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Experience with Height Interpolation by Finite Elements

Practical results are presented, obtained with the HIFI minicomputer program package for DEM interpolation and digital contouring.

Abstract: *The method of height interpolation by finite elements was suggested by the authors at the DTM Symposium in St. Louis in 1978. Based on this concept, a general minicomputer program package, "HIFI", was developed and presented at the 1980 ISP Congress in Hamburg. HIFI interpolates a grid Digital Elevation Model (DEM) from arbitrarily distributed reference points and points along break lines. Digital profiles and digital contours can be derived from the DEM. The program package is written in FORTRAN and implemented on a Hewlett-Packard HP 1000 minicomputer. Alternative implementations are available for Prime computers and Digital Equipment computers.*

After a brief description of HIFI, the paper gives a review of the experience gained with height interpolation by finite elements. The presented examples refer to the accuracy of DEM interpolation and to the operational use of HIFI. Main applications are the derivation of digital profiles for orthophoto production with the Zeiss Orthocomp Z 2 analytical orthoprojector and the derivation of digital contours for automatic plotting. The required computing times are reasonable and the resulting computing costs are low.

Introduction

This paper refers to the generation of Digital Elevation Models (DEM) and to the derivation of digital profiles and digital contours from the DEM.

As generally agreed, a DEM is understood as a description of a surface, in our case the terrain surface, by digital data. Two concepts are used in practice today:

- irregular DEMS, consisting of a network of plane or curved triangles. The given reference points represent the nodes of this network.
- regular DEMS, built of meshes, which form a square shaped grid in planimetry. The heights of the grid points are interpolated from the heights of given reference points.

In photogrammetry grid models are more frequently applied than triangle models. Their main advantage is the regular structure of the data, which makes the use of the DEM more simple.

DEM Interpolation by Finite Elements

The DEM, referred to in this paper, is of the square grid type. The heights of grid points are interpolated from arbitrarily distributed reference points and points along break lines. The concept of interpolation used fits into the general Finite Element Method, which has been successfully applied in numerical mathematics and several fields of engineering over more than two decades (Strang and Fix, 1973; Zienkiewicz, 1977). In contrast to most of these applications, however, the DEM interpolation used here is based on simple and regular structures.

For terrain description, an interpolation surface is defined. It is formed of local surfaces, or finite

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Elements, which are tied together accordingly. Two types of Finite Elements are used, either bilinear or bicubic.

If the terrain surface is rough and break lines have to be considered, bilinear finite elements are used. In this case, a separate bilinear polynomial is defined for each grid mesh, and continuity along the borders of adjacent elements is guaranteed. The interpolation surface is determined by a minimization of the weighted sum of squares of the discrepancies of the given reference points from the interpolation surface and of the second differences of the heights of neighboring grid points. At break lines the minimization of the second differences is altered in such a way that connections across the break lines are avoided. By this means a definite interpolation surface is obtained which approximates the available reference points with optional filtering and represents the given break lines adequately (Figure 1a).

Bicubic finite elements are used if the terrain surface is smooth and no break lines have to be considered. In this case a separate bicubic polynomial is defined for each grid mesh and continuity along the common borders of neighboring elements is extended up to the second derivatives. The interpolation surface is now determined by a minimization of the weighted sum of squares of the discrepancies of the given reference points from the interpolation surface and of the second derivatives at the grid points (Figure 1b).

The basic idea of height interpolation by finite elements is not new, but was described earlier in a paper on surface approximation by piecewise polynomials (Kubik, 1971). The realization, chosen in the present paper, however, shows some clear distinctions concerning the mathematical formulation as well as the used minimum condition. Moreover, Kubik did not take break lines into consideration.

The bilinear variant of height interpolation by finite elements was presented at the DTM Symposium of the ASP in St. Louis (Ebner and Reiss, 1978). Work on the bicubic variant was started in 1979.

Finally, a paper on surface modeling by means of an elastic grid should be mentioned (de Masson d'Autume, 1979). This independently developed method shows close similarities to the bilinear variant described above.

THE HIFI MINICOMPUTER PROGRAM PACKAGE

Based on the method described in the preceding chapter, a general program package, HIFI (Height Interpolation by FInite elements) was developed in Munich and presented at the 1980 Congress of the ISP in Hamburg. HIFI interpolates a grid DEM from arbitrarily distributed reference points and points along break lines. Digital profiles and digital contours can be derived from the DEM. A detailed description is given in Ebner *et al.* (1980).

The HIFI program package was developed for use with minicomputers and is written in FORTRAN. The development was supported by Zeiss, Oberkochen. Accordingly, HIFI can be used in combination with the Planicomp C 100 analytical plotter and the Orthocomp Z 2 analytical orthoprojector (Hobbie, 1976; Faust, 1980). In these cases HIFI runs on a Hewlett-Packard HP 1000 minicomputer and requires a minimum central memory capacity of 128 K words of 16 bit.

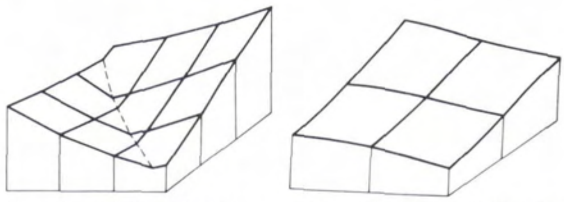

FIG. 1. (a) A grid DEM consisting of bilinear finite elements and representing a break line. (b) A grid DEM consisting of bicubic finite elements.

A transfer of the program package to other minicomputers and to large computers is possible, however, and has already been carried out. Alternative HIFI implementations are available for Prime computers and Digital Equipment computers.

HIFI is available in four versions:

The HIFI-P version interpolates a DEM using either bilinear or bicubic finite elements. The mesh width of the DEM can be chosen by the user. If reference points and points along break lines are given, the bilinear variant is applied. Without break lines, either the bilinear, or the bicubic variant can be used.

Moreover, HIFI-P allows for optional derivation of digital profiles from the DEM which can be directly used for orthophoto production with the Orthocomp Z 2 analytical orthoprojector.

The HIFI-PS version is an extension of HIFI-P, allowing for derivation of digital profiles and partner profiles, which can be directly used for stereo orthophoto production with the Orthocomp Z 2.

The HIFI-PC version includes HIFI-P and allows for optional derivation of digital contours from the DEM. The contour interval is chosen by the user. The computed contours are prepared for subsequent automatic plotting on a screen or a digital tracing table.

The HIFI-PSC version combines the capabilities of HIFI-PS and HIFI-PC. A block diagram of this version is shown in Figure 2.

EXPERIENCE

The following examples give a review of the experience gained with height interpolation by finite elements. The first example is an accuracy test for DEM interpolation; the other examples demonstrate the operational use of HIFI. One case refers to the derivation of digital orthophoto profiles from recorded points along contours using HIFI-P. In the other cases HIFI-PC was used for DEM interpolation and derivation of digital contours from measurements of arbitrarily distributed topographical points or grid points and break lines.

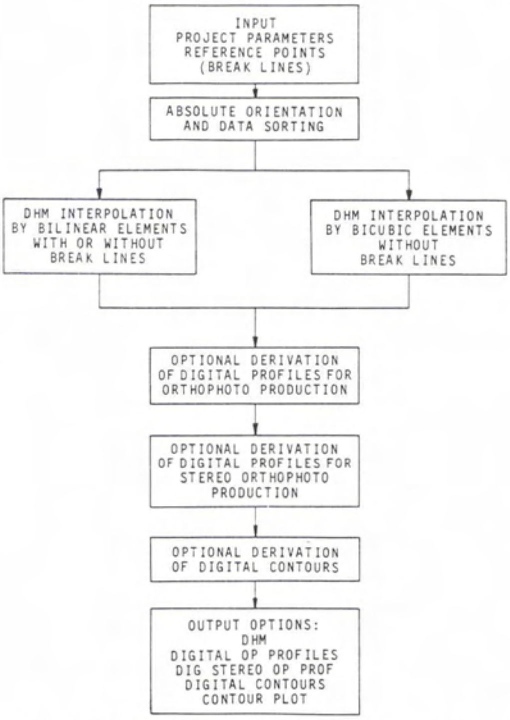

FIG. 2. Block diagram of the HIFI-PSC version.

DONAUWOERTH TEST

The purpose of this DEM test was to investigate the effect of the type and density of reference points and of the mesh width of the DEM on the accuracy of interpolated heights. The test is part of an extensive investigation on the efficiency of DEM interpolation by finite elements, which is the topic of the second co-author's doctoral thesis.

The results presented here refer to an area of 1200 by 1200 m^2, located in one model. The terrain is steep with quite varied slopes and distinct break lines. Photographs were taken with a Zeiss RMK A 15/23 camera at a scale of 1:14 400 and were made available by the Board of Surveying in Munich.

For data acquisition, the Planicomp C 100 analytical plotter of the Chair for Photogrammetry at the Munich Technical University was used. The following data were measured and recorded (Operator M. Spindler):

- grid heights (10 by 10 m^2)
- profile heights (spacing between profiles 20 m)
- contours (interval 2.5 m; partly 1 m)
- break lines and spot heights.

Bilinear finite elements were used for DEM interpolation, and the given break lines were taken into consideration. Subsets of the recorded data were used as reference points, and the data density of the subsets was varied accordingly. The mesh width of the DEM was 20 m, 40 m, and 60 m, respectively. From the DEM the heights of a 10 by 10 m^2 grid were interpolated and compared with the directly measured and recorded grid heights in order to check the accuracy of DEM interpolation. As far as grid measurements were used as input for DEM interpolation, of course only the remaining grid heights were used for checking.

Table 1 shows the corresponding RMS values of the height differences which, however, include the random errors of the check heights. The standard deviation of these heights (grid heights) was estimated from repeated measurements and amounted to about $\sigma = 0.25$ m. The standard deviation of the profile and contour heights was about $\sigma = 0.50$ m.

The three columns in Table 1 show, for a given mesh width, how the accuracy of DEM interpolation is effected by the type and density of reference points.

Greatest accuracy is obtained from grid measurements where the floating mark stops at the respective point until the operator has executed the measurement. The dynamic procedures of profiling and contouring are less accurate.

It is remarkable that, for DEMs of 20-m and 40-m mesh width, the relatively sparse 60-m grid leads to about the same accuracy as 20-m profiles or contours do. The required time for data acquisition, however, is much shorter. These results confirm the earlier findings of Rüdenauer (1980). For practical projects, therefore, the replacement of dynamic profiling by static grid measurements can be recommended.

By using a denser grid of 40 by 40 m^2 or 20 by 20 m^2, accuracy is improved accordingly. Unfortunately, the full improvement doesn't show up in Table 1 because of the limited accuracy of the check heights which has a greater effect on the RMS values the smaller they are.

The rows in Table 1 show the effect of the mesh width of the DEM on the accuracy of interpolated heights. In all cases, greatest accuracy is obtained with 20-m meshes, but larger meshes don't reduce the accuracy drastically.

It should be emphasized, however, that the RMS

TABLE 1. RMS VALUES OF THE DIFFERENCES BETWEEN INTERPOLATED AND DIRECTLY MEASURED GRID HEIGHTS (10 BY 10 m^2 GRID)

Type and Density of Reference Points	Mesh Width of the DEM		
	20 m	40 m	60 m
20-m grid	0.35 m	0.35 m	0.41 m
40-m grid	0.38 m	0.44 m	0.46 m
60-m grid	0.46 m	0.49 m	0.61 m
20-m profiles	0.47 m	0.48 m	0.51 m
40-m profiles	0.49 m	0.51 m	0.56 m
60-m profiles	0.52 m	0.55 m	0.59 m
contours	0.51 m	0.53 m	0.57 m

values only describe the average accuracy of DEM interpolation. They don't indicate that details are lost, when using larger meshes. Detail information, however, is essential if digital contours are derived from the DEM. For such applications, a mesh width of about one half of the distance between neighboring reference points can be recommended.

THE LOHNWEILER EXAMPLE

This project is an example of the use of HIFI-P in conjunction with the Orthocomp Z 2 analytical orthoprojector. At the Board of Surveying in Koblenz, contours had been measured photogrammetrically and recorded. The contours covered an area of 2 by 2 km^2, corresponding to one sheet of the German Base Map (scale 1:5000). The terrain was hilly and rather rough. The total number of points along contours was 14671.

Because of the roughness of the terrain, the bilinear variant of HIFI was used for DEM interpolation. Table 2 represents the results for a mesh width of 20 m (orthophoto slit length 4 mm) and of 40 m (orthophoto slit length 8 mm), using all contour points or only every second or third point. The heights of the omitted contour points were compared with the interpolated heights at the same locations x, y. The corresponding RMS values of the height differences are listed in Table 2 and make it possible to check the accuracy of the respective DEM interpolation. It should be emphasized, however, that the check points are of the same accuracy as the reference points and that the space between contours is not checked in this way.

With 20-m meshes the use of every second contour point still leads to an average accuracy of about 0.5 m but reduces the computing time from 92 min to 77 min. With 40-m meshes an average accuracy of about 1.0 m is obtained by using only every third contour point. The corresponding time is reduced to 21 min on the HP 1000.

For pure orthophotography, both the 20-m meshes and the 40-m meshes can be considered as adequate. The resulting average accuracy of an orthophoto (scale 1:5000) from wide angle photography is significantly below 0.1 mm in x and y.

LOWER MKOMAZI IRRIGATION PROJECT

An agricultural study was performed in Lower Mkomazi Valley, Tanzania, on behalf of the Technical Foreign Aid Society of West Germany (GTZ). This study was executed by the working group Instrupa-Rodeco-Hydroplan, and Prof. Linkwitz and Dr. Preuss (Stuttgart) were in charge of the topographic work.

The project area of 220 km^2 was covered by 44 models, from wide angle and super wide angle photographs, taken with Wild cameras at scales of 1:12 000 to 1:47 000. The terrain was smooth but covered with bushes, which didn't allow for direct contouring. Instead, about 6000 irregularly distributed topographic points, corresponding to five map sheets, were measured by the private firm Photogrammetrie GmbH, Munich, and contours were derived by means of DEM, using HIFI.

The bicubic variant was used for DEM interpolation, and the chosen mesh width was 100 m. Subsequently, the DEM was densified to 50 m for contouring. The total number of densified points was 113539 and the computing time amounted to 4 h 08 min on the HP 1000 minicomputer.

Digital contours at an interval of 2 m were derived from the DEM in 29 min. The contour plotting to a scale of 1:10 000 on five sheets took about 10h on a Zeiss DZ 7 digital tracing table.

THE HINTERSCHMIDING EXAMPLE

The data for this project originate from an investigation which was performed on behalf of the Bavarian Department of Agriculture and Forestry. An area of 2335 by 2335 m^2, corresponding to one sheet of the Bavarian Base Map (scale 1:5000), was covered by two models. Photographs were taken with an RMK A 30/23 camera at a scale of 1:13 500.

On the Planicomp C 100 analytical plotter, points of a grid of 40 by 40 m^2 and points along break lines were measured and recorded. The required time was about 6 hours, including the orientation of the two models (operator H. Rentsch).

Using these data, a DEM of 20-m mesh width was computed, using the bilinear variant of HIFI. The number of interpolation points was 14875, and the required computing time on the HP 1000 amounted to 81 min.

From the DEM digital contours at an interval of 5 m were computed using HIFI, and were plotted to a scale of 1:5000 using the DZ 7 digital tracing table. The required computing time was 4 min on the HP

TABLE 2. PROJECT DATA FROM THE LOHNWEILER EXAMPLE

Mesh Width of the DEM	Reference Points	Check Points	RMS Value	Interpol. Points	Comp. Time
20 m	14671	—	—	11881	92 min
20 m	7323	7348	0.51 m	11881	77 min
20 m	4898	9773	0.66 m	11881	76 min
40 m	14671	—	—	3025	31 min
40 m	7323	7348	0.91 m	3025	23 min
40 m	4898	9773	0.96 m	3025	21 min

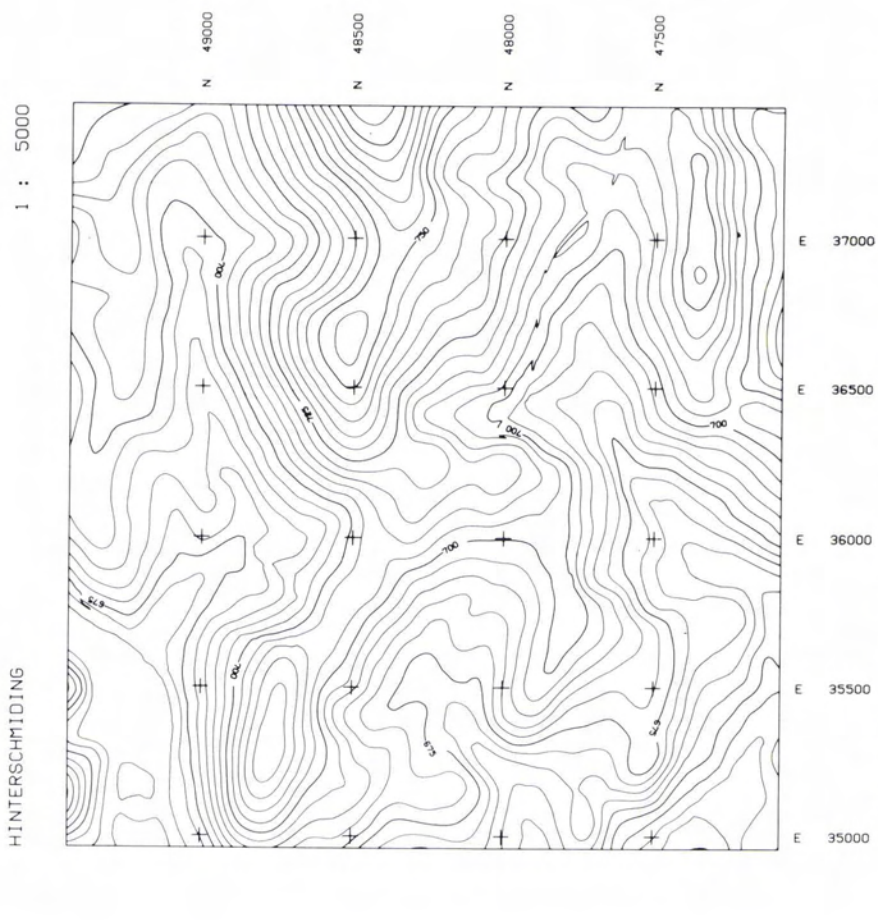

FIG. 3. The Hinterschmiding example; contour map derived from measured grid points and break lines using HIFI-PC.

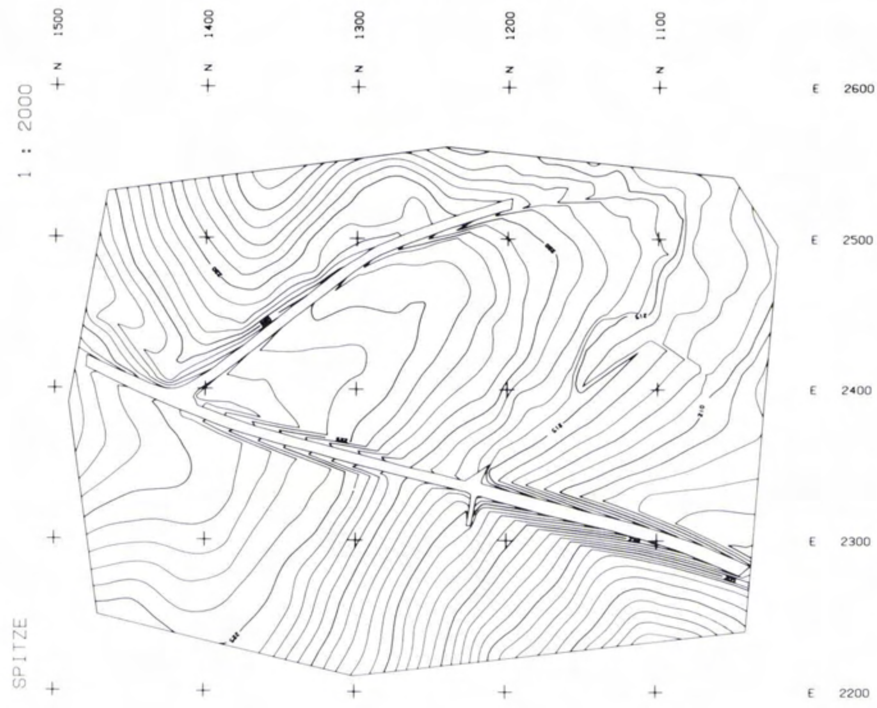

FIG. 4. The Spitze example; contour map derived from measured grid points and break lines using HIFI-PC.

1000, and 29 min were needed for plotting. Figure 3 shows the contour plot for the whole map sheet.

The DEM was further used for automatic production of slope maps, using a preliminary program on a large CDC Cyber 175 computer (Ebner and Reiss, 1980).

THE SPITZE EXAMPLE

Spitze is a test demonstrating the use of DEM in road construction. An area of about 700 by 700 m^2 was covered by two models. Photographs were taken with an RMK A 15/23 camera at a scale of 1:4000.

At the Institute of Photogrammetry and Engineering Surveying at Hannover University, grid points (8 by 8 m^2) and points along break lines were measured and recorded using the O.M.I. AP/C-3 analytical plotter. The data were made available by Prof. Konecny. For details see Rüdenauer (1980).

For the test described here, a subset of grid points (16 by 16 m^2) was selected. From these data and all available break lines, a DEM of 8-m mesh width was interpolated using the bilinear variant of HIFI. The total number of interpolation points was 7480, and a computing time of 44 min was required on the HP 1000.

The derivation of digital contours of 1-m interval from the DEM and the omission of contours in predefined areas required 10 min on the HP 1000. Figure 4 shows a plot of the contours to a scale of 1:2000 for a part of the area. The plotting time on the DZ 7 digital tracing table was 20 min.

Conclusion

The examples presented demonstrate the efficiency of the method of height interpolation and of the HIFI program package.

DEM interpolation by finite elements is a real two-dimensional method and allows for consideration of break lines in a simple and effective way.

HIFI realizes this concept on minicomputers which are widely used nowadays. The required computing times are reasonable and the resulting computing costs are low.

For DEM interpolation with the bilinear variant of HIFI, which can be applied to any type of reference data including break lines, about 0.3 sec per interpolation point are required on an HP 1000 minicomputer. Thus, a DEM of 10 000 points can be generated in less than one hour. Using the bicubic variant of HIFI, about 0.6 sec are required per interpolation point.

The computing time for derivation of orthophoto profiles from the DEM, using the HIFI-P program version, is below 0.01 sec per point. Derivation of profiles for stereo orthophoto production is possible with HIFI-PS and requires about 0.06 sec per point on an HP 1000.

HIFI-PC allows for derivation of digital contours from the DEM. The required computing time is usually in the order of 10 percent of the computing time for the preceding DEM interpolation.

The program package has already been implemented on many HP 1000 computers by Zeiss, Oberkochen. Further implementations on Prime and Digital Equipment minicomputers were made from Munich.

Acknowledgment

In addition to the authors, Dr. Hofmann-Wellenhof (now in Graz) and Dr. Steidler (now in Heerbrugg) have made important contributions to the development of the program package. Theoretical support concerning the bicubic version and the derivation of digital contours from the DEM was given by Prof. Meissl, Graz.

Further development and operational use of HIFI in Munich are in the hands of Mr. Hoessler, Dr. Stephani, Mr. Unsleber, and Mr. Eder.

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