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Photogrammetric Densification of Control in Ada County, Idaho: Data Processing and Results

High accuracies were obtained using a recursive partitioning technique which provided for determining the covariance between ground points.

Abstract: *The need to integrate all land surveys in the United States into a unified coordinate system cannot be denied, but densification of existing geodetic control networks is an expensive and time consuming prerequisite. To meet this need, the National Ocean Service (NOS) has developed a photogrammetric control densification system. This system was used by NOS to determine the horizontal positions of 346 section corners in Ada County, Idaho with standard errors of approximately 50 mm. The distance, azimuth, and elevation difference of each pair of intervisible densification points were also computed using an adjustment program designed for use with this system. A description of the program is presented, in addition to the results obtained from the Ada County project.*

Introduction

In many urban areas there is no single coordinate system to which all land surveys are referenced, but a hodgepodge of unrelated local datums. The continual cost and confusion created by this situation are among the factors that motivated a report by the Committee on Geodesy of the National Research Council (1980) calling for the establishment of a national multipurpose cadastre. The National Geodetic Horizontal Control Network is a logical framework on which to build the national cadastre, but in most urban areas the geodetic control tied to the national network consists of a few points separated by distances of 3 to 6 miles. Consequently, only a small percentage of local surveys are tied to the national network, because land surveyors cannot be expected to tranverse the several miles that may be required to make a proper connection.

The NRC report states that densification of existing geodetic networks must be the first priority, and it suggests high precision photogrammetric surveys as one method of accomplishing this densification in a timely and economic manner. Not long ago, such a suggestion would have been considered an impractical dream by most geodesists, but photogrammetric geodesy has gained many supporters during the last decade.

The capability of photogrammetric control densification was demonstrated by Brown (1977) in a pilot project in the City of Atlanta. The horizontal coordinates of densification points, with a nominal spacing of 800 metres, were determined with a standard error of less than 70 mm from photography taken at a scale of 1:17,500. At about the same time a precise photogrammetric survey system developed by NOS was being tested at the Casa Grande, Arizona, test range. The results of this test, reported by Slama (1978), provided horizontal coordinates with standard errors of approximately 50 mm from a photo scale of 1:24,000.

In 1978, in cooperation with Ada County, Idaho, NOS undertook the densification of approximately 350 square miles in northern Ada County. The project area, shown in Figure 1, included the City of Boise and surrounding countryside. In contrast to the near ideal environment of a test range, this proj-

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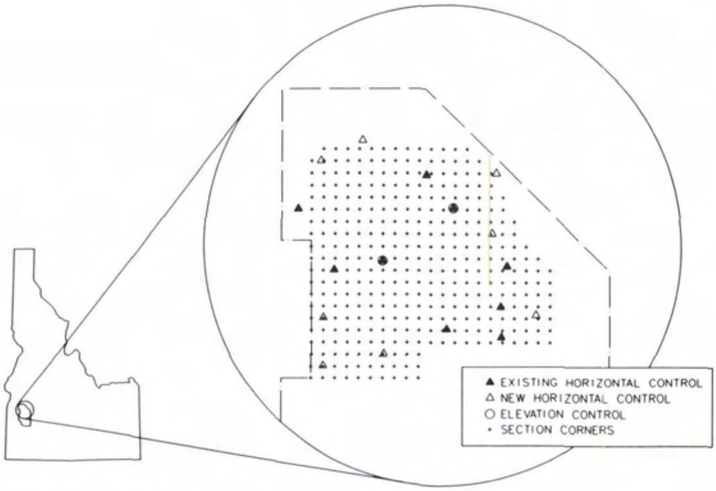

FIG. 1. Geodetic control in project area.

ect included a variety of conditions, ranging from urban streets to moderately rugged terrain, and provided a good operational test of the system.

DATA ACQUISITION

The NOS photogrammetric densification system has already been described in some detail by Slama (1978), and the data acquisition phase of the Ada County project is discussed by Perry (1984) on pages 563-567 of this issue of the Journal. Therefore, only a brief description of the data acquisition process will be presented here in order that more space can be devoted to the data reduction phase, which is discussed in the next section.

Figure 1 provides a summary of the distribution of geodetic control in northern Ada County before, during, and after the project. The nine existing geodetic control points in the project area were not adequate to support photogrammetric densification, so NGS had to establish eight new points. Ada County personnel saw to the monumentation of the section corners and the placement of photovisible targets on all ground points. This was a slow process because nearly one-third of the 346 section corner targets had to be offset, due to some form of obstruction. The photography, mensuration, correction for systematic errors, and preliminary adjustments were all handled by the Photogrammetry Division of NOS.

The metric camera lens cone used in the NOS system is a modified Wild Aviogon II with a focal length of 152 mm and a maximum aperture of *f*/4. This lens has been modified by the manufacturer to provide optimum resolution in a narrow band in the orange portion of the spectrum. Because each ground point of interest is identified by a photovisible target, consisting of a blaze orange disc 0.76 m in diameter on a black background, the imagery to be measured is always crisp and sharp. In addition, there is a 10 by 10 mm projected reseau, an integral part of the lens cone, which provides a means for removing the effects of film distortion from the image measurements. The importance of a reseau in high precision densification cannot be over emphasized.

Complete coverage of the project area, composed of 285 frames of usable photography, was obtained from 16 flight lines in an east-west direction. The nominal flying height was 3600 metres (12,000 ft) above mean terrain, or slightly more than twice the distance between targets. A nominal value of 67 percent was used for both forward and side overlap. This configuration ensures that, except at the edges of the coverage or where targets are obstructed from view, every photo will image at least nine targets and every densification point will be observed from at least nine photographs. An additional 153 photographs of secondary coverage were taken from nine north-south flight lines on which side overlap was reduced to 33 percent. This back-up coverage does not add much to the cost of the photographic mission, but may be invaluable if there are gaps in the primary coverage.

Five independent pointings on the Mann Automatic Stellar Comparator at NOS were used in the measurement of all target images and reseau intersections. Six reseau intersections were measured along with each target image—the four surrounding the image and the next two in order of increasing distance. An eight-parameter transformation is used to restore the reseau intersections to their calibrated positions relative to the optical axis in order to remove the effects of film distortion. The redundancy supplied by six reseau marks, rather than four, is a safeguard against induced distortions that could arise from measurement errors or local deformations, such as dimpling caused by the platten vacuum in the vicinity of one of the reseau marks.

At this time the image measurements are also corrected for both radial and tangential (decentering) lens distortion, using parameters derived from an elaborate preflight calibration, designed and performed by the Photogrammetry Division of NOS. Extreme care and more than the usual number of observation sets have been used in performing this preflight calibration, because the NGS densification adjustment program has no provision for self-calibration. We hope to add this important feature in the near future.

DATA REDUCTION

Most photogrammetric bundle adjustment programs employ the recursive partitioning algorithm, proposed by Brown (1958), which enables them to handle very large adjustments. Any of these programs can be used for densification with only minor modifications. However, to serve the needs of land surveyors, the National Geodetic Survey (NGS) wanted to be able to compute the distance, azimuth, and elevation difference between each pair of intervisible densification points. The computation of these additional parameters from the adjusted point positions is a trivial matter, but the logic of recursive partitioning programs precludes the covariance propagation needed for estimating the standard errors of the extra parameters.

It would be counterproductive to modify an existing program to incorporate the desired covariance propagation, because a significant portion of the efficiency gained through recursive partitioning would be lost. Therefore, NGS was forced to develop a new adjustment program for densification. A listing of this program, called DENCIPHI, is available in Lucas (1981). DENCIPHI is based on an unconventional application of recursive partitioning, which enables it to perform the total densification adjustment, including the additional covariance propagation that motivated its development, using less computer time and storage than a conventional bundle adjustment program.

Before describing the algorithm used in DENCIPHI, it is convenient to review conventional recursive partitioning. The normal equations for bundle adjustment can be expressed in the partitioned matrix form

$$\begin{bmatrix} \dot{\mathbf{N}} & \overline{\mathbf{N}} \\ \overline{\mathbf{N}}^{\mathrm{T}} & \ddot{\mathbf{N}} \end{bmatrix} \begin{bmatrix} \dot{\boldsymbol{\delta}} \\ \ddot{\boldsymbol{\delta}} \end{bmatrix} = \begin{bmatrix} \dot{\mathbf{c}} \\ \ddot{\mathbf{c}} \end{bmatrix} \qquad (1)$$

where $\dot{\boldsymbol{\delta}}$ and $\ddot{\boldsymbol{\delta}}$ are corrections to the exterior orientation parameters and ground point positions, respectively. While recursive partitioning is equally important in obtaining the solution of Equation 1, this discussion will be limited to the method for computing the covariance matrix

$$\begin{bmatrix} \dot{\mathbf{N}} & \overline{\mathbf{N}} \\ \overline{\mathbf{N}}^{\mathrm{T}} & \ddot{\mathbf{N}} \end{bmatrix}^{-1} = \begin{bmatrix} \dot{\mathbf{M}} & \overline{\mathbf{M}} \\ \overline{\mathbf{M}}^{\mathrm{T}} & \ddot{\mathbf{M}} \end{bmatrix} \qquad (2)$$

of the adjusted parameters.

Consider a small densification network, consisting of 25 ground points and an equal number of photographs, in which both forward and side overlap are 67 percent. The structure of the coefficient matrix for such a network is illustrated by Figure 2. Associated with each ground point is a 3 by 3 submatrix, $\ddot{\mathbf{N}}_i$, of the block-diagonal ground point partition, $\ddot{\mathbf{N}}$, and three columns of $\overline{\mathbf{N}}$ which contain the connections between that ground point and all photos on which it is imaged. If the photos are ordered by cross-flight numbering, all non-zero connection elements for a particular ground point will be contained in a b by 3 submatrix, $\overline{\mathbf{N}}_i$, of $\overline{\mathbf{N}}$. When both side and forward overlap are 67 percent, $b = 6(2n + 3)$, where n is the number of flight strips in the network. Hence, the forward reduction by recursive partitioning proceeds a ground point at a time to accumulate the banded matrix

$$\tilde{\mathbf{N}} = \dot{\mathbf{M}}^{-1} = \dot{\mathbf{N}} - \Sigma\, \overline{\mathbf{N}}_i\, \ddot{\mathbf{N}}_i^{-1}\, \overline{\mathbf{N}}_i^{\mathrm{T}} \qquad (3)$$

without ever constructing $\overline{\mathbf{N}}$ or $\ddot{\mathbf{N}}$.

The contribution to $\tilde{\mathbf{N}}$ of all ground points seen by the first photograph will be restricted to the b by b submatrix illustrated in Figure 3. When all image data from the first photograph have been pro-

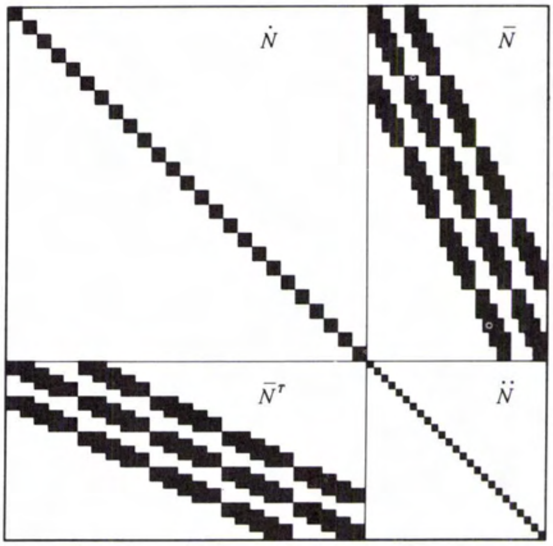

FIG. 2. Structure of normal equations for small densification project.

cessed, the first six rows and columns of $\tilde{\mathbf{N}}$ can be eliminated (as in Gaussian elimination) and moved to peripheral storage. By this device, the core storage required to hold the banded matrix $\tilde{\mathbf{N}}$ during its accumulation and forward reduction is diminished to a *b* by *b* array—in fact, by virtue of symmetry, only the triangular part of a *b* by *b* is needed.

It is obvious from Figure 3 that computation of the 3 by 3 covariance matrix of the adjusted position of any ground point *i* from

$$\ddot{\mathbf{M}}_{ii} = \ddot{\mathbf{N}}_i^{-1} + \ddot{\mathbf{N}}_i^{-1}\,\overline{\mathbf{N}}_i^{\mathrm{T}}\,\dot{\mathbf{M}}\,\overline{\mathbf{N}}_i\,\ddot{\mathbf{N}}_i^{-1} \qquad (4)$$

involves only a *b* by *b* block of $\dot{\mathbf{M}}$. Therefore, only those elements of $\tilde{\mathbf{N}}^{-1}$ that are inside the bandwidth need be computed, and the core storage allocated for the forward reduction is sufficient for computing all of the $\ddot{\mathbf{M}}_{ii}$. This core storage block can be visualized as moving down the banded matrix, $\tilde{\mathbf{N}}$, during the forward reduction and moving back up as the inverse elements and covariance matrices are computed.

The primary virtue of recursive partitioning is efficiency in the use of core storage. The resident normal equations, those portions of the normal equations that must reside in core simultaneously, are limited to the triangular part of a symmetric 3 by 3 submatrix of $\ddot{\mathbf{N}}$ (or $\ddot{\mathbf{M}}$), a *b* by 3 submatrix of $\overline{\mathbf{N}}$, and the triangular part of a symmetric *b* by *b* submatrix of $\tilde{\mathbf{N}}$ (or $\dot{\mathbf{M}}$). Without this conservation of core storage, the large photogrammetric adjustments that are now commonplace would be impossible.

Another advantage of recursive partitioning is economy. No computer time is devoted to computing inverse elements that do not contribute directly to the covariance matrices of the adjusted ground point positions. For most applications, this saving in computation is not only economical, but an important part of the esthetic beauty of the algorithm. However, the computation of standard errors of distance, azimuth, and elevation difference, which relate two ground points, requires the off-diagonal matrix of covariance between the two point positions. It is immediately apparent that some economy must be sacrificed in order to satisfy this requirement, but there are more serious consequences that are less obvious.

It is necessary to assume that two adjacent ground points may be intervisible. Therefore, we will want to be able to compute the 3 by 3 matrix of covariance between them, which can be expressed as

$$\ddot{\mathbf{M}}_{ij} = -\ddot{\mathbf{N}}_i^{-1}\,\overline{\mathbf{N}}_i^{\mathrm{T}}\,\dot{\mathbf{M}}\,\overline{\mathbf{N}}_j\,\ddot{\mathbf{N}}_j^{-1}. \qquad (5)$$

Although the non-zero elements of $\overline{\mathbf{N}}_i$ and $\overline{\mathbf{N}}_j$ each span only *b* rows of $\dot{\mathbf{M}}$, together they may span $6(3n + 4)$ rows. Therefore, the computation indicated by Equation 5 will require that core storage be provided for the triangular part of a square submatrix of $\dot{\mathbf{M}}$ with dimension $6(3n + 4)$, as shown in Figure 4.

In order to obtain the desired off-diagonal covariance blocks by modifying an existing program, the bandwidth of $\tilde{\mathbf{N}}$ would have to be increased artificially. As a result, the amount of core storage reserved for resident normal equations would be doubled, even if intervisibility were restricted to neighboring points. But there is a much more efficient means of computing the off-diagonal covariance blocks.

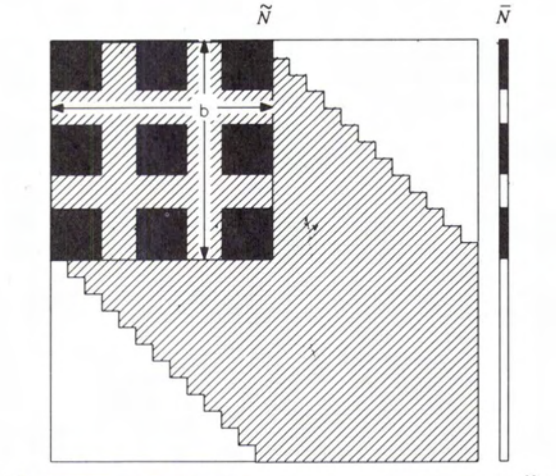

FIG. 3. Core storage allocation for recursive partitioning.

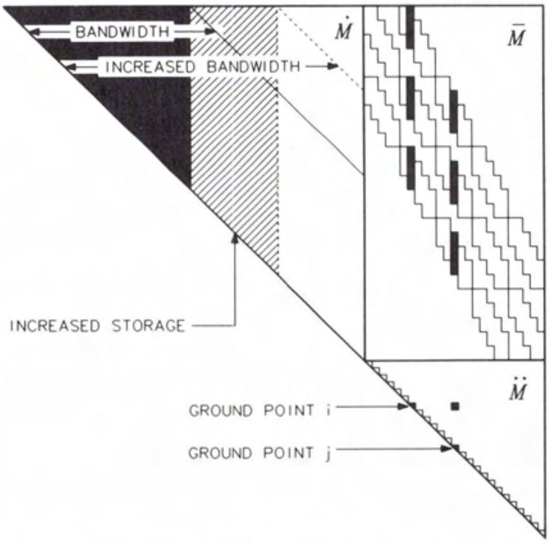

FIG. 4. Increased storage resulting from minimal modifications to recursive partitioning algorithm.

In a conventional bundle adjustment by recursive partitioning, the ground points are eliminated first, resulting in a banded matrix for the photo parameters. Because $\dot{\mathbf{N}}$ is block-diagonal, it is possible, and somewhat more convenient, to eliminate the photo parameters first in order to accumulate the banded matrix

$$\ddot{\mathbf{M}}^{-1} = \ddot{\mathbf{N}} - \Sigma\, \overline{\mathbf{N}}_i^{T}\, \dot{\mathbf{N}}_i^{-1}\, \overline{\mathbf{N}}_i. \qquad (6)$$

In this expression $\overline{\mathbf{N}}_i$ is a 6 by b' submatrix of $\overline{\mathbf{N}}$, where $b' = 3(2n + 3) = b/2$. Because the bandwidth, b', of $\ddot{\mathbf{M}}^{-1}$ is only half that of $\ddot{\mathbf{N}}$, the amount of core storage that must be provided to hold the resident normal equation is reduced by a factor of almost four. As in the conventional application of recursive partitioning, it is necessary and sufficient to compute only those inverse elements that are within the bandwidth, but the number of these elements is reduced to less than half, so there is a significant improvement in both storage and computing time. Moreover, the off-diagonal covariance blocks, that are so difficult to obtain from conventional recursive partitioning, are included among the inverse elements that are within the bandwidth (Figure 5) and computed without extra effort or expense.

Furthermore, having computed $\ddot{\mathbf{M}}$, which contains the covariance matrices of all the adjusted ground point positions and the off-diagonal covariance blocks relating intervisible ground points, the matrix inversion process can be terminated. There is no need to compute $\dot{\mathbf{M}}$, the covariance matrix of the photo parameters. If the 6 by 6 covariance matrix associated with each set of photo parameters is desired for analysis or quality control, these submatrices can be computed sequentially by the technique used to obtain ground point covariance matrices in conventional recursive partitioning. For most applications, however, avoidance of this computational step is a welcome saving in computer time and money.

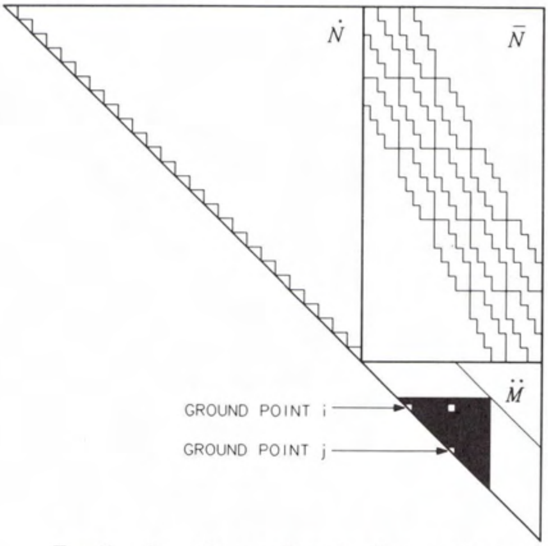

FIG. 5. Core storage allocation for DENCIPHI.

This unconventional application of recursive partitioning, from which Program DENCIPHI has been developed, requires considerably less core storage and computer time than a conventional program. It also obtains the off-diagonal covariance blocks between intervisible densification points at no extra cost. From analyses of ideal networks, it has been estimated that use of the DENCIPHI algorithm reduces both core storage and computing time by a factor of almost four. While this should be regarded as a rough estimate, it proved to be reasonably accurate in the Ada County adjustment, where the core storage reduction factor (based on resident normal equations alone) was approximately 3.3.

RESULTS

The flight plan used by NOS for densification provides for each ground point to be intersected from at least nine photographs whenever possible. If this level of redundancy is to be carried to the perimeter of the project, however, it is necessary to target an additional row of points outside the area of interest in order to provide attitude control for the edge photographs. The positions of these extra points are computed as a part of the adjustment, but with lesser precision, nominally 80 to 100 mm in each coordinate. Ada County elected to accept this reduced accuracy along the borders of the project to avoid the trouble and expense of targeting so many extra points. Therefore, the precision estimates for this project will generally be given in two forms: An overall figure and an estimate for all points that are not contaminated by edge effects.

The primary coverage of the project area, consisting of east-west flights with two-thirds forward overlap and two-thirds side overlap, was measured first. When these data became available, a preliminary adjustment was made. Overall the fit was good, but analysis of the residuals indicated that the secondary coverage would strengthen some areas of the network.

The preliminary adjustment caused some concern over the compatibility of the photogrammetry with the control points established by conventional geodetic methods. This concern was alleviated by an adjustment in which all of the geodetic control was constrained according to reasonable estimates of its precision. In this adjustment, which included the secondary coverage, standard deviations of 50 mm in both latitude and longitude were assigned to all control points. The elevations of two control points were known from geodetic leveling and were, therefore, assigned standard deviations of 50 mm. All other geodetic control point elevations were assigned 250 mm. As a result of the large number of photogrammetric observations of each geodetic point, the imposed constraints had little effect, allowing these points to shift to positions dictated by the photogrammetry. Nonetheless, the maximum shift in the geodetic control was less than 60 mm, and most points shifted less than 50 mm. It was very encouraging to find that the compatibility of the photogrammetry and geodesy was consistent with their assumed precisions.

Having determined that there were no significant discrepancies between the photogrammetric and geodetic positions, the final adjustment was made with all geodetic control held fixed. Lacking information on the intervisibility of ground points, the distance, azimuth, and elevation difference were computed from each point to all other points within a radius of 2.5 km. For a section corner, this included each of its eight neighbors.

By comparison, the elevation differences had both the largest standard errors and the greatest range of standard errors—from 50 mm to a maximum of 240 mm. Approximately 90 percent of the computed elevation differences had sigmas less than 120 mm, and edge effects were responsible for nearly all that were greater than 100 mm. The precion of azimuths is a function of distance, but when distances shorter than one mile and lines influenced by edge effects are eliminated, all sigmas were between 2 and 5 arc seconds.

Ninety-five percent of the standard errors associated with computed distances between section corners were less than 60 mm. One of the section corners was obstructed from view on all but three photographs, which resulted in a weak position determination. Distances involving this point or points on the periphery of the project area accounted for virtually all of the standard errors in distance that exceeded 50 mm.

In January 1981, a field check was performed, as shown in Figure 6. Geodetic traverse station IDA 80 60 1958, labeled A in the figure, and two section corners labeled B and C, were occupied to form a triangle. Traverses were run to four additional section corners over distances of 1.5 to 5 km. All distance measurements were made with a Hewlett Packard electronic distance measuring instrument, model 3808, on loan from the Pacific Marine Center. Unfortunately, station IDA 80 60 1958 was not included in the photogrammetric adjustment and the HP 3808 had not been calibrated for distances greater than 1400 m. Considering these deficiencies, the geodetic measurements compared favorably with the photogrammetric survey results shown in Table 1.

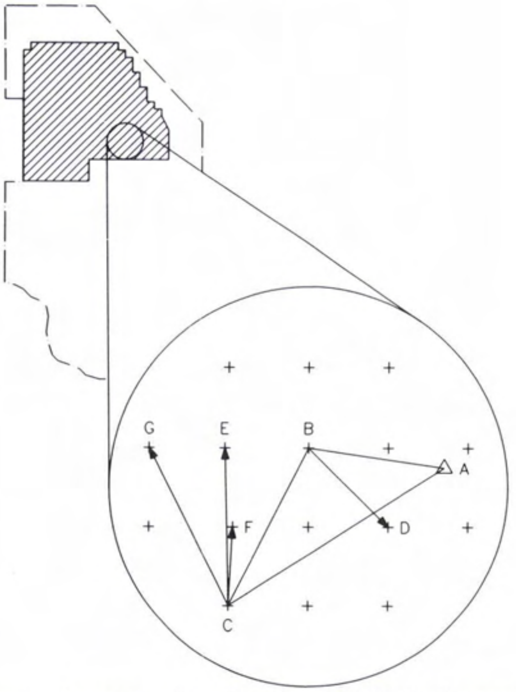

FIG. 6. Location and configuration of field check survey.

TABLE 1. COMPARISON OF DISTANCES AND AZIMUTHS DETERMINED BY PHOTOGRAMMETRIC DENSIFICATION AND BY FIELD SURVEY

Line	Distance		Difference (m)	Azimuth						Difference
	Photo (m)	Geodetic (m)		Photo			Geodetic			″
				°	′	″	°	′	″	
AB	2551.216	2551.186	0.030	100	12	05.4	100	12	08.2	−2.8
AC	4980.958	4980.973	−0.015	55	56	22.4	55	56	21.8	0.6
BC	3621.813	3621.815	−0.002	26	28	13.1	26	28	14.0	−0.9
BD	2291.520	2291.505	0.015	315	13	18.8	315	13	21.9	−3.1
CF	1609.837	1609.818	0.019	180	26	49.6	180	26	47.6	2.0
CE	3247.790	3247.881	−0.091	179	54	52.4	179	54	53.2	−0.8
CG	3634.623	3634.667	−0.044	153	29	59.4	153	29	57.5	1.9

All azimuth differences are small compared to the standard errors predicted from the photogrammetric adjustment and all distance differences, except line CE, are below the 50 mm level. The 90 mm difference in distance CE, although large in comparison with the other distance differences, is less than twice the standard deviation for the computed length of this line.

CONCLUSION

There is no question that densification of geodetic control can be accomplished more economically by photogrammetry than by conventional ground survey. This project has shown that a precision of 50 mm is obtainable, which should satisfy most requirements. In urban areas, where a higher degree of precision and greater density of control may be required, 20 to 30 mm should be attainable by increasing the photo scale by a factor of two, although this extrapolation remains to be verified.

The Ada County project was successful in verifying the utility of photogrammetric control densification, but the ultimate measure of the success of this endeavor depends upon its impact on the geodetic community. If other state and local governments are stimulated to densify inadequate control networks, and if other agencies and contractors are motivated to develop a capability for performing this service, both the community of land surveyors and the general public will reap a substantial benefit.

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BOOK REVIEW

Textbook of Photogrammetry, by K. K. Rampal. Oxford & IBH Publishing Co., New Delhi, Bombay, Calcutta, 1982, price unknown.

I READ THIS BOOK with great expectation. It reminded me, by its title, of the now classical *Text Book of Photogrammetry* by Zeller.

This book contains sixteen chapters and four appendices for a total of 335 pages. The chapters are (1) Introduction, (2) Geometry of Aerial Photos, (3) Geometry of Two Overlapping Photographs, (4) Flight Planning or Design, (5) Aerial Cameras, (6) Stereoscopy, (7) Mapping from Air Photographs, (8) Mosaicking, (9) Stereo Mapping, (10) Rectification, (11) Analytical Methods, (12) Aerial Triangulation, (13) Terrestrial Photogrammetry, (14) Photogrammetric Applications, (15) Computers in Stereo Mapping, and (16) Elements of Remote Sensing.

When one reads this book, the first impression is that it is unfinished. Pages 80 to 97 are missing. The author does not provide the reader with a bibliography at the end of the book or with references at the end of each chapter, in spite of frequent references made to various scientists in connection with photogrammetric methods. For example, on page 129, it is written: "Of the several three-point methods developed by Professor Church, the most important is described by Baker. A modification (by Church himself) is included in the *Manual of Photogrammetry* (p. 41) published by the American Society of Photogrammetry." Which edition?

The book does not describe photogrammetric instruments other than rectifiers, and in a number of places refers to "Wild A-7," "Kelsh plotter," etc.

Seven pages are devoted to aerial triangulation adjustment by graphical method and two pages to analytical aerotriangulation.

The figures and definitions occasionally are confusing. For example: "In absolute control the ground control is typically a triangulation traversing on the trilateration network" (p. 141).

Photogrammetry as a science or as a tool is so beautiful and so useful: it deserves better treatment than this.

—*S. A. Veress*