove Point*. The difference between the two values is shown in the last column. The first two columns give the tape number and the approximate location of the tape on the tank. The diagonal and transverse horizontal tapes shown on the Table 6, numbers 6, 7, and 8, were stretched underneath the tank.

Similar results to those shown in Table 1 were obtained for the tapes on the tanks of the *El Paso Columbia* and *El Paso Savannah*.

During the tests of the 15 tanks, eight tapes were photographed about 70 times. The 260 total targets attached provided about 190 measured target separations. The vast majority of the target spacings measured by NBS and the photogrammetrist disagreed by less than 2 mm. Some of the exceptions had obvious explanations. In two cases horizontal tapes were placed along the side of the tank separated by about 7 m from another tank. The camera was positioned such that the angle between the viewing direction and the side of the tank was only about 20°. This increases the position uncertainty of the targets in the horizontal direction parallel to the tank wall but it does not affect the accuracy of the tank calibration. Only target coordinate uncertainties perpendicular to the tank wall affect the volume uncertainty for plane walls, and that uncertainty is not increased by the narrow camera angle. Tape 5 on tank 4 of the *Cove Point* (Table 1) showed an apparent scale factor error differing from all the other tapes on that tank. Inadequate tensioning of this tape, probably due to the tape hanging up on a support near the weighted end, is the suspected source of this difference. Laboratory tests after the fact showed that a 6-mm error in 31 m is possible without visible evidence that the tensioning weight is not providing the correct tension.

Errors in the photogrammetric scaling produced some larger length differences on the *Savannah* tank. Even though reasons exist to discard some of the data, the analysis presented here includes even the questionable tape length data. Slightly larger magnitudes of calibration uncertainties result.

METHOD OF ANALYZING TAPE DATA

The photogrammetrically determined i^{th} length from the j^{th} tape on a particular tank was related to the corresponding NBS length by

$$l'_{ij} - l_{ij} = c + s\,(l_{ij} - \bar{l}_j) + \epsilon_{ij} \tag{2}$$

TABLE 1. COMPARISON OF NBS DETERMINED TARGET SPACINGS AND PHOTOGRAMMETRICALLY MEASURED TARGET SPACINGS ON THE TANKS OF THE *EL PASO COVE POINT*.

Tape # and Location		NBS Length (m) d_{NBS}	Photogrammetric Length (m) d_P	$d_P - d_{\text{NBS}}$ (mm)
	Tank #3		Temperature, 26.8°C 8-17-78	
1	For. Vert.	1.7188	1.7182	−0.6
		20.5985	20.5969	−1.6
2	Aft Vert.	1.2673	1.2674	+0.1
		20.5812	20.5801	−1.1
3	Port Vert.	0.5826	Target center out	
		16.0113	16.0105	−0.8
4	For. Horiz.	2.6679	Target obscured	
		13.0921	13.0906	−1.5
		16.0220	16.0210	−1.0
		20.2112	20.2095	−1.7
		20.8869	20.8851	−1.7
5	Port Horiz.	5.9795	5.9798	+0.3
		Target not recorded	7.0734	
		12.0630	12.0626	−0.4
		19.7401	19.7388	−1.3
		Target not recorded	20.4795	
		25.7966	25.7953	−1.3
		31.9756	31.9736	−2.0
6	Diag. Horiz.	0.3278	Target obscured	
		38.9965	Target obscured	
		39.1896	Target obscured	
7	Trans. Horiz.	0.1989	0.1985	−0.4
		30.1526	30.1493	−3.3
		30.4457	30.4428	−2.9
8	Diag. Horiz.	0.1690	0.1690	0
		39.6625	39.6599	−2.6
		39.8910	39.8887	−2.3
	Tank # 4		Temperature, 26°C 9-15-78	
1	For. Vert.	1.7256	1.7252	−0.4
		20.6195	20.6144	−5.1
2	Aft Vert.	1.2304	1.2301	−0.3
		20.9810	20.9808	−0.2
3	Stbd. Vert.	0.5672	0.5673	+0.1
		16.6537	16.6532	−0.5
4	For. Horiz.	2.6532	2.6537	+0.5
		13.0551	13.0562	+1.1
		15.9874	15.9890	+1.6
		20.8413	20.8430	+1.7
5	Stbd. Horiz.	6.0658	6.0645	−1.3
		7.0668	7.0650	−1.8
		12.1546	12.1521	−2.5
		19.7142	19.7108	−3.4
		25.7845	25.7800	−4.5
		32.0028	31.9968	−6.0
	Tank #5		Temperature, 24°C 10-12-78	
1	For. Vert.	1.7407	1.7410	+0.3
		20.1578	20.1580	+0.2
2	Aft Vert.	1.3327	1.3331	+0.4
		21.1277	21.1271	−0.6
3	Stbd. Vert.	0.5746	0.5747	+0.1
		16.6067	16.6077	+1.0
4	For. Horiz.	2.6450	2.6449	−0.1
		13.0582	13.0581	−0.1
		15.9717	15.9714	−0.3
		20.1938	20.1933	−0.5
		20.8499	20.8498	−0.1

TABLE 1. CONTINUED

Tape # and Location		NBS Length (m) d_{NBS}	Photogrammetric Length (m) d_P	$d_P - d_{NBS}$ (mm)
5	Stbd. Horiz.	5.9948	5.9949	+0.1
		7.0890	7.0894	+0.4
		12.0673	12.0679	+0.6
		19.7514	19.7523	+0.9
		23.0463	23.0469	+0.6
6	Diag. Horiz.	0.3263	0.3264	+0.1
		29.2902	29.2919	+1.7
		29.6015	29.6033	+1.9
7	Port Horiz.	6.0835	6.0836	+0.1
		12.0287	12.0192	−9.5
		21.2385	21.2358	−2.8
		24.1893	24.1913	+2.0
8	Diag. Horiz.	0.1652	0.1652	0
		29.5943	29.5933	−1.0
		29.8706	29.8719	+1.3
	Tank #2		Temperature, 18.5°C 11-20-79	
1	For. Horiz.	1.7128	1.7139	+1.1
		8.9399	8.9405	+0.6
		14.0045	14.0052	+0.7
		20.1844	20.1844	0
		20.6216	20.6222	+0.6
2	For. Vert.	1.2420	1.2424	+0.4
		20.5848	20.5849	+0.1
		21.0513	21.0517	+0.4
3	Port Vert.	0.5550	0.5547	−0.3
		16.5642	16.5645	+0.3
4	Aft Vert.	0.6678	0.6674	−0.4
		20.8501	20.8516	+1.5
	Tank # 1		Temperature, 18.7°C 11-21-79	
1	Aft Vert.	1.7224	1.7223	−0.1
		20.1815	20.1819	+0.4
		20.6383	20.6389	+0.6
2	For. Vert.	1.2825	1.2823	−0.2
		20.5991	20.5982	−0.9
		21.0468	21.0463	−0.5
3	Port Vert.	0.5545	0.5548	+0.3
		16.7708	16.7720	+1.2

where $\bar{l}_j$ is the average of the l_{ij} for the j^{th} tape. The scale factor difference s is ideally 0, as would the value for c $(=s\bar{l}_j)$ which is the offset of target 1. The form of Equation (2) provides that the estimate for s is unaffected by an error in the position of target 1. The quantity ϵ_{ij} is the random error attributed to the photogrammetric measurement. The information derived from fitting to the l'_{ij} data are an estimated value for s, an estimate of how this s value affects the volume, and an estimated value for the standard deviation of the l'_{ij}.

The scale factor error, s, did not vary significantly from tape to tape on the same tank for the first two ship sets calibrated. For the final ship set, the evidence indicated s was not the same for all tapes on the same tank; so, s was assumed to change in a random fashion from tape to tape. Permitting s to vary in the analysis ascribes more of the uncertainty to the scale error and less to relative coordinate positions. The analyses of all the fifteen tanks summarized in Tables 3 and 4 used Equation (2) with s permitted to vary from tape to tape on the same tank. The estimated volume bias and volume uncertainty resulting from the s values in column 2 is given in the last column.

The larger scale errors in the *Savannah* data, Table 2, are attributed to the method of applying the photogrammetric scale to the tank. The targeted

TABLE 2. VALUES OF s WHEN ALLOWED TO VARY FOR EACH TAPE ON THE TANKS OF THE *SAVANNAH*.

Tank	s Value with 95% C.I.*	Number of Tapes	Volume Bias with 95% C.I. (%)
1	$-14.5 \pm 6.4\ (10^{-5})$	3	-0.04 ± 0.02
2	$-22.6 \pm 6.4\ (10^{-5})$	3	-0.06 ± 0.02
3	$2.4 \pm 7.9\ (10^{-5})$	2	0.01 ± 0.02
4	$-9.8 \pm 7.9\ (10^{-5})$	2	-0.03 ± 0.02
5	$-9.8 \pm 7.9\ (10^{-5})$	2	-0.03 ± 0.02

* Confidence intervals based on overall standard deviation of 4.8×10^{-5} with seven degrees of freedom.

tape method of providing the scale gives more accurate results, as can be seen by comparing Table 2 with 3 and 4.

RANDOM UNCERTAINTY OF THE LENGTH MEASUREMENTS

The random uncertainty in d_p for each tank set, assuming d_{NBS} correct and after correcting for scale error, is given in Table 5. The standard deviations for the first two ships sets were estimated by assuming a single value for s over a tank, while s was allowed to vary from tape to tape over a tank in the analysis of the *Columbia* data. The estimated standard deviation in this case is smaller because more uncertainty appears in the scale error. The photogrammetrist estimates the uncertainty of his length measurements at 0.4 to 0.8 mm, depending on the target locations and the number of photographs in which each target is visible. Because a large number of targets define the planar side of a tank, this random error can be expected to average out to a negligible contribution to the volume error.

The diagonal tapes showed no evidence that the photogrammetric dimensioning distorts the tank diagonally as it could do without affecting the tapes applied to the tank surfaces. An independent survey of one tank by NBS personnel using a laser plane method (Hoken and Haight, 1978) showed no evidence of any other distortion in the measurement of the tank by the photogrammetric calibration (Siegwarth and LaBrecque, 1981).

TABLE 3. SCALING ERROR ESTIMATES, s, FOR EACH TAPE ON THE TANKS OF THE *COVE POINT*.

Tank	s Value with 95% C.I.*	Number of Tapes	Volume Bias with 95% C.I. (%)
1	$2.0 \pm 5.5\ (10^{-5})$	3	0.006 ± 0.016
2	$2.1 \pm 4.8\ (10^{-5})$	4	0.006 ± 0.014
3	$-6.7 \pm 1.7\ (10^{-5})$	6	-0.021 ± 0.005
4	$-7.6 \pm 16.1\ (10^{-5})$	5	-0.024 ± 0.050
5	$1.3 \pm 3.4\ (10^{-5})$	8	0.004 ± 0.010

* Confidence intervals based on overall standard deviation of 4.4×10^{-5} with 12 degrees of freedom.

TABLE 4. SCALE ERROR ESTIMATES, s, FOR EACH TAPE ON THE TANKS OF THE *COLUMBIA*.

Tank	s value with 95% C.I.*	Number of tapes	Volume Bias with 95% C.I. (%)
1	$-1.4 \pm 8.4\ (10^{-5})$	3	-0.004 ± 0.025
2	$-0.8 \pm 8.4\ (10^{-5})$	3	-0.002 ± 0.025
3	$-1.6 \pm 8.4\ (10^{-5})$	3	-0.005 ± 0.025
4	$-1.5 \pm 8.4\ (10^{-5})$	3	-0.004 ± 0.025
5	$7.0 \pm 8.4\ (10^{-5})$	3	0.021 ± 0.025

* Confidence intervals are based on overall standard deviations of the 6.6 (10^{-5}) with 10 degrees of freedom.

The standard deviations of the scale factor s for each of the three sets of tanks are 4.8×10^{-5} (12 d.f.) for the *Savannah*, 4.4×10^{-5} (12 d.f.) for the *Cove Point*, and 6.6×10^{-5} (10 d.f.) for the *Columbia*. The predicted scale uncertainty for future photogrammetric surveys is the pooled value of these three results, 5.3×10^{-5} with 29 d.f. A two-sided 99 percent confidence interval for the true standard deviation s is 3.9×10^{-5} to 7.8×10^{-5}. Taking the upper end of the confidence interval and using six calibration tapes as employed in the NBS tests, the uncertainty in the volume calibration is 0.025 percent for a 99 percent confidence interval. Using three tapes, as did the photogrammetrist (two lengths on each), the uncertainty in the volume calibration is 0.035 percent.

TOTAL UNCERTAINTY OF THE CALIBRATION

So far, only the accuracy of the photogrammetric coordinate determination has been considered. Other sources of uncertainty enter the tank calibration and must be included along with the uncertainty of the photogrammetric measurement. Some of these other sources of uncertainty have been estimated from NBS measurements and are given in Tables 6, 7, and 8 for each of the fifteen tanks of the three ships (Siegwarth and LaBrecque, 1981). The photogrammetry associated errors appear in the scale factor and scale bias column. Note that, if the scale bias values in the last column of Tables 2, 3, and 4 are less in magnitude than the random uncertainty of the scale factor, the scale bias is assumed to be zero.

TABLE 5.

Ship	Standard Deviation of Length Measurement	Degrees of Freedom
Savannah	0.58 mm	27
Cove Point	0.65 mm	57
Columbia	0.37 mm	34

TABLE 6. ERROR SUMMARY (PERCENT) FOR THE *SAVANNAH* TANKS†.

Tank	Random Uncertainty				Limits of Systematic Error for Thermal Coef.	Scale Bias	Scale Bias ± Systematic and Random Error
	Volume Calculation	Target Spacing	Scale Factor	RMS Total			
1	±0.01	±0.014	±0.02	±0.026	±0.03	−0.04	−0.04 ± 0.056
2	±0.01	±0.014	±0.02	±0.026	±0.03	−0.06	−0.06 ± 0.056
3	±0.01	+0.00	±0.02	±0.022	±0.03	0.01	0.01 ± 0.052
4	±0.01	0.00	±0.02	±0.022	±0.03	−0.03	−0.03 ± 0.052
5	±0.01	0.00	±0.02	±0.022	±0.03	−0.03	−0.03 ± 0.052

The volume calculation column of Tables 6, 7, and 8 is an estimate of the uncertainty introduced by the model fit to the coordinate data. The tank was modeled both by fitting planes to the coordinate data of each side and by estimating cross-sectional areas at the heights of the various levels of targets. The second method agreed most closely with the results of the photogrammetrist, who used a similar method. The difference between the two methods and the photogrammetrist's results provide an estimate of 0.01 percent for the uncertainty due to the mathematical model of the tank (Siegwarth and LaBrecque, 1981).

The first three tanks calibrated, 3, 4, and 5 of the *Savannah*, had twice as many targets as the remaining tanks. The larger number of targets should give a better detail of the tank shape. The estimated uncertainty added by reducing the number of targets is given in the "target spacing" column (Siegwarth and LaBrecque, 1981).

The root mean square total of the first three columns of uncertainty is given in the "RMS Total" column. Except for tank 4 of the *Cove Point*, all the RMS totals are no larger than the 0.03 percent estimated by the photogrammetrist (Brown, 1981).

Thermal expansion of the tank material adds uncertainty in two ways. The tank was not isothermal at the time the photographs were taken. The inside quadrant walls were as much as 8°C warmer than the outer walls near the top center. This causes the outer walls to bow out slightly at their junction with the quadrant walls. The systematic uncertainty added by this effect is conservatively estimated to be no larger than an 0.01 percent.

A relatively large error is introduced into the operating temperature calibration by the ±3 percent uncertainty in the expansion coefficient of aluminum (Mann, 1977). This contributes a ±0.03 percent uncertainty to the tank table, assuming the level gage is an aluminum coaxial capacitance gage with the same uncertainty in its coefficient of expansion.

The volume of aluminum in the walls and structural members inside the tank must be subtracted as a function of height from the exterior volume of the tank to obtain the liquid capacity as a function of height. This correction, called the deadwood correction, also introduces an uncertainty in the volume calibration because the plate thicknesses of the various tank components were, to varying degrees, generally larger than the plate thicknesses on the drawings (Siegwarth and LaBrecque, 1981) used by the photogrammetrist to do this correction. The volume of the one tank for which NBS personnel measured the plate dimensions was overestimated by 0.03 percent from that using the nominal plate dimensions (Siegwarth and LaBrecque, 1981). The deviations of the plates from nominal dimensions vary so that the size of the correction to the other tanks could be larger or smaller. Including an estimated uncertainty to the systematic error, the volume of the largest tanks could be overestimated

TABLE 7. ERROR SUMMARY (PERCENT) FOR THE *COVE POINT* TANKS†.

Tank	Random Uncertainty				Limits of Systematic Error for Thermal Coef.	Scale Bias	Scale Bias ± Systematic and Random Error
	Volume Calculation	Target Spacing	Scale Factor	RMS Total			
1	±0.01	±0.014	±0.016	±0.023	±0.03	0.00	0.00 ± 0.053
2	±0.01	±0.014	±0.014	±0.022	±0.03	0.00	0.00 ± 0.052
3	±0.01	±0.014	±0.005	±0.018	±0.03	−0.02	−0.02 ± 0.048
4	±0.01	±0.014	±0.050	±0.053	±0.03	0.00	0.00 ± 0.103*
5	±0.01	±0.014	±0.010	±0.02	0.03	0.00	0.00 − 0.05

* Test tape errors suspected.

TABLE 8. ERROR SUMMARY (PERCENT) FOR THE *COLUMBIA* TANKS.

	Random Uncertainty				Limits of Systematic Error for Thermal Coef.		
Tank	Volume Calculation	Target Spacing	Scale Factor	RMS Total		Scale Bias	Systematic and Random Error Limit
1	±0.01	±0.014	±0.025	±0.03	±0.03	0.00	±0.06
2	±0.01	±0.014	±0.025	±0.03	±0.03	0.00	±0.06
3	±0.01	±0.014	±0.025	±0.03	±0.03	0.00	±0.06
4	±0.01	±0.014	±0.025	±0.03	±0.03	0.00	±0.06
5	±0.01	±0.014	±0.025	±0.03	±0.03	0.00	±0.06

by as much as 0.04 percent and the smallest by as much as 0.06 percent.

NBS personnel made measurements on two tanks to obtain an estimate of the effect lifting the tank and installing it on the ship might have on the volume uncertainty and made measurements on yet another tank to estimate the effect of hydrostatic loading (Siegwarth and LaBrecque, 1981). No effect on tank volume was detected in either case.

The inclusion of all the additional uncertainties into Tables 6, 7, and 8 does not raise the estimated uncertainty above the 0.2 percent maximum value of uncertainty required.

CONCLUSIONS

By using some accurately calibrated iron-36 percent nickel surveyors tapes, NBS independently estimated the uncertainty of the calibration of some large LNG ship transport tanks. The dimensioning of the tank provided a volume uncertainty of 0.03 percent or less, assuming that the small systematic errors detected have been corrected. This estimated uncertainty agrees with the uncertainty reported by the photogrammetrist (Brown, 1981).

Including some additional sources of uncertainty unrelated to the photogrammetric measurement does not increase the uncertainty above the ±0.2 percent uncertainty required by the ship owner.

The random uncertainty of the scale factor is the leading uncertainty in the photogrammetric portion of the calibration. This uncertainty could be reduced if the photogrammetrist used more and longer calibration tapes to provide the scale. However, the other uncertainties not associated with the photogrammetric method are sufficiently large so the total calibration uncertainty would not be reduced significantly by adding more calibration tapes.

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† Some of the values in the scale factor, RMS total, scale bias, and scale bias + systematic and random error columns are slightly higher in Table 6 and 7 than in the corresponding tables in Siegwarth and La Brecque, (1981). This results because s_j was allowed to vary from tape to tape on a tank in the analysis presented here, which gives slightly larger values for the estimated uncertainties.