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A Step-by-Step Strategy for Gross-Error Detection*

A four-step technique, based upon the magnitude and type (image coordinate or ground control coordinate) of error, is described.

Introduction

The least-squares estimators, for unknown parameters, are defined to be those estimators that minimize the weighted sum of squared residuals. By assuming that the residuals are random variables with zero mean, the condition of least squares also forces the weighted sum of residuals to be zero. In bundle adjustment, this is not only true for the sum of residuals of all observations, but also for the residuals of the observations in each photograph. In addition, the sum of residuals of the same point in the different photographs where it appears is also zero. Therefore, an observational blunder at a certain point affects not only the residuals for that point but also the residuals for the other points in the same photograph, and for the same point in other photographs, in order to maintain the zero sums. The latter effect is reduced by the increase in the number of photographs in which the point appears. When there are more gross errors than one and when they have vastly different magnitudes, it becomes almost impossible to locate these errors by simply analyzing the residuals after the adjustment of the whole block. Therefore, most of these errors should be detected and removed before starting the block adjustment. The techniques used for this purpose depend on the type and the magnitude of gross errors. The technique used at NRC for gross-error detection divides these errors into four types. A special procedure is applied for each type. Non-statistical approaches are employed for the first three types of gross errors and a rigorous statistical method (data snooping) for the fourth type.

Abstract: *The technique developed at NRC for gross-error detection is composed of four steps. Each step deals with errors of a certain magnitude and nature such as errors in point identification or errors in the control points. The order of applying this procedure is of great importance. The final step, which deals with small observation errors of magnitudes larger than the random range, consists in applying a sensitive statistical test (data snooping). A special bundle adjustment program has been developed with a built-in automatic gross-error detector employing the data snooping approach. The program computes the exact values of the redundancy numbers for each image point. Using the OEEPE test blocks, which include all types of errors (intentionally introduced), the above procedure was applied and proved to be effective.*

Types of Gross Errors

One can suspect, or expect, all kind of errors from many possible sources in the input data for a photogrammetric triangulation. In particular, the following types of gross errors could exist:

Gross errors type A

These are very large blunders in image coordinates. These errors occur, for example, when the image coordinates are exchanged or when a point number is wrongly introduced. If the wrong point

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number happens to be identical to one belonging to an existing point somewhere else in the block, the effect is most severe. The size of the error is larger than the dimensions of a photograph. If a complete block adjustment is carried out with errors of this type in the data, all the residuals will be affected. However, the residuals of the points in the photographs directly affected will be much larger than other residuals.

GROSS ERRORS TYPE B

These are very large blunders in the coordinates of the control points resulting from many possible causes, for example, recording or copying mistakes, wrong point numbers, transposition of digits, the mixing of different ground coordinate systems, or the use of a ground coordinate system with a rotational direction for azimuths different from that for the photo coordinate system.

GROSS ERRORS TYPE C

These are errors in either the image coordinates or the control coordinates but of smaller magnitudes than types A and B. For example, errors in image coordinates could occur if the numbers of two adjacent points in a photograph are exchanged. The same sources causing errors of types A and B could also cause Type C errors but with smaller effect.

GROSS ERRORS TYPE D

The main characteristics of these types of errors is that they are much smaller than the previous three types. In fact, they are not very distinct from systematic errors in size; however, their characteristics are different and therefore they should be treated differently. These are mainly observation errors of magnitudes larger than the random range and occur if the point is difficult to identify.

Identification and Elimination of Gross Errors

Although all the four types of errors occur at the same time and affect one another in a complex way, they are best searched for and eliminated separately. They also must be detected in a certain order as follows:

GROSS ERRORS TYPE A

A first processing step checks the image coordinates measured in adjacent photographs for compatibility. One photograph is held as reference, and all other photographs having more than two points in common with the reference photograph are subjected to a linear conformal transformation to that photograph with its proper and its mirror imaged coordinates. The transformation coefficients, transformation residuals, and the transformed coordinates of all points of the non-reference photograph are listed. An incompatible point will be recognized by its large residuals, and an incompatibility of the two image coordinate systems by comparing the residuals for the two transformations. A comparison of the transformation results of all pairs of photographs in which a certain point occurs enables the identification of that photograph in which a wrong point is located. The transformation coefficients may indicate other problems by scale changes and/or rotations differing significantly from expected values, for example, an unknown break in a strip consisting of two sections flown at different times.

GROSS ERRORS TYPE B

At this stage, large errors in the image coordinates have been eliminated and any large residuals remaining after a block adjustment will probably be due to errors in the control points. However, it may not be clear, from an analysis of the residuals, which of the control point coordinates is in error. Therefore, a complete bundle adjustment is not advised at this stage. Instead, a simple strip and block adjustment by Schut's polynomial transformation (Schut, 1980), using the lowest possible degrees of polynomial and the minimum number of control points, is employed. The remaining control points are used as check points. Errors of Type B in the check points will be easily identified, unless the points used as control contain such errors. In this case all check points will have very large discrepancies. Different control points are chosen, in such a case, and the adjustment is repeated.

GROSS ERRORS TYPE C

The errors remaining after elimination of the gross errors of types A and B are small enough to permit bundle block adjustment. Yet errors large enough to be detected without rigorous statistical tests may still be present. After completing a bundle block adjustment, once with minimum control and once with good control distribution, the image residuals are examined to detect errors of Type C. Errors in the control points will have a global effect on the residuals while errors in the photo coordinates will have a somewhat more local effect. Also, examining the residuals of the same point in different photographs will show that the residual of the faulty observation usually has the opposite sign from the remaining residuals.

GROSS ERRORS TYPE D

Rigorous statistical testing is needed to detect gross errors type D, that is, those gross errors undetected during the already described data checking procedures. In the past few years the method of data snooping has found application in photogrammetry. This approach is regarded as much more sensitive for gross-error type D detection than classical statistical tests. The theory behind the approach is presented by Baarda (1967; 1968), while the application to photogrammetry can, for example, be found in publications of Ackermann (1979), El-Hakim

(1981b), Förstner (1980), Mikhail (1979), and Grün (1979, 1980). A summary of the theory follows without proof.

The relationship between residual, v, and observational error, d , is given by

$$v = -\mathbf{Q}_{vv}\,\mathbf{P}_{\ell\ell}\,d\ell \tag{1}$$

where $\mathbf{Q}_{vv}$ is the weight cofactor matrix of the residuals and $\mathbf{P}_{\ell\ell}$ is the weight matrix of the observations. The effect of a gross error, $\Delta\ell_i$, on the residual, v_i, of an observation, ℓ_i, is therefore

$$\Delta v_i = -r_i \Delta \ell_i \tag{2}$$

where r_i is the diagonal element of the ith column of matrix $\mathbf{Q}_{vv}\,\mathbf{P}_{\ell\ell}$. Element r_i, whose value must be between 0 and 1, is called the redundancy number. The redundancy number indicates the reliability of the adjustment of a particular observation. Zero redundancy means no reliability whatever, while increasing redundancy means increasing reliability.

The standard error, σ_{v_i}, of the residual v_i is given by

$$\sigma_{v_i} = \sigma_0\sqrt{q_i} \tag{3}$$

where q_i is the ith diagonal element of matrix $\mathbf{Q}_{vv}$, and is equal to r_i if the observation has a unit weight. In the data snooping approach, the standardized residual, which is the residual divided by its own standard deviation, is tested for gross errors. This is a more sensitive quantity than the residual itself because it takes into consideration the geometry of the intersecting rays.

Assuming that the standardized residual w_i of the observation,

$$w_i = v_i/\sigma_{v_i} = v_i/(\sigma_0\sqrt{q_i}), \tag{4}$$

is a standardized normally distributed variable, the null hypothesis, H_0, that no gross error exists in observation ℓ_i is rejected if

$$|w_i| > C \tag{5}$$

where C is a critical value chosen for a specific confidence level. The choice should be made so that the probability (α) of type I error (rejecting H_0 when true) and the probability (β) of type II error (accepting H_0 when false) are as small as possible. However, the two types of errors are related and it is not possible to make them both arbitrarily small. A reduction of α will increase β, assuming other things equal. Thus, the choice of these probabilities is really a matter of judgement and experience. How much loss is involved in rejecting good observations (type I error) compared to accepting bad observations (type II error)? The answer to this question differs from one project to another. A procedure for computing C, given α and β, can be found in Baarda (1968). Baarda suggested that $\alpha = 0.1$ percent and $\beta = 20$ percent, for which $C = 4.1$, while Förstner used $C = 3.29$. A special bundle adjustment program, BADS (Bundle Adjustment - Data Snooping), was developed at NRC (El-Hakim, 1981b). The program computes the exact value of the redundancy number for each coordinate of each image point, and tests the standardized residual statistically.

In general, the matrix $\mathbf{Q}_{vv}$ is computed as follows:

$$\mathbf{Q}_{vv} = \mathbf{Q}_{\ell\ell} - \mathbf{A}\mathbf{N}^{-1}\mathbf{A}^{T} \tag{6}$$

where $\mathbf{Q}_{\ell\ell}$ is the weight cofactor matrix of the observations, $\mathbf{A}$ is the design matrix with respect to the unknowns, and $\mathbf{N}$ is the normal equation matrix of the unknowns. Usually, in bundle adjustment programs, the normal equations are partitioned into

$$\mathbf{N} = \begin{bmatrix} \mathbf{N}_{11} & \mathbf{N}_{12} \\ \mathbf{N}_{21} & \mathbf{N}_{22} \end{bmatrix} \tag{7}$$

where subscript 1 refers to the orientation parameters and subscript 2 refers to the object coordinates. Also, matrix $\mathbf{A}$ is partitioned accordingly. Applying this to the general form (Equation 6), and introducing the design matrix $\mathbf{B}$ with respect to the observations, the final form of $\mathbf{Q}_{vv}$ will be

$$\begin{aligned}\mathbf{Q}_{vv} = {} & \mathbf{P}^{-1}\mathbf{B}^{t}\mathbf{M}^{-1}\mathbf{B}\mathbf{P}^{-1} \\ & - \mathbf{P}^{-1}\mathbf{B}^{t}\mathbf{M}^{-1}\mathbf{A}_2\mathbf{N}_{22}^{-1}\mathbf{A}_2^{t}\mathbf{M}^{-1}\mathbf{B}\mathbf{P}^{-1} \\ & - \mathbf{P}^{-1}\mathbf{B}^{t}\mathbf{M}^{-1}\mathbf{A}_2\mathbf{N}_{22}^{-1}\mathbf{N}_{21}\boldsymbol{\xi}^{-1}\mathbf{N}_{12}\mathbf{N}_{22}^{-1}\mathbf{A}_2^{t}\mathbf{M}^{-1}\mathbf{B}\mathbf{P}^{-1} \\ & + \mathbf{P}^{-1}\mathbf{B}^{t}\mathbf{M}^{-1}\mathbf{A}_2\mathbf{N}_{22}^{-1}\mathbf{N}_{21}\boldsymbol{\xi}^{-1}\mathbf{A}_1^{t}\mathbf{M}^{-1}\mathbf{B}\mathbf{P}^{-1} \\ & - \mathbf{P}^{-1}\mathbf{B}^{t}\mathbf{M}^{-1}\mathbf{A}_1\boldsymbol{\xi}^{-1}\mathbf{A}_1^{t}\mathbf{M}^{-1}\mathbf{B}\mathbf{P}^{-1} \\ & + \mathbf{P}^{-1}\mathbf{B}^{t}\mathbf{M}^{-1}\mathbf{A}_1\boldsymbol{\xi}^{-1}\mathbf{N}_{12}\mathbf{N}_{22}^{-1}\mathbf{A}_2^{t}\mathbf{M}^{-1}\mathbf{B}\mathbf{P}^{-1}\end{aligned} \tag{8}$$

where

$$\boldsymbol{\xi} = \mathbf{N}_{11} - \mathbf{N}_{12}\mathbf{N}_{22}^{-1}\mathbf{N}_{21},\ \mathbf{M} = \mathbf{B}\mathbf{P}^{-1}\mathbf{B}^{t},\ \mathbf{N}_{ij} = \mathbf{A}_i^{t}\mathbf{M}^{-1}\mathbf{A}_j$$

and

$$\mathbf{A}_i = \frac{\partial F}{\partial x_i},\ \mathbf{B} = \frac{\partial F}{\partial \ell}. \qquad (i = 1,2;\ j = 1,2)$$

Here, x_1 is the vector of the orientation parameters, x_2 is the vector of the object coordinates, $\mathbf{P}$ is the weight matrix of the observations, and F is the mathematical model.

Results from the OEEPE-Test on Gross-Error Detection

The above strategy has been applied to photogrammetric blocks provided by WG III/1 (Gross and Systematic Errors in Photogrammetric Point Determination) to its participants. The blocks contained errors which were known only to the distributor.

Block Description

Two blocks, BI and BII, are employed in this test. Both blocks consist of four strips with 13 photographs per strip. Block BII contains more points than block BI; however, the extra points are located in groups distributed in the same locations as the single points in block BI. This gives about 6 to 15 points per image for BI and 12 to 30 points per image for BII. The photo scales are 1:14 000 and 1:5 000, respectively.

APPLICATION OF GROSS-ERROR DETECTION STRATEGY

The following is a summary of the application of the above described strategy:

Error Type	Applied Adjustment	Errors Found
A	Conformal transformation between photos with at least three common points	1. Some points have their point numbers exchanged. These points were recognized by the size of the residuals, and an examination of the transformed coordinates with the block layout indicated the correct point numbers. 2. Some points were located in the wrong photograph. They were easily discovered by a comparison of the transformed coordinates and the layout of the block. 3. x and y coordinates had to be exchanged in a group of successive photographs.
B	Polynomial strip and block adjustment with minimum control (2 planimetry, 3 height in first strip)	1. Some control point coordinates had obvious copying mistakes. Examination of the coordinate discrepancies at check points, suggested rounded out coordinate errors (e.g., 1000 m). Only large errors can be corrected with certainty. 2. Point numbers of some control points were exchanged. These points had large discrepancies. Examination of the given and the transformed coordinates enabled the detection of the errors.
C	Bundle adjustment with minimum control (4 full points at corners + 2 height inside block)	By examining check point coordinate discrepancies, errors of smaller magnitudes than type B were detected. These points had reasonable image coordinate residuals; therefore, ground coordinates were suspected. Coordinates were corrected by rounded-out values (usually between 10 to 100 m) where this appeared appropriate and then used as control points in the next adjustment. The adjustment was repeated three times until all ground control points were corrected.
C	Bundle adjustment with all given control points	Errors in image coordinates with smaller magnitude than errors type A were detected by this adjustment by examining their residuals. Most were eliminated by exchanging their point number with close-by points within the same point group, others by giving them a different number in one of the strips. A few points could not be properly relocated and were taken out completely.

Error Type	Applied Adjustment	Errors Found
D	Bundle adjustment + data snooping	Not useful in block BI because many photographs contained too few points (less than nine) and others had weak geometry. For block BII, it was applied successfully and reasonable redundancy members were computed (see Table 1). Image coordinate errors between 20 μm and 100 μm were detected, depending on the redundancy number of the point.

Table 1. Range of Redundancy Numbers for Block BII

Type of point	No. of Rays	Range of Redundancy Numbers	
		r_x	r_y
Control	2	0.566–0.665	0.563–0.688
Control	3	0.602–0.753	0.600–0.801
Control	4	0.608–0.681	0.552–0.677
Tie	2	0.000–0.000	0.273–0.438
Tie	3	0.079–0.578	0.056–0.566
Tie	4	0.174–0.601	0.177–0.538
Tie	5	0.217–0.650	0.036–0.596
Tie	6	0.150–0.679	0.040–0.596

FINAL RESULTS USING CORRECTED DATA

In order to evaluate the accuracy rigorously, the variance-covariance matrices of the adjusted ground coordinates and the equivalent error ellipsoids at the 95 percent confidence level are computed for all points (El-Hakim, 1981a). The precision of measurements is estimated by the standard error of unit weight (σ_0). Table 2 displays the accuracy obtained with the cleaned data.

The accuracy values obtained for block BII compare favorably with values for fully-controlled test-field photography (Ziemann and El-Hakim, 1982). The values for block BI are, when all accuracy values are considered given in the scale of the photographs, about twice as large throughout, probably because some smaller cross errors were not detected. The main reason for this is that block BI contains many fewer points than block BII: in many of the photographs less than nine points were available, which results in a low reliability (El-Hakim, 1981b).

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Table 2. Accuracy Evaluation
(Values in parenthesis are in μm at photo scale)

Block	σ_o (μm)	Mean of all standard errors—(cm)			Mean of all semi-major axes of E.E.		
		σ_x	σ_y	σ_z	X	Y	Z
BI	7.4	8.3 (5.9)	10.1 (7.2)	21.1 (15.1)	19.0 (13.6)	24.2 (17.3)	60.9 (43.5)
BII	5.1	1.6 (3.2)	1.9 (3.8)	3.8 (7.6)	3.6 (7.2)	4.5 (9.0)	10.8 (21.6)

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