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Orientation Problem of Two-Media Photographs with Curved Boundary Surfaces

The analytical orientation problem of two-media photographs is discussed for the case where the air/water interface is composed of simple harmonic waves.

ABSTRACT: *In the usual analysis of photographs taken of coastal underwater areas, the sea surface may be approximated by a horizontal plane at the mean height of the wave crests and troughs. However, position errors due to this approximation are not negligibly small, in particular, in deep underwater areas or for photography from low altitude (Okamoto, 1982b). Thus, this paper discusses the analytical orientation problem of two-media photographs having wave surface so as to correct the approximation errors. First, this problem is considered theoretically by representing the sea surface with simple mathematical functions. The orientation methods proposed, however, can readily be extended to the actual sea surface expressed with complicated mathematical functions such as a Fourier series. Next, the orientation techniques presented are tested with simulated two-media photographs in order to clarify the difficulties when applying them to practical case.*

INTRODUCTION

TWO-MEDIA PHOTOGRAMMETRIC MAPPING is difficult to carry out using conventional plotting instruments, first because corresponding imaging rays in air generally do not intersect, and second because we cannot observe optically-mechanically the true position of underwater points. Thus, analog methods provide approximate solutions only, except when using modified double-projector instruments (Kreiling, 1970). On the other hand, two-media photographs can be analyzed rigorously by means of analytical plotters. Also, to this end, we need exterior orientation parameters of the two-media photographs and elements describing the boundary surface, both of which can be determined from orientation theories in two-media photogrammetry.

The orientation problem of two-media photographs has been investigated mainly for placid water conditions. Also, many orientation techniques have been developed by Rinner (1948, 1969), Schmutter and Bonfiglioli (1967), Hoehle (1971, 1972), Okamoto and Hoehle (1972), Okamoto and Mori (1973), and Girndt (1973). By classifying the orientation parameters of a two-media photograph into six exterior ones and elements defining the water surface, the orientation theories already derived can readily be extended to the case of a curved boundary surface such as a wave surface. In the following chapters, analytical orientation methods of two-media photographs are presented for the case where the refractive interface is composed of sinusoidal waves. The interior orientation parameters of the two-media photographs and the refractive index of water are assumed to be known. Further, tests with numerical examples are reported so as to access the practicability of the methods proposed, and the results obtained are discussed.

SELECTION OF ORIENTATION PARAMETERS OF A TWO-MEDIA PHOTOGRAPH

This chapter discusses how to select orientation elements of a two-media photograph for the case

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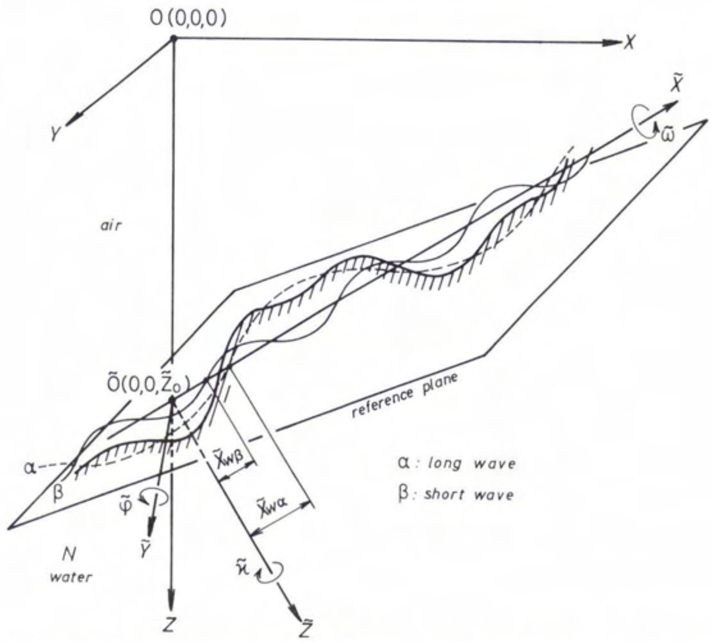

FIG. 1. Relationship between orientation and wave coordinate systems for the general case.

where the boundary surface (the refractive interface) is a wave surface. The orientation coordinate system (X, Y, Z) is taken as a right-handed, rectangular-Cartesian system with its origin at an arbitrary point over the water surface. Also, a plane with the mean height of the wave crests and troughs is assumed not to be parallel to the X-Y plane of the orientation coordinate system (see Figure 1). Furthermore, the wave coordinate system $(\tilde{X}, \tilde{Y}, \tilde{Z})$ is selected with its $\tilde{X}$-$\tilde{Y}$ plane on the reference plane of the wave and with its origin at the point where the Z-axis of the orientation coordinate system intersects the plane with the mean height of the wave crests and troughs. Then, the relationship between the orientation and wave coordinate systems can be given in the form

$$\begin{bmatrix} \tilde{X} \\ \tilde{Y} \\ \tilde{Z} \end{bmatrix} = \mathbf{D}_{\tilde{\kappa}}^{\mathrm{T}}\mathbf{D}_{\tilde{\omega}}^{\mathrm{T}}\mathbf{D}_{\tilde{\phi}}^{\mathrm{T}} \begin{bmatrix} X \\ Y \\ Z - \tilde{Z}_0 \end{bmatrix} \qquad (1)$$

in which $\mathbf{D}_{\tilde{\phi}}$, $\mathbf{D}_{\tilde{\omega}}$, and $\mathbf{D}_{\tilde{\kappa}}$ are rotation matrices of rotation parameters $\tilde{\phi}$, $\tilde{\omega}$, and $\tilde{\kappa}$ about the wave coordinate axes $\tilde{Y}$, $\tilde{X}$, and $\tilde{Z}$, respectively, and $(0, 0, \tilde{Z}_0)$ denote the space coordinate of the origin $\tilde{O}$ in the orientation coordinate system (X, Y, Z). Equation 1 is rewritten as

$$\begin{aligned} \tilde{X} &= \tilde{e}_{11}X + \tilde{e}_{12}Y + \tilde{e}_{13}(Z - \tilde{Z}_0) \\ \tilde{Y} &= \tilde{e}_{21}X + \tilde{e}_{22}Y + \tilde{e}_{23}(Z - \tilde{Z}_0) \\ \tilde{Z} &= \tilde{e}_{31}X + \tilde{e}_{32}Y + \tilde{e}_{33}(Z - \tilde{Z}_0) \end{aligned} \qquad (2)$$

in which

$$\mathbf{D}_{\tilde{\kappa}}^{\mathrm{T}}\mathbf{D}_{\tilde{\omega}}^{\mathrm{T}}\mathbf{D}_{\tilde{\phi}}^{\mathrm{T}} = \begin{bmatrix} \tilde{e}_{11} & \tilde{e}_{12} & \tilde{e}_{13} \\ \tilde{e}_{21} & \tilde{e}_{22} & \tilde{e}_{23} \\ \tilde{e}_{31} & \tilde{e}_{32} & \tilde{e}_{33} \end{bmatrix}$$

A sinusoidal wave can be expressed in the wave coordinate system in the form

$$\tilde{Z} = A \sin[-k(\tilde{X} - \tilde{X}_w)] \qquad (3)$$

where

$$A = H_w/2 \quad (H_w\text{: wave height})$$
$$k = 2\pi/\lambda_w \quad (\lambda_w\text{ : wave length})$$

and $\tilde{X}_w$ indicates the $\tilde{X}$-coordinate of the point where the vertical displacement of the wave is equal to zero. By substituting Equation 2 into Equation 3, we obtain the wave equation constructed in the orientation coordinate system, i.e.,

$$\begin{aligned} &\tilde{e}_{31}X + \tilde{e}_{32}Y + \tilde{e}_{33}(Z - \tilde{Z}_0) \\ &= A \sin[-k\{\tilde{e}_{11}X + \tilde{e}_{12}Y + \tilde{e}_{13}(Z - \tilde{Z}_0) - \tilde{X}_w\}]. \end{aligned} \qquad (4)$$

Parameters describing the sinusoidal wave in the orientation coordinate system are $\tilde{\phi}$, $\tilde{\omega}$, $\tilde{\kappa}$, $\tilde{Z}_0$, A, k, and $\tilde{X}_w$. However, the element $\tilde{X}_w$ is not adequate to take as an orientation parameter to be solved, because this element cannot be determined uniquely in the orientation problem of two-media photographs. Hence, Equation 4 will be modified as follows:

$$\begin{aligned} &\tilde{e}_{31}X + \tilde{e}_{32}Y + \tilde{e}_{33}(Z - \tilde{Z}_0) \\ &= A \sin[-k\{\tilde{e}_{11}X + \tilde{e}_{12}Y + \tilde{e}_{13}(Z - \tilde{Z}_0)\} + k\tilde{X}_w] \\ &= -A \sin[k\{\tilde{e}_{11}X + \tilde{e}_{12}Y + \tilde{e}_{13}(Z - \tilde{Z}_0)\}] \cos(k\tilde{X}_w) \\ &\quad + A \cos[k\{\tilde{e}_{11}X + \tilde{e}_{12}Y + \tilde{e}_{13}(Z - \tilde{Z}_0)\}] \sin(k\tilde{X}_w) \\ &= a \sin(kq) + b \cos(kq) \end{aligned} \tag{5}$$

in which

$$\begin{aligned} a &= -A \cos(k\tilde{X}_w),\ b = A \sin(k\tilde{X}_w), \\ A &= \sqrt{a^2 + b^2}, \text{ and} \\ q &= \tilde{e}_{11}X + \tilde{e}_{12}Y + \tilde{e}_{13}(Z - \tilde{Z}_0). \end{aligned}$$

It follows from Equation 5 that $\tilde{\phi}$, $\tilde{\omega}$, $\tilde{\kappa}$, $\tilde{Z}_0$, a, b, and k can also be selected as orientation parameters of the sinusoidal wave. Accordingly, a two-media photograph with a sinusoidal wave surface has 13 orientation elements (six exterior orientation parameters and seven parameters describing the sinusoidal wave in the orientation coordinate system).

A wave superposed of two sinusoidal waves (a long wave and a short wave) can be described in the wave coordinate system $(\tilde{X}, \tilde{Y}, \tilde{Z})$ as

$$\begin{aligned} \tilde{Z} = \tilde{Z}_\alpha + \tilde{Z}_\beta &= A_\alpha \sin[-k_\alpha(\tilde{X} - \tilde{X}_{w\alpha})] \\ &\quad + A_\beta \sin[-k_\beta(\tilde{X} - \tilde{X}_{w\beta})] \end{aligned} \tag{6}$$

under the assumption that the long and short waves propagate in the same direction. By substituting Equation 2 into Equation 6, we get

$$\begin{aligned} &\tilde{e}_{31}X + \tilde{e}_{32}Y + \tilde{e}_{33}(Z - \tilde{Z}_0) \\ &= A_\alpha \sin[-k_\alpha\{\tilde{e}_{11}X + \tilde{e}_{12}Y + \tilde{e}_{13}(Z - \tilde{Z}_0) - \tilde{X}_{w\alpha}\}] \\ &\quad + A_\beta \sin[-k_\beta\{\tilde{e}_{11}X + \tilde{e}_{12}Y + \tilde{e}_{13}(Z - \tilde{Z}_0) - \tilde{X}_{w\beta}\}] \\ &= a_\alpha \sin(k_\alpha q) + b_\alpha \cos(k_\alpha q) + a_\beta \sin(k_\beta q) \\ &\quad + b_\beta \cos(k_\beta q) \end{aligned} \tag{7}$$

in which

$$\begin{aligned} a_\alpha &= -A_\alpha \cos(k_\alpha \tilde{X}_{w\alpha}),\ b_\alpha = A_\alpha \sin(k_\alpha \tilde{X}_{w\alpha}), \\ A_\alpha &= \sqrt{a_\alpha^2 + b_\alpha^2}, \\ a_\beta &= -A_\beta \cos(k_\beta \tilde{X}_{w\beta}),\ b_\beta = A_\beta \sin(k_\beta \tilde{X}_{w\beta}), \\ A_\beta &= \sqrt{a_\beta^2 + b_\beta^2}, \text{ and} \\ q &= \tilde{e}_{11}X + \tilde{e}_{12}Y + \tilde{e}_{13}(Z - \tilde{Z}_0). \end{aligned}$$

Consequently, it can be understood that the boundary surface can be determined with ten parameters ($\tilde{\phi}$, $\tilde{\omega}$, $\tilde{\kappa}$, $\tilde{Z}_0$, a_α, b_α, k_α, a_β, b_β, k_β).

In the usual case, where the reference plane of a wave is horizontal and given, the orientation coordinate system can be replaced by the ground coordinate system. Then, we can assume that

$$\begin{aligned} \tilde{\phi} &= \tilde{\omega} = 0 \text{ and} \\ \tilde{Z}_0 &= H \ (H\text{: given}). \end{aligned} \tag{8}$$

By using the conditions above, the equation of the boundary surface is simplified to

$$\begin{aligned} Z - H &= A \sin[-k(X \cos \tilde{\kappa} + Y \sin \tilde{\kappa} - \tilde{X}_w)] \\ &= a \sin(kq') + b \cos(kq') \end{aligned} \tag{9}$$

for the simple harmonic wave, and

$$\begin{aligned} Z - H &= A_\alpha \sin[-k_\alpha(X \cos \tilde{\kappa} + Y \sin \tilde{\kappa} - \tilde{X}_{w\alpha})] \\ &\quad + A_\beta \sin[-k_\beta(X \cos \tilde{\kappa} + Y \sin \tilde{\kappa} - \tilde{X}_{w\beta})] \\ &= a_\alpha \sin(k_\alpha q') + b_\alpha \cos(k_\alpha q') + a_\beta \sin(k_\beta q') \\ &\quad + b_\beta \cos(k_\beta q') \end{aligned} \tag{10}$$

for the superposed wave, respectively. The symbol q' in Equations 9 and 10 denotes

$$q' = X \cos \tilde{\kappa} + Y \sin \tilde{\kappa}.$$

Also, parameters describing the boundary surface are reduced to a, b, k, and $\tilde{\kappa}$ for the former, and to a_α, b_α, k_α, a_β, b_β, k_β, and $\tilde{\kappa}$ for the latter, respectively.

Analytical Orientation Problem of Two-Media Photographs

We have three different solutions approaches to the analytical orientation problem of photographs:

- Methods to calculate orientation parameters of individual photographs by means of the space resection theory;
- Techniques to divide the orientation procedure of a stereopair of photographs into the two main phases; relative orientation and absolute orientation; and
- Methods to determine all orientation parameters of photographs overlapped simultaneously.

The orientation problem of two-media photographs can also be solved by applying any of the three orientation approaches above. This chapter treats the analytical orientation problem of two-media photographs for the case where the boundary surface is composed of sinusoidal waves.

Orientation Problem of Individual Two-Media Photographs

We will assume that the ground coordinate system can be adopted as the orientation coordinate system, because a plane with the mean height of the wave crests and troughs is usually considered to be horizontal and known. Thus, the boundary surface is expressed by Equation 9 for a simple harmonic wave, and by Equation 10 for a superposed wave, respectively. By employing a superposed wave surface as an example of the air/water interface, the orientation procedure will be, briefly outlined, as follows (see Figure 2). (The detailed discussion on the mathematical treatment of two-media photographs with a curved boundary surface has been made by Okamoto (1982b).)

An imaging ray **g** in air is expressed in the orientation coordinate system (X, Y, Z) as

$$\mathbf{g}\text{:}\ \frac{X - X_0}{l} = \frac{Y - Y_0}{m} = \frac{Z - Z_0}{n} = \rho \tag{11}$$

in which l, m, n indicate direction cosines of **g**. Substituting Equation 11 into Equation 10, we have an equation with respect to the auxiliary parameter ρ in Equation 11, i.e.,

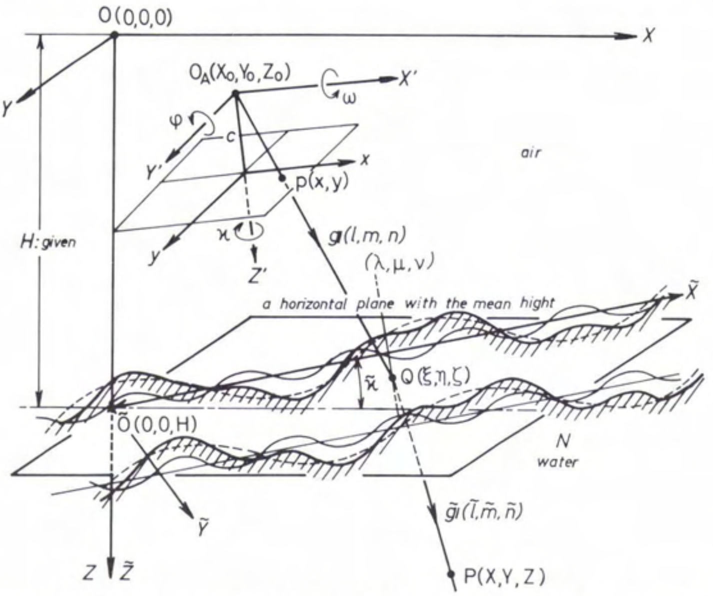

FIG. 2. Orientation problem of a two-media photograph having a superposed wave surface for the usual case.

$$H = \rho n + Z_0 - a_\alpha \sin(k_\alpha q') - b_\alpha \cos(k_\alpha q') - a_\beta \sin(k_\beta q') - b_\beta \cos(k_\beta q') \tag{12}$$

in which

$$q' = (\rho l + X_0)\cos\tilde{\kappa} + (\rho m + Y_0)\sin\tilde{\kappa}.$$

By solving Equation 12 with respect to ρ, the refraction point $Q(\xi, \eta, \zeta)$ can be calculated in the following form:

$$\xi = \rho l + X_0,\ \eta = \rho m + Y_0,\ \zeta = \rho n + Z_0. \tag{13}$$

However, it is mathematically impossible to express the solution ρ in terms of orientation parameters (ϕ, ω, κ, X_0, Y_0, Z_0, a_α, b_α, k_α, a_β, b_β, k_β, $\tilde{\kappa}$) of the two-media photograph. Hence, ρ is determined with the orientation elements together.

Next, the equation of the imaging ray $\tilde{\mathbf{g}}$ in water will be constructed in the orientation coordinate system. For this purpose, we must find the direction cosines (λ, μ, ν) of the normal to the air/water interface. They are given as follows (see Okamoto (1982b)).

$$(\lambda, \mu, \nu) = ((h\cos\tilde{\kappa})/A,\ (h\sin\tilde{\kappa})/A,\ 1/A) \tag{14}$$

in which

$$\begin{aligned} h &= -k_\alpha[a_\alpha\cos(k_\alpha q') - b_\alpha\sin(k_\alpha q')] \\ &\quad - k_\beta[a_\beta\cos(k_\beta q') - b_\beta\sin(k_\beta q')] \\ A &= \sqrt{h^2 + 1}. \end{aligned}$$

By means of λ, μ, ν and the direction cosines (l, m, n) of the imaging ray $\mathbf{g}$ in air, we can calculate the direction cosines ($\tilde{l}$, $\tilde{m}$, $\tilde{n}$) of the imaging ray $\tilde{\mathbf{g}}$ in water in the form

$$\begin{aligned} N\tilde{l} &= l - \lambda[\cos i - \sqrt{N^2 - 1 + \cos^2 i}] \\ N\tilde{m} &= m - \mu[\cos i - \sqrt{N^2 - 1 + \cos^2 i}] \\ N\tilde{n} &= n - \nu\,[\cos i - \sqrt{N^2 - 1 + \cos^2 i}] \end{aligned} \tag{15}$$

where N denotes the refractive index of water, and

$$\cos i = l\lambda + m\mu + n\nu.$$

Then, the imaging ray $\tilde{\mathbf{g}}$ in water can be expressed in the orientation coordinate system as

$$\tilde{\mathbf{g}}\text{:}\ \frac{X - \xi}{\tilde{l}} = \frac{Y - \eta}{\tilde{m}} = \frac{Z - \zeta}{\tilde{n}}. \tag{16}$$

The determination equations in the analytical orientation problem of individual two-media photographs are obtained from the condition that an underwater control point P(X, Y, Z) must lie on the imaging ray $\tilde{\mathbf{g}}$ in water. Hence, they are given from Equations 13 and 16 as

$$\begin{aligned} X &= \rho l + X_0 + \frac{\tilde{l}}{\tilde{n}}(Z - \rho n - Z_0) \\ Y &= \rho m + Y_0 + \frac{\tilde{m}}{\tilde{n}}(Z - \rho n - Z_0). \end{aligned} \tag{17}$$

However, in the actual orientation calculation, auxiliary unknowns ρ_i($i = 1, \ldots, t$ (t: the number of underwater control points)) must be determined with orientation parameters (ϕ, ω, κ, X_0, Y_0, Z_0, a_α, b_α, k_α, a_β, b_β, k_β, $\tilde{\kappa}$) of the two-media photograph together. Thus, the following equations may be required:

$$X_i = \rho_i l_i + X_0 + \frac{\tilde{l}_i}{\tilde{n}_i}(Z_i - \rho_i n_i - Z_0)$$

$$
\begin{aligned}
Y_i &= \rho_i m_i + Y_0 + \frac{\tilde{m}_i}{\tilde{n}_i}(Z_i - \rho_i n_i - Z_0) \\
H &= \rho_i n_i + Z_0 - a_\alpha \sin(k_\alpha q_i') - b_\alpha \cos(k_\alpha q_i') \\
&\quad - a_\beta \sin(k_\beta q_i') - b_\beta \cos(k_\beta q_i'). \qquad (18)
\end{aligned}
$$

The 13 orientation parameters and auxiliary unknowns ρ_i ($i = 1, \ldots, t$) are provided by solving Equation 18 with a least-squares adjustment. The number of underwater control points mathematically required is seven, because the parameters describing the boundary surface must be determined from given coordinates in this orientation method.

ORIENTATION PROBLEM OF A STEREOPAIR OF TWO-MEDIA PHOTOGRAPHS

The orientation problem of a stereopair of two-media photographs having a wave surface differs comparatively from that in the preceding paragraph. The reasons are almost the same as in the case of placid water conditions and can be explained as follows. The relationship between the model and ground coordinate systems is always unknown, and thus three parameters ($\tilde{\phi}$, $\tilde{\omega}$, $\tilde{Z}_0$) describing the reference plane of the wave must also be determined in the relative orientation, even when it is horizontal and given. Furthermore, the known reference plane plays the roll of ground control points. Hence, the number of underwater control points mathematically required can be reduced. In this paragraph, the orientation procedure will be described in detail for the case of simultaneous photography.

Relative orientation. First, the relative orientation of a stereopair of two-media photographs will be discussed. The model coordinate system (X_M, Y_M, Z_M) is taken as a right-handed, rectangular-Carte-

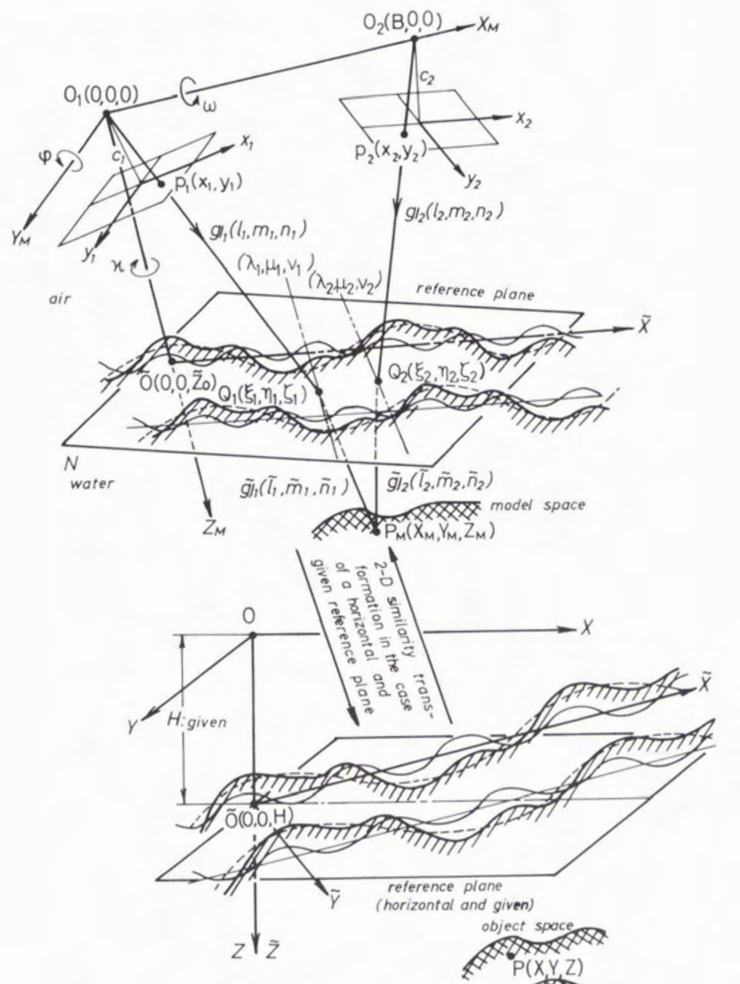

FIG. 3. Orientation problem of a stereopair of two-media photographs having a superposed wave surface for the general and usual cases.

sian system with its origin at the projection center of the left picture and with its X_M-axis through that of the right photograph (see Figure 3). Also, five rotation parameters (ϕ_1, κ_1, ϕ_2, ω_2, κ_2) are employed as relative orientation elements of the stereopair. Corresponding imaging rays $\mathbf{g}_1(l_1, m_1, n_1)$ and $\mathbf{g}_2(l_2, m_2, n_2)$ in air are expressed in the model coordinate system (X_M, Y_M, Z_M) as

$$\mathbf{g}_1: \frac{X_M}{l_1} = \frac{Y_M}{m_1} = \frac{Z_M}{n_1} = \rho_1 \qquad (19)$$

$$\mathbf{g}_2: \frac{X_M - B}{l_2} = \frac{Y_M}{m_2} = \frac{Z_M}{n_2} = \rho_2 \qquad (20)$$

in which (l_1, m_1, n_1) and (l_2, m_2, n_2) denote direction cosines of the corresponding rays $\mathbf{g}_1$ and $\mathbf{g}_2$, respectively, and B is the model base.

The boundary surface (a superposed wave surface) is described in the model coordinate system in the form of Equation 7, if all parameters ($\bar{\phi}$, $\bar{\omega}$, $\bar{\kappa}$, $\bar{Z}_0$, a_α, b_α, k_α, a_β, b_β, k_β) of the air/water interface are defined newly with respect to the model coordinate system (X_M, Y_M, Z_M). The corresponding refraction points $Q_1(\xi_1, \eta_1, \zeta_1)$ and $Q_2(\xi_2, \eta_2, \zeta_2)$ can be calculated by means of Equations 7, 19, and 20 in the form

$$\xi_1 = \rho_1 l_1, \qquad \eta_1 = \rho_1 m_1, \quad \zeta_1 = \rho_1 n_1 \quad (21)$$

$$\xi_2 = \rho_2 l_2 + B, \quad \eta_2 = \rho_2 m_2, \quad \zeta_2 = \rho_2 n_2. \quad (22)$$

However, as was explained in the previous paragraph, auxiliary parameters ρ_1 and ρ_2 must be calculated with orientation elements required for the construction of a model similar to the object. For this purpose, we need the following equations:

$$\begin{aligned} &{}_1\bar{e}_{31}\rho_1 l_1 + {}_1\bar{e}_{32}\rho_1 m_1 + {}_1\bar{e}_{33}(\rho_1 n_1 - \bar{Z}_0) \\ &= a_\alpha \sin(k_\alpha q_1) + b_\alpha \cos(k_\alpha q_1) + a_\beta \sin(k_\beta q_1) \\ &\quad + b_\beta \cos(k_\beta q_1),\ q_1 = {}_1\bar{e}_{11}\rho_1 l_1 + {}_1\bar{e}_{12}\rho_1 m_1 \\ &\quad + {}_1\bar{e}_{13}(\rho_1 n_1 - \bar{Z}_0) \end{aligned} \qquad (23)$$

for the left photograph, and

$$\begin{aligned} &{}_2\bar{e}_{31}(\rho_2 l_2 + B) + {}_2\bar{e}_{32}\rho_2 m_2 + {}_2\bar{e}_{33}(\rho_2 n_2 - \bar{Z}_0) \\ &= a_\alpha \sin(k_\alpha q_2) + b_\alpha \cos(k_\alpha q_2) + a_\beta \sin(k_\beta q_2) \\ &\quad + b_\beta \cos(k_\beta q_2),\ q_2 = {}_2\bar{e}_{11}(\rho_2 l_2 + B) \\ &\quad + {}_2\bar{e}_{12}\rho_2 m_2 + {}_2\bar{e}_{13}(\rho_2 n_2 - \bar{Z}_0) \end{aligned} \qquad (24)$$

for the right photograph, respectively.

The direction cosines (λ, μ, ν) of the normal to the boundary surface are given in the form

$$(\lambda, \mu, \nu) = \left(\frac{e_{31} + e_{11}h}{\sqrt{h^2 + 1}}, \frac{e_{32} + e_{12}h}{\sqrt{h^2 + 1}}, \frac{e_{33} + e_{13}h}{\sqrt{h^2 + 1}} \right)$$

$$\begin{aligned} h = &-k_\alpha[a_\alpha \cos(k_\alpha q) - b_\alpha \sin(k_\alpha q)] \\ &- k_\beta[a_\beta \cos(k_\beta q) - b_\beta \sin(k_\beta q)]. \end{aligned} \qquad (25)$$

In the above expression, the subscripts 1 and 2 are omitted, because the formula is the same for the left and right pictures. Also, direction cosines $(\bar{l}_1, \bar{m}_1, \bar{n}_1)$ and $(\bar{l}_2, \bar{m}_2, \bar{n}_2)$ of the corresponding imaging rays $\bar{\mathbf{g}}_1$ and $\bar{\mathbf{g}}_2$ in water can be calculated by means of Equation 15.

The corresponding refraction points $Q_1(\xi_1, \eta_1, \zeta_1)$, $Q_2(\xi_2, \eta_2, \zeta_2)$ and direction cosines $(\bar{l}_1, \bar{m}_1, \bar{n}_1)$, $(\bar{l}_2, \bar{m}_2, \bar{n}_2)$ of the corresponding imaging rays $\bar{\mathbf{g}}_1$, $\bar{\mathbf{g}}_2$ in water having been provided, the equations of $\bar{\mathbf{g}}_1$ and $\bar{\mathbf{g}}_2$ are constructed in the model coordinate system (X_M, Y_M, Z_M) in the form

$$\bar{\mathbf{g}}_1: \frac{X_M - \xi_1}{\bar{l}_1} = \frac{Y_M - \eta_1}{\bar{m}_1} = \frac{Z_M - \zeta_1}{\bar{n}_1} \qquad (26)$$

$$\bar{\mathbf{g}}_2: \frac{X_M - \xi_2}{\bar{l}_2} = \frac{Y_M - \eta_2}{\bar{m}_2} = \frac{Z_M - \zeta_2}{\bar{n}_2}. \qquad (27)$$

By means of Equations 26 and 27, the coplanarity condition of corresponding imaging rays in water can be derived as

$$\begin{vmatrix} \xi_2 - \xi_1 & \bar{l}_1 & \bar{l}_2 \\ \eta_2 - \eta_1 & \bar{m}_1 & \bar{m}_2 \\ \zeta_2 - \zeta_1 & \bar{n}_1 & \bar{n}_2 \end{vmatrix} = 0. \qquad (28)$$

The coplanarity condition (Equation 28) includes 15 orientation parameters (ϕ_1, κ_1, ϕ_2, ω_2, κ_2, $\bar{\phi}$, $\bar{\omega}$, $\bar{\kappa}$, $\bar{Z}_0$, a_α, b_α, k_α, a_β, b_β, k_β). Also, all of these parameters must be determined only from the coplanarity condition, because the relationship between a photographed underwater object point and its image point becomes non-linear by the presence of the refractive interface (see Okamoto (1982a)). In the actual relative orientation calculation, Equations 23, 24, and 28 are solved with respect to the 15 orientation unknowns above and auxiliary unknowns ρ_{1i}, $\rho_{2i}(i = 1, \ldots, u$ (u: the number of orientation points in water)) by least-squares adjustment with conditions having unknowns (Gotthardt (1968)).

Absolute orientation. In the orientation problem of a stereopair of two-media photographs having a superposed wave surface, we must determine 22 orientation parameters, if the plane with the mean height of the wave crests and troughs is perfectly unknown. They are 12 exterior orientation elements of the stereopair and ten parameters describing the boundary surface. In the relative orientation process, 15 among these 22 parameters have been provided from the coplanarity condition of corresponding imaging rays in water. Thus, the remaining seven elements are determined during the phase of absolute orientation by the three-dimensional similarity transformation between the model and object spaces. For the unique determination of these seven orientation elements, we must have the "minimum" number of control points, namely, seven coordinates such as two planimetric and three vertical points.

On the other hand, in the case where the reference plane of the wave is horizontal and given, 19 orientation elements (12 exterior orientation parameters of a stereopair of two-media photographs and seven elements defining the wave) must be provided. However, also in this case, 15 among these 19 parameters can be calculated mathematically from the coplanarity condition of corresponding imaging rays in water. Hence, the absolute orientation has only four parameters to be solved. Also, these four orientation elements can be determined by the two-dimensional similarity transformation (Rinner, 1948).

The geometrical meaning of the absolute orientation in the second case can also be understood from the following facts:

- Three absolute orientation elements (Φ, Ω, Z_0) can be calculated from three parameters ($\bar{\phi}$, $\bar{\omega}$, $\bar{Z}_0$) describing the reference plane of the wave, which have already been determined during the phase of relative orientation (see Okamoto and Hoehle (1972)).
- In the case where we have surface points imaged, Z-coordinates of three surface points can be used mathematically as those of control points.

SIMULTANEOUS DETERMINATION OF ALL ORIENTATION UNKNOWNS OF TWO-MEDIA PHOTOGRAPHS OVERLAPPED

The simultaneous determination of all orientation unknowns of photographs overlapped can be formulated in some different ways. When introducing the well-known technique in bundle adjustment, namely, the method to employ directly the collinearity equations for pictures overlapped as the determination equations, we can simplify the orientation procedure by treating space coordinates of object points (not ground control points) as unknowns. In this paragraph, this method will be discussed in two-media photogrammetry with a superposed wave surface (see Figure 4). Furthermore, simultaneous photography is assumed to be taken.

We can adopt the ground coordinate system as the orientation coordinate system under the assumption that the reference plane of the wave is horizontal and known. Thus, we have 19 orientation parameters to be determined for a stereopair of two-media photographs, the six exterior orientation elements (ϕ_1, ω_1, κ_1, X_{01}, Y_{01}, Z_{01}) of the left photograph, those (ϕ_2, ω_2, κ_2, X_{02}, Y_{02}, Z_{02}) of the right photograph, and parameters (a_α, b_α, k_α, a_β, b_β, k_β, $\bar{\kappa}$) describing the boundary surface. The equations required for determining these 19 orientation elements are derived from Equation 17, because the collinearity equations are very difficult to construct in two-media photogrammetry. Writing down Equation 17 for the left and right two-media photographs together, we obtain

$$
\begin{aligned}
X &= \rho_1 l_1 + X_{01} + \frac{\bar{l}_1}{\bar{n}_1}(Z - \rho_1 n_1 - Z_{01}) \\
Y &= \rho_1 m_1 + Y_{01} + \frac{\bar{m}_1}{\bar{n}_1}(Z - \rho_1 n_1 - Z_{01}) \\
X &= \rho_2 l_2 + X_{02} + \frac{\bar{l}_2}{\bar{n}_2}(Z - \rho_2 n_2 - Z_{02}) \\
Y &= \rho_2 m_2 + Y_{02} + \frac{\bar{m}_2}{\bar{n}_2}(Z - \rho_2 n_2 - Z_{02}). \quad (29)
\end{aligned}
$$

Equation 29 includes mathematically one equation equivalent to the coplanarity condition of corresponding imaging rays in water. Also, the copla-

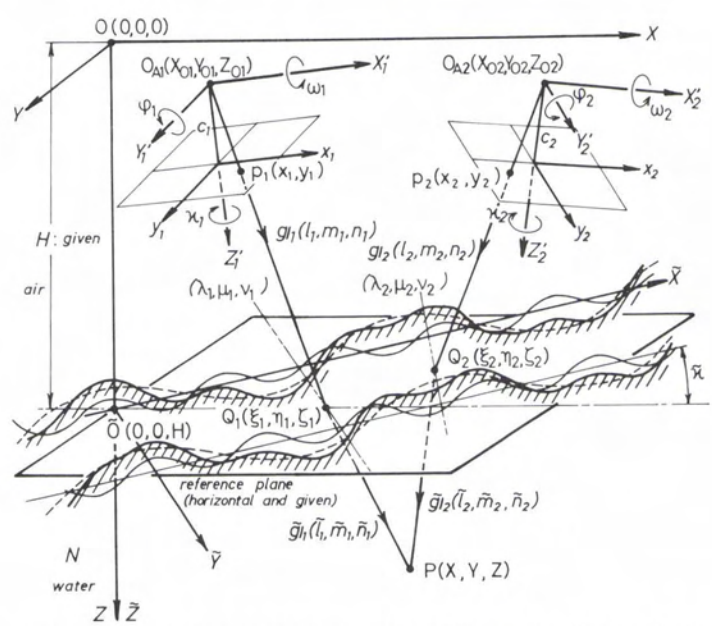

FIG. 4. Simultaneous determination of all orientation parameters of two-media photographs having a superposed wave surface for the usual case.

narity condition provides mathematically 15 independent orientation parameters. Accordingly, we must set up Equation 29 for 15 underwater points for the unique determination of the 19 orientation elements of the left and right two-media photographs. Then, we have 60 equations.

By solving these 60 equations with respect to 60 unknowns (the 19 orientation elements and 41 unknown coordinates of the 15 underwater points containing two ground control points with the planimetric coordinates given), we can perform the analytical orientation of a stereopair of two-media photographs having a superposed wave surface.

In the actual orientation calculation, the following equations may be required, because auxiliary parameters ρ_{1i} and ρ_{2i} ($i = 1, \ldots, 15$) must be determined with the 60 unknowns together:

$$\begin{aligned}
X_i &= \rho_{1i} l_{1i} + X_{01} + \frac{\bar{l}_{1i}}{\bar{n}_{1i}} (Z_i - \rho_{1i} n_{1i} - Z_{01}) \\
Y_i &= \rho_{1i} m_{1i} + Y_{01} + \frac{\bar{m}_{1i}}{\bar{n}_{1i}} (Z_i - \rho_{1i} n_{1i} - Z_{01}) \\
H &= \rho_{1i} n_{1i} + Z_{01} - a_\alpha \sin(k_\alpha q'_{1i}) - b_\alpha \cos(k_\alpha q'_{1i}) \\
&\quad - a_\beta \sin(k_\beta q'_{1i}) - b_\beta \cos(k_\beta q'_{1i}) \\
X_i &= \rho_{2i} l_{2i} + X_{02} + \frac{\bar{l}_{2i}}{\bar{n}_{2i}} (Z_i - \rho_{2i} n_{2i} - Z_{02}) \\
Y_i &= \rho_{2i} m_{2i} + Y_{02} + \frac{\bar{m}_{2i}}{\bar{n}_{2i}} (Z_i - \rho_{2i} n_{2i} - Z_{02}) \\
H &= \rho_{2i} n_{2i} + Z_{02} - a_\alpha \sin(k_\alpha q'_{2i}) - b_\alpha \cos(k_\alpha q'_{2i}) \\
&\quad - a_\beta \sin(k_\beta q'_{2i}) - b_\beta \cos(k_\beta q'_{2i}) \qquad (30)
\end{aligned}$$

in which

$$\begin{aligned}
q'_{1i} &= (\rho_{1i} l_{1i} + X_{01}) \cos \bar{\kappa} + (\rho_{1i} m_{1i} + Y_{01}) \sin \bar{\kappa} \\
q'_{2i} &= (\rho_{2i} l_{2i} + X_{02}) \cos \bar{\kappa} + (\rho_{2i} m_{2i} + Y_{02}) \sin \bar{\kappa}.
\end{aligned}$$

Further Considerations Regarding the Orientation Problem of Two-Media Photographs

The shape of a wave surface varies steadily, because the wave is propagating with the time. In the preceding chapter, we have assumed simultaneous photography to be taken from two aircraft so as to keep the shape of the boundary surface unchanged between photographs. However, theoretically, the air/water interface can be determined separately for a stereopair of two-media photographs taken with non-simultaneous photography by means of the orientation theories presented previously, if the subscript 1 is added to parameters describing the boundary surface at the exposure instant of the left photograph, and the subscript 2 to those of the right picture, respectively. In addition, no difficulties occur even when long and short waves propagate in different directions.

The coplanarity condition of corresponding imaging rays in water provides mathematically five exterior orientation elements of a stereopair of two-media photographs and parameters defining the boundary surface in the orientation coordinate system. Practically, however, all of these orientation parameters cannot always be determined accurately because of high correlations among them (see Okamoto and Hoehle (1972)). In order to overcome such difficulties, the following techniques may be adopted:

Use of More Than the Underwater Control Points Mathematically Required

As is obvious from the orientation theory of individual two-media photographs, parameters describing the boundary surface in the orientation coordinate system can also be calculated from given coordinates of underwater control points. Thus, the potential to determine accurately orientation parameters (containing the elements defining the air/water interface) of a stereopair of two-media photographs may be increased, if we have more than the underwater control points mathematically required. Also, the simultaneous determination of all orientation unknowns may be the most pertinent method to this end, because the orientation approach with the two main phases has a disadvantage that the elements describing the boundary surface must be calculated only from the coplanarity condition of corresponding imaging rays in water, even when we have many underwater control points.

Utilization of Imaged Surface Points

If imaged surface points are available, such points may be effectively utilized in the orientation calculation of a stereopair of two-media photographs taken with simultaneous photography. The reason is explained as follows. In the orientation method having the relative and absolute orientation processes, the coplanarity condition of corresponding imaging rays in air can be set up for the surface points imaged, which contains only five relative exterior orientation parameters. These five elements having been provided, we can construct a stereo model similar to the object. It follows that the shape of the water surface can be determined from the surface points, if it is represented by a functional form.

In the actual relative orientation calculation, the five exterior orientation elements and the parameters describing the boundary surface in the model coordinate system may be determined by solving the three conditions (the coplanarity condition of corresponding imaging rays in air, that of corresponding imaging rays in water, and the condition that surface points imaged exist on the water surface) simultaneously with a least-squares adjustment.

In the case where the reference plane of the wave is horizontal and known, the simultaneous determination of all orientation unknowns may be more effective, because three parameters describing the

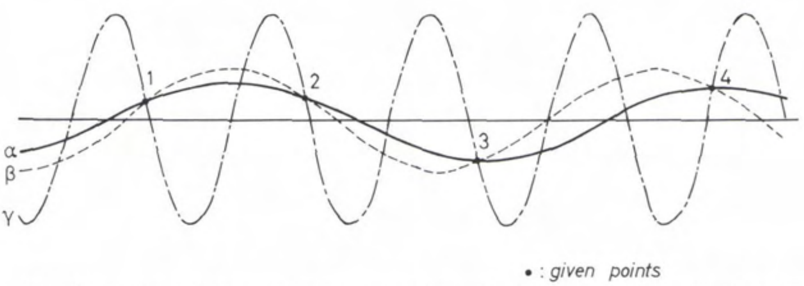

Fig. 5. Many sinusoidal waves propagating through a limited number of points given.

reference plane need not be provided. The determination equations may be (a) the coplanarity condition of corresponding imaging rays in air for surface points imaged, which includes nine exterior orientation parameters of a stereopair of two-media photographs (see Finsterwalder and Hofmann (1969)); (b) the coplanarity condition of corresponding imaging rays in water for underwater points (not control points), which has 12 exterior orientation elements of the stereopair and parameters defining the boundary surface; (c) the equation of the water surface expressed in terms of the same parameters as in (b), and (d) Equation 30 for underwater control points. This is because the simultaneous determination technique proposed previously has the large number of unknowns to be solved.

INTRODUCTION OF ASSUMED VALUES TO THE PARAMETERS DESCRIBING THE WATER SURFACE

Parameters describing a wave surface in the orientation coordinate system may be classified into elements defining the reference plane of the wave and those determining the wave itself. Also, the fundamental wave parameters are H_w (the wave height), λ_w (the wave length), and $\bar{X}_w$ (the $\bar{X}$-coordinate of the point where the vertical displacement of the wave is equal to zero). For the limited number of points given, we may not determine these wave parameters uniquely, because there may be many sinusoidal waves which propagate through the points (see Figure 5). Therefore, the wave length λ_w is assumed to be given in the orientation problem of two-media photographs having a wave surface.

Test with Simulation Models

The foregoing chapters have presented some orientation methods of two-media photographs with a curved boundary surface such as a wave surface. Among these orientation techniques, the simultaneous determination of all orientation unknowns using Equation 30 may be applied most easily to the case where we have comparatively many underwater control points with the space coordinates known. This orientation method will be tested with simulated two-media photographs having a superposed wave surface.

In order to construct the simulation models, the refraction points of underwater points are sought with the general refraction calculation (Okamoto, 1982b), and the plate coordinates are calculated by means of the collinearity equations. Theoretically, perfect plate coordinates are given in micrometres for an array of 231 underwater points (see Figure 6). Then, perturbed photo coordinates are provided in which the perturbation consists of random normal deviates having a standard deviation of 10 micrometres. The flying height is selected as $H + h = 500$ m in order that the refraction effect may appear as remarkably as possible. The number of underwater control points is ten (see Figure 6). The further conditions of the fictitious two-media pictures are the same as those in the previous paper (Okamoto, 1982b).

The orientation calculation is carried out for simultaneous photography and also for non-simultaneous photography. Orientation parameters of a stereopair of two-media photographs are taken as in Table 1. In addition, the next constraints must be considered among the parameters of superposed waves for the left and right photographs taken with non-simultaneous photography:

$$A_\alpha = \sqrt{a_{\alpha1}^2 + b_{\alpha1}^2} = \sqrt{a_{\alpha2}^2 + b_{\alpha2}^2}$$
$$A_\beta = \sqrt{a_{\beta1}^2 + b_{\beta1}^2} = \sqrt{a_{\beta2}^2 + b_{\beta2}^2}$$

under the assumption that the sinusoidal waves are propagating with their form unchanged. The constraints above can also be regarded as observation equations. However, it has been clarified from the orientation calculation with various sinusoidal waves that the solution often does not converge, even when (perfect) photo coordinates (in micrometres) of underwater control points are employed. Also, careful investigation of the problem has revealed that the convergency of the solution depends on the following two ratios:

- the ratio of the water depth to the flying height. The greater this ratio becomes, the better convergency we have, because the refraction effect increases with this ratio.
- the ratio of the wave length to the water depth.

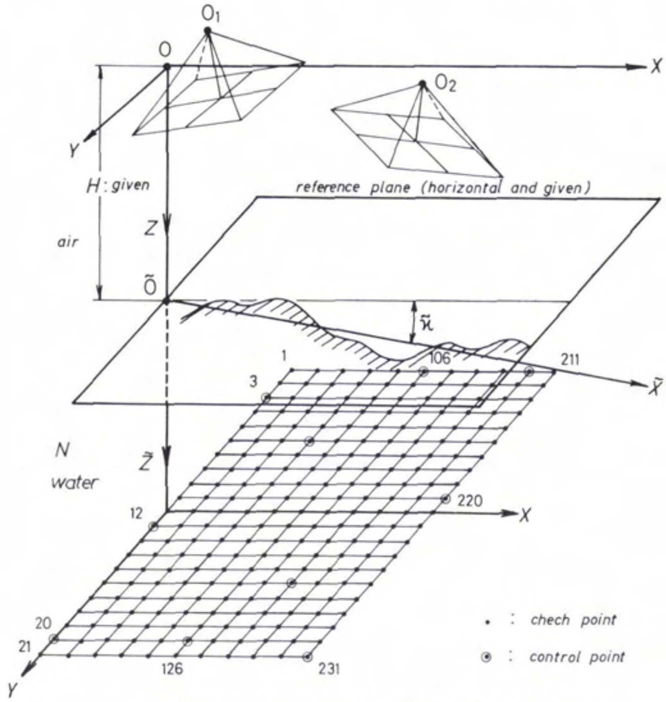

FIG. 6. Position of underwater control and check points.

When the wave length is smaller than the water depth, we may have many refraction points for an underwater point, which results in the ill convergency.

In coastal waters, the first ratio is usually very small because of water penetration. The second ratio also becomes small for short waves. Thus, the wave parameters (a_α, b_α, a_β, b_β, $\bar{\kappa}$) may not be provided accurately in the orientation calculation with (practical) two-media photographs. In order to overcome such difficulty, we will assume that the direction angle $\bar{\kappa}$ of wave propagation is known, because this element may be measured with respect to the ground coordinate system rather easily.

The obtained results regarding the standard errors of the orientation parameters, the correlation coefficients among them, and the standard errors of the 221 underwater check points are shown in Tables 2 through 7.

We can find in Tables 2 through 7 the following characteristics of the orientation problem of two-media photographs having superposed wave surface:

- The planimetric accuracy of calculated underwater points corresponds to that of the perturbed photo coordinates.
- In conventional photogrammetry, the height accuracy may be two times poorer than the planimetric accuracy. However, in two-media photogrammetry, the height errors are about four times as large as the planimetric errors, which is due to the presence of the boundary surface determined with error.
- Correlations between the exterior orientation elements and the wave parameters are not so high.
- Because one of the wave parameters (a, b) often has the very small value, its standard error becomes larger than this element itself. However, this fact does not cause the large orientation error.

TABLE 1. ORIENTATION PARAMETERS OF A STEREOPAIR OF TWO-MEDIA PHOTOGRAPHS HAVING A SUPERPOSED WAVE SURFACE FOR SIMULTANEOUS AND NON-SIMULTANEOUS PHOTOGRAPHY

Photography	Picture	Exterior Orientation Elements	Wave Parameters	
Simultaneous	Left picture	ϕ_1, ω_1, κ_1, X_{o1}, Y_{o1}, Z_{o1}	a_α, b_α, a_β, b_β	$\bar{\kappa}$
	Right picture	ϕ_2, ω_2, κ_2, X_{o2}, Y_{o2}, Z_{o2}	a_α, b_α, a_β, b_β	$\bar{\kappa}$
Non-simultaneous	Left picture	ϕ_1, ω_1, κ_1, X_{o1}, Y_{o1}, Z_{o1}	$a_{\alpha1}$, $b_{\alpha1}$, $a_{\beta1}$, $b_{\beta1}$	$\bar{\kappa}$
	Right picture	ϕ_2, ω_2, κ_2, X_{o2}, Y_{o2}, Z_{o2}	$a_{\alpha2}$, $b_{\alpha2}$, $a_{\beta2}$, $b_{\beta2}$	$\bar{\kappa}$

TABLE 2. ORIENTATION RESULT FOR A SUPERPOSED WAVE SURFACE (LONG WAVE WITH H_w = 1.33 M AND T = 15 SECONDS AND SHORT WAVE WITH H_w = 0.14 M AND T = 4 SECONDS) IN THE CASE OF SIMULTANEOUS PHOTOGRAPHY

	ϕ_1	ω_1	κ_1	X_{o1}	Y_{o1}	Z_{o1}	a_α	b_α	a_β	b_β	ϕ_2	ω_2	κ_2	X_{o2}	Y_{o2}	Z_{o2}	obtained orientation elements	σ_u
ϕ_1		0.25	-0.20	-0.99	0.24	0.70		-0.20		-0.37							0.052590 rad	0.000193 rad
ω_1			-0.41	-0.25	0.99		0.21										-0.069880 rad	0.000127 rad
κ_1				0.22	-0.45												-0.122119 rad	0.000043 rad
X_{o1}					-0.24	-0.67		0.21		0.37							29.872 m	0.107 m
Y_{o1}																	-30.045 m	0.083 m
Z_{o1}							-0.33	-0.23									-24.945 m	0.036 m
a_α								0.28				0.21					0.0120 m	0.0559 m
b_α									-0.24		-0.33	-0.20		0.33	-0.20	0.24	1.2636 m	0.0670 m
a_β										0.32						0.21	0.0017 m	0.0255 m
b_β																	0.1270 m	0.0252 m
ϕ_2														-0.99		-0.78	-0.104721 rad	0.000156 rad
ω_2													0.29		0.99		0.087228 rad	0.000119 rad
κ_2														0.21	0.33		0.104726 rad	0.000041 rad
X_{o2}																0.73	310.019 m	0.081 m
Y_{o2}																	14.971 m	0.076 m
Z_{o2}																	10.016 m	0.035 m

correlation coefficient $\rho > 0.2$

water depth h = 5m

number of underwater control points :10

number of underwater check points : 221

standard errors of underwater check points

σ_X, σ_Y, σ_Z : object scale

σ_x, σ_y, σ_z : picture scale

σ_X = 0.032 m, σ_x = 9.6 µm

σ_Y = 0.042 m, σ_y = 12.7 µm

σ_Z = 0.145 m, σ_z = 43.5 µm

σ_u : standard errors of orientation unknowns

TABLE 3. ORIENTATION RESULT FOR A SUPERPOSED WAVE SURFACE (LONG WAVE WITH H_w = 1.14 M AND T = 15 SECONDS AND SHORT WAVE WITH H_w = 0.15 M AND T = 4 SECONDS) IN THE CASE OF SIMULTANEOUS PHOTOGRAPHY

	ϕ_1	ω_1	κ_1	X_{o1}	Y_{o1}	Z_{o1}	a_α	b_α	a_β	b_β	ϕ_2	ω_2	κ_2	X_{o2}	Y_{o2}	Z_{o2}	obtained orientation elements	σ_u
ϕ_1		0.35	-0.29	-0.99	0.35	0.66				0.28							0.052496 rad	0.000176 rad
ω_1			-0.36	-0.36	0.99			0.22	-0.24								-0.069829 rad	0.000124 rad
κ_1				0.34	-0.42					-0.32							-0.122095 rad	0.000042 rad
X_{o1}					-0.36	-0.64				-0.28							29.925 m	0.096 m
Y_{o1}									-0.20								-30.019 m	0.081 m
Z_{o1}								0.40									-24.945 m	0.035 m
a_α										-0.59	0.22						0.0751 m	0.0802 m
b_α																0.37	1.2135 m	0.0686 m
a_β											0.39		-0.31	0.36		-0.37	0.0019 m	0.0180 m
b_β											0.24		-0.29	-0.25		-0.32	0.1188 m	0.0214 m
ϕ_2													-0.36	-0.99		-0.80	-0.104747 rad	0.000173 rad
ω_2													0.32		0.98		0.087221 rad	0.000104 rad
κ_2														0.40	0.36	0.32	0.104737 rad	0.000043 rad
X_{o2}																0.76	310.032 m	0.086 m
Y_{o2}																	14.967 m	0.067 m
Z_{o2}																	10.039 m	0.038 m

correlation coefficient $\rho > 0.2$

water depth h = 10m

number of underwater control points :10

number of underwater check points : 221

standard errors of underwater check points

σ_X, σ_Y, σ_Z : object scale

σ_x, σ_y, σ_z : picture scale

σ_X = 0.037 m, σ_x = 11.1 µm

σ_Y = 0.046 m, σ_y = 13.8 µm

σ_Z = 0.165 m, σ_z = 49.4 µm

σ_u : standard errors of orientation unknowns

TABLE 4. ORIENTATION RESULT FOR A SUPERPOSED WAVE SURFACE (LONG WAVE WITH $H_w = 1.00$ M AND $T = 15$ SECONDS AND SHORT WAVE WITH $H_w = 0.15$ M AND $T = 4$ SECONDS) IN THE CASE OF SIMULTANEOUS PHOTOGRAPHY

	ϕ_1	ω_1	κ_1	X_{o1}	Y_{o1}	Z_{o1}	a_α	b_α	a_β	b_β	ϕ_2	ω_2	κ_2	X_{o2}	Y_{o2}	Z_{o2}	obtained orientation elements	σ_u
ϕ_1		0.35	-0.28	-0.99	0.34	0.67	-0.23	-0.35	-0.34								0.052608 rad	0.000208 rad
ω_1			-0.42	-0.36	0.99												-0.069849 rad	0.000123 rad
κ_1				0.30	-0.46				0.30	-0.23							-0.122133 rad	0.000044 rad
X_{o1}					-0.34	-0.64	0.22	0.33	0.34								29.864 m	0.112 m
Y_{o1}																	-30.025 m	0.081 m
Z_{o1}								-0.26									-24.958 m	0.034 m
a_α											-0.30			0.28		0.37	-0.0341 m	0.0454 m
b_α																	1.0166 m	0.0393 m
a_β											0.55		-0.37	-0.53		-0.50	-0.0170 m	0.0092 m
b_β												0.20	-0.20			-0.31	0.1477 m	0.0078 m
ϕ_2													-0.32	-0.99		-0.86	-0.104808 rad	0.000182 rad
ω_2													0.20		0.99	-0.21	0.087251 rad	0.000112 rad
κ_2														0.35	0.26	0.33	0.104744 rad	0.000044 rad
X_{o2}																0.81	310.062 m	0.092 m
Y_{o2}																	14.987 m	0.071 m
Z_{o2}																	10.030 m	0.040 m

correlation coefficient $\rho > 0.2$

water depth h = 20m

number of underwater control points : 10

number of underwater check points : 221

standard errors of underwater check points

σ_X, σ_Y, σ_Z : object scale

σ_x, σ_y, σ_z : picture scale

$\sigma_X = 0.032$ m, $\sigma_x = 9.5$ µm

$\sigma_Y = 0.042$ m, $\sigma_y = 12.7$ µm

$\sigma_Z = 0.140$ m, $\sigma_z = 41.9$ µm

σ_u : standard errors of orientation unknowns

TABLE 5. ORIENTATION RESULT FOR A SUPERPOSED WAVE SURFACE (LONG WAVE WITH $H_w = 1.33$ M AND $T = 15$ SECONDS AND SHORT WAVE WITH $H_w = 0.14$ M AND $T = 4$ SECONDS) IN THE CASE OF NON-SIMULTANEOUS PHOTOGRAPHY

	ϕ_1	ω_1	κ_1	X_{o1}	Y_{o1}	Z_{o1}	$a_{\alpha1}$	$b_{\alpha1}$	$a_{\beta1}$	$b_{\beta1}$	ϕ_2	ω_2	κ_2	X_{o2}	Y_{o2}	Z_{o2}	$a_{\alpha2}$	$b_{\alpha2}$	$a_{\beta2}$	$b_{\beta2}$	obtained orientation elements	σ_u
ϕ_1			-0.21	-0.99		0.71	-0.30	-0.31		-0.48							0.22	0.26	-0.20	-0.43	0.052471 rad	0.000139 rad
ω_1			-0.40		0.99	-0.24	0.30														-0.069800 rad	0.000087 rad
κ_1				0.23	-0.44																-0.122113 rad	0.000029 rad
X_{o1}						-0.68	0.29	0.32		0.48							-0.22	-0.27	0.20	0.42	29.939 m	0.077 m
Y_{o1}						-0.21	0.25			0.20										0.20	-30.014 m	0.059 m
Z_{o1}							-0.46	-0.29		-0.23							0.22	0.26		-0.24	-24.981 m	0.026 m
$a_{\alpha1}$								0.32									-0.28	-0.33		0.21	0.1562 m	0.0535 m
$b_{\alpha1}$										0.27	0.36			-0.36			-0.66	-0.76	0.20	0.24	1.3623 m	0.0487 m
$a_{\beta1}$										0.34											-0.0238 m	0.0235 m
$b_{\beta1}$											0.23			-0.25				-0.23	0.37	0.90	0.1342 m	0.0203 m
ϕ_2												0.20		-0.99		-0.75		-0.43		0.25	-0.104748 rad	0.000110 rad
ω_2													0.25		0.99	-0.28	0.36	-0.33	-0.20		0.087127 rad	0.000088 rad
κ_2															0.31						0.104733 rad	0.000027 rad
X_{o2}																0.70		0.43		-0.26	310.033 m	0.057 m
Y_{o2}																-0.21	0.30	-0.32			14.917 m	0.054 m
Z_{o2}																	-0.22	0.28	0.20		10.035 m	0.023 m
$a_{\alpha2}$																					-0.9612 m	0.0459 m
$b_{\alpha2}$																			-0.21		-0.9779 m	0.0522 m
$a_{\beta2}$																					0.0488 m	0.0208 m
$b_{\beta2}$																					0.1272 m	0.0188 m

correlation coefficient $\rho > 0.2$

water depth h = 5m

number of underwater control points : 10

number of underwater check points : 221

standard errors of underwater check points

σ_X, σ_Y, σ_Z : object scale

σ_x, σ_y, σ_z : picture scale

$\sigma_X = 0.034$ m, $\sigma_x = 10.3$ µm

$\sigma_Y = 0.045$ m, $\sigma_y = 13.6$ µm

$\sigma_Z = 0.155$ m, $\sigma_z = 46.5$ µm

σ_u : standard errors of orientation unknowns

TABLE 6. ORIENTATION RESULT FOR A SUPERPOSED WAVE SURFACE (LONG WAVE WITH H_w = 1.14 M AND T = 15 SECONDS AND SHORT WAVE WITH H_w = 0.15 M AND T = 4 SECONDS) IN THE CASE OF NON-SIMULTANEOUS PHOTOGRAPHY

	ϕ_1	ω_1	κ_1	X_{o1}	Y_{o1}	Z_{o1}	$a_{\alpha1}$	$b_{\alpha1}$	$a_{\beta1}$	$b_{\beta1}$	ϕ_2	ω_2	κ_2	X_{o2}	Y_{o2}	Z_{o2}	$a_{\alpha2}$	$b_{\alpha2}$	$a_{\beta2}$	$b_{\beta2}$	obtained orientation elements	σ_u
ϕ_1		0.37	-0.30	-0.99	0.37	0.66				0.33									0.28		0.052519 rad	0.000124 rad
ω_1			-0.39	-0.38	0.99				-0.24												-0.069902 rad	0.000086 rad
κ_1				0.31	-0.45					-0.28									-0.25		-0.122118 rad	0.000029 rad
X_{o1}					-0.37	-0.64				-0.32									-0.27		29.920 m	0.067 m
Y_{o1}									-0.21												-30.050 m	0.055 m
Z_{o1}								0.52		0.30							-0.44	-0.21	0.21	0.26	-24.977 m	0.026 m
$a_{\alpha1}$								-0.33	0.35	-0.39									-0.30	-0.20	0.1425 m	0.0672 m
$b_{\alpha1}$										0.33							-0.82	-0.45		0.39	1.0401 m	0.0586 m
$a_{\beta1}$																					0.0106 m	0.0167 m
$b_{\beta1}$											0.44		-0.38	-0.42		-0.44	-0.32		0.86	0.52	0.1282 m	0.0139 m
ϕ_2													-0.42	-0.99		-0.82	0.28		0.35	0.27	-0.104889 rad	0.000129 rad
ω_2													0.27		0.98					0.20	0.087232 rad	0.000074 rad
κ_2														0.43	0.32	0.39			-0.24	-0.35	0.104761 rad	0.000031 rad
X_{o2}																0.79	-0.23		-0.34	-0.27	310.096 m	0.063 m
Y_{o2}																					14.970 m	0.047 m
Z_{o2}																		0.35	-0.29	-0.38	10.072 m	0.028 m
$a_{\alpha2}$																				-0.46	-0.7899 m	0.0659 m
$b_{\alpha2}$																					-0.6915 m	0.0476 m
$a_{\beta2}$																					0.1170 m	0.0131 m
$b_{\beta2}$																					0.0535 m	0.0168 m

correlation coefficient $\rho > 0.2$

water depth h = 10m

number of underwater control points : 10

number of underwater check points : 221

standard errors of underwater check points

σ_X, σ_Y, σ_Z : object scale

σ_x, σ_y, σ_z : picture scale

σ_X = 0.037 m, σ_x = 11.2 μm

σ_Y = 0.046 m, σ_y = 13.7 μm

σ_Z = 0.141 m, σ_z = 42.2 μm

σ_u : standard errors of orientation unknowns

TABLE 7. ORIENTATION RESULT FOR A SUPERPOSED WAVE SURFACE (LONG WAVE WITH H_w = 1.00 M AND T = 15 SECONDS AND SHORT WAVE WITH H_w = 0.15 M AND T = 4 SECONDS) IN THE CASE OF NON-SIMULTANEOUS PHOTOGRAPHY

	ϕ_1	ω_1	κ_1	X_{o1}	Y_{o1}	Z_{o1}	$a_{\alpha1}$	$b_{\alpha1}$	$a_{\beta1}$	$b_{\beta1}$	ϕ_2	ω_2	κ_2	X_{o2}	Y_{o2}	Z_{o2}	$a_{\alpha2}$	$b_{\alpha2}$	$a_{\beta2}$	$b_{\beta2}$	obtained orientation elements	σ_u
ϕ_1		0.34	-0.44	-0.99	0.34	0.60	-0.32	-0.33	-0.54	0.28								0.26		0.23	0.052411 rad	0.000185 rad
ω_1			-0.41	-0.35	0.99																-0.069864 rad	0.000090 rad
κ_1				0.46	-0.46				0.55	-0.33						0.21			-0.24	-0.21	-0.122105 rad	0.000037 rad
X_{o1}					-0.34	-0.58	0.31	0.32	0.55	-0.27								-0.25		-0.23	29.968 m	0.100 m
Y_{o1}																					-30.036 m	0.059 m
Z_{o1}							-0.21	-0.32										0.23			-24.965 m	0.025 m
$a_{\alpha1}$								0.28													-0.0054 m	0.0484 m
$b_{\alpha1}$											0.33			-0.30		-0.29	-0.64	-0.66			1.0286 m	0.0327 m
$a_{\beta1}$																					-0.0030 m	0.0119 m
$b_{\beta1}$											0.33	0.26	-0.38	-0.28	0.23	-0.49		0.20	0.65	0.64	0.1433 m	0.0062 m
ϕ_2													-0.27	-0.99		-0.82	-0.31		0.51		-0.104770 rad	0.000130 rad
ω_2													0.22		0.99	-0.23	-0.28	0.25			0.087251 rad	0.000084 rad
κ_2														0.29	0.27	0.30			-0.30		0.104741 rad	0.000031 rad
X_{o2}																0.76	0.29		-0.50		310.045 m	0.066 m
Y_{o2}																	-0.26	0.23			14.986 m	0.053 m
Z_{o2}																	0.37		-0.39	-0.25	10.019 m	0.028 m
$a_{\alpha2}$																					-0.6036 m	0.0422 m
$b_{\alpha2}$																				0.21	-0.8329 m	0.0314 m
$a_{\beta2}$																					0.1139 m	0.0061 m
$b_{\beta2}$																					0.0871 m	0.0079 m

correlation coefficient $\rho > 0.2$

water depth h = 20m

number of underwater control points : 10

number of underwater check points : 221

standard errors of underwater check points

σ_X, σ_Y, σ_Z : object scale

σ_x, σ_y, σ_z : picture scale

σ_X = 0.034 m, σ_x = 10.1 μm

σ_Y = 0.045 m, σ_y = 13.5 μm

σ_Z = 0.151 m, σ_z = 45.4 μm

σ_u : standard errors of orientation unknowns

- There are no clear differences between the orientation results for simultaneous photography and non-simultaneous photography.
- The orientation accuracy is almost constant regardless of the water depth in coastal waters.

Concluding Discussions

The analytical orientation problem of two-media photographs has been studied for the case where the air/water interface is not a plane but a wave surface. First, this problem has been considered theoretically, and the following orientation methods have been presented:

- orientation method of individual two-media pictures;
- orientation technique to divide the orientation procedure into the two main phases: relative orientation and absolute orientation; and
- simultaneous determination technique of all orientation unknowns of two-media photographs overlapped.

Ground control points and orientation points are selected mainly in water. However, all orientation methods above can also be applied easily to the case where we have orientation points both in air and in water.

Next, the characteristics of the orientation problem have been investigated with simulated two-media photographs. Also, the wave length and the direction angle of wave propagation have been clarified to be difficult to provide in coastal waters. Hence, these two wave parameters have been treated as known so as to obtain the satisfactory accuracy in the orientation calculation with the simulated two-media photographs.

The actual sea surface may deviate greatly from a superposed wave form (Masry and MacRitchie, 1980). Thus, the desired results may not be obtained by applying the proposed orientation techniques themselves to practical two-media photographs. In such cases, however, we may use many sinusoidal waves in order to model the complicated sea surface. Also, when surface points imaged are available, the complicated boundary surface could be traced accurately by employing the orientation method described earlier.

If the actual sea surface has a very complicated form, the least-squares interpolation (Kraus and Mikhail, 1972) may be effectively applied to two-media photogrammetric mapping in coastal waters. Also, the well-known technique in block adjustment, namely, a method to introduce additional parameters for correcting systematic position errors, may also be adopted in two-media photogrammetry with complicated boundary surface. The orientation theories discussed in this paper may provide a mathematical background to such problems.

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