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Some Theoretical and Practical Aspects of the OR1/SORA-PR System

Perspective or combined perspective and affine transformations are accomplished by means of digitally controlled differential rectifiers.

Introduction

Despite the fact that in recent years digitally controlled differential rectification has become increasingly important for orthophoto and stereo-orthophoto production, simple rectification remains in many cases a very important task.

In simple rectification, it is generally assumed that the surface of the object to be rectified is a plane or can be approximated by a plane. This assumption relieves photogrammetry of the need to record the surface of the object by means of stereophotographs and to perform costly, time-consuming restitution processes.

Abstract: *The Avioplan OR1 orthophoto system was shown for the first time to the public at the 1976 ISP Congress in Helsinki. The SORA-PR program, used for the preparation of digital data for carrying out differential rectifications of oblique photographs of plane objects by means of the OR1, was developed at the Institute for Photogrammetry of the Technical University Vienna and it was presented at the 1980 ISP Congress in Hamburg. Since then, the OR1/SORA-PR system has been in practical use world-wide.*

This paper deals with the theory and practical details of the manner in which this system is used, followed by information, based on practical examples, of the quality and accuracy obtainable in the product and the economics of the method.

Finally an analysis of the accuracy obtained is given.

Except for certain fields of activity, such as industry, architecture, the conservation of monuments, archaeology, etc., the surface of an object is rarely plane. Thus, the use of simple rectification results in a loss of accuracy. It is often necessary to strike a compromise between the accuracy of the product and the economy of the method used. The cost of simple rectification in preference to other restitution procedures must be considered. The following situation may be regarded as a typical example:

> Assume that a map is urgently needed of an approximately plane area for which no information is available on heights or, perhaps, of an area over which flying is not permitted so that oblique photographs must be taken. In practice, there are many situations where oblique photographs must be used and which cannot therefore be restituted by normal stereorestitution (Vozikis, 1979).

In such situations, conventional rectification equipment has been used ever since the beginnings of photogrammetry. The capability of this type of instrument for handling gross tilts is limited, and the constraints which this imposes on working are inconvenient and time-consuming. Because they make dark-room conditions necessary and control points have to be marked and corrected graphically, they constitute a source of inaccuracy and error.

This paper presents the possibility of using digitally controlled instruments for differential rectification in order to carry out perspective or combined perspective and affine transformations of substantially inclined photographs.

Because the Wild Avioplan OR1 orthophoto instrument (Stewardson, 1976) and the SORA-PR program (Vozikis and Loitsch, 1980) were available, the characteristics of the OR1/SORA-PR system are presented first. This is then followed by showing the simplicity, economy, quality, and accuracy obtain-

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able in such an application by reference to practical examples.

The OR1/SORA-PR System

The Avioplan OR1 is a digitally controlled differential rectifier for offline operation. The data required for its control are transmitted by magnetic tape.

The functional principle of the Avioplan OR1 can be formulated by reference to Figure 1 (Kraus, 1976). This instrument can carry out a photographic transformation from two predetermined projections I and II, based on the mathematical relationship, f, of the two projections. The geometrical definition of this transformation is that the OR1 can transform an original document in such a way that any deformed grid in the original will correspond to a square grid in the rectified photograph used as the transformed copy.

In mathematical terms, the OR1 carries out the following transformation:

$$\mathbf{\Xi}' = \mathbf{G} \cdot \mathbf{X}' \qquad (1)$$

where $\mathbf{\Xi}' = [\xi\ \eta\ 0]^{\mathrm{T}}$ are the coordinates of the corner points of the deformed grid in projection I;

$$\mathbf{G} = \begin{bmatrix} a_0 & a_1 & a_2 & a_3 \\ b_0 & b_1 & b_2 & b_3 \end{bmatrix}, \mathbf{X}' = [1\ X\ Y\ X{\cdot}Y]^{\mathrm{T}}; \text{ and}$$

a_i, b_i($\mathbf{i}$ = 0,1,2,3) are the parameters of the transformation (Equation 1).

The coordinates $\mathbf{\Xi}'$ can be prepared in a number of different ways (Vozikis and Loitsch, 1980).

Function f may be a determinate methematical expression (Bormann and Vozikis, 1982) or may be replaced by interpolation algorithms (Otepka, 1976; Jansa, 1980), depending on the specific transformation to be carried out. In the context of aerial photographs for the production of orthophotos or stereo-orthophotos, function f is very difficult to be determined. In this case, projection I is the original photograph (central perspective) and projection II is the orthophoto or stereomate.

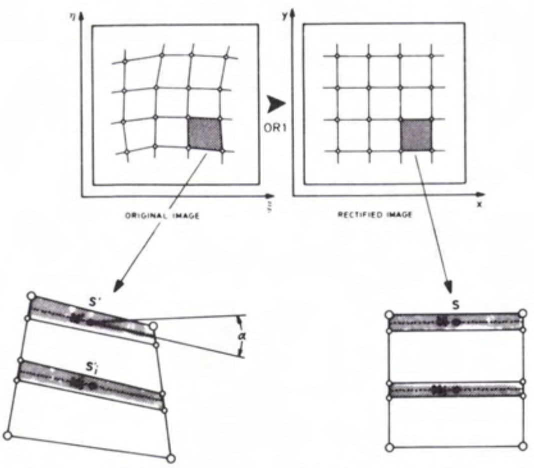


Fig. 1. Basic operating principle of the OR1.

For this purpose, function f can be determined indirectly as follows:

$$\left.\begin{array}{l} \mathbf{X} = f_1(\mathbf{x}) \\ \mathbf{\Xi} = f_2(\mathbf{x}) \end{array}\right\} \rightarrow \mathbf{\Xi} = f_2(f_1^{-1}(\mathbf{X})) = f(\mathbf{X}) \qquad (2)$$

where $\mathbf{x} = [x\ y\ z]^{\mathrm{T}}$ are the coordinates of a number of points on the surface, in a Cartesian system (usually the model system), and $\mathbf{X} = [X\ Y\ Z]^{\mathrm{T}}$ are the coordinates of the grid points in projection II corresponding to the corner points $\mathbf{\Xi}$ of projection I (Equation 1).

In order to carry out the transformation in this case, some knowledge is required of the shape of the ground surface, the image-forming laws (mathematical definition of the central-perspective transformation f_2), and the orientation (relative position to the ground) of the original photograph. The ground shape is given in this case by means of points whose spatial positions within a Cartesian system are known (coordinates $\mathbf{x}$). The orientation (spatial resection) is determined by points whose coordinates $\mathbf{X}$ and $\mathbf{\Xi}'$ are known.

The SORA-PR program executes all the required procedures in two main steps:

In the first step, the plane e, which replaces and approximates the surface photographed, is determined first (Figure 2). Function f_1 is then defined and, depending on the type of transformation (perspective or combined perspective and affine), the square grid is placed in the XY or the e plane.

In the second step, function f_2 is determined. In perspective rectification, the eight parameters of the central-perspective transformation describe the mathematical relationship between the object plane and the image plane. However, even in this case, the more general relationship of the central projection is taken, with six elements of outer (three rotations and three translations) and three elements of inner orientation (focal length and translation to the principal point): i.e.,

$$\mathbf{X} = m \cdot \mathbf{R} \cdot (\mathbf{\Xi} - \mathbf{\Xi}_0) + \mathbf{X}_0 \qquad (3)$$

where $\mathbf{X} = [X\ Y\ Z]^{\mathrm{T}}$ are the ground coordinates of a point,

m	is the scale factor,
$\mathbf{R}\ (r_1\ r_2\ r_3)$	is the rotation matrix with the elements $r_{ij}(i,j = 1,2,3)$
$\mathbf{\Xi} = [\xi\ \eta\ 0]^{\mathrm{T}}$	are the image coordinates of this point,
$\mathbf{\Xi}_0 = [\xi_0\ \eta_0\ -c]^{\mathrm{T}}$	are the image coordinates of the principal point, and
$\mathbf{X}_0 = [X_0\ Y_0\ Z_0]^{\mathrm{T}}$	are the ground coordinates of the projection center.

The grid points of the XY or e plane are now transformed by means of f to projection II (Figure

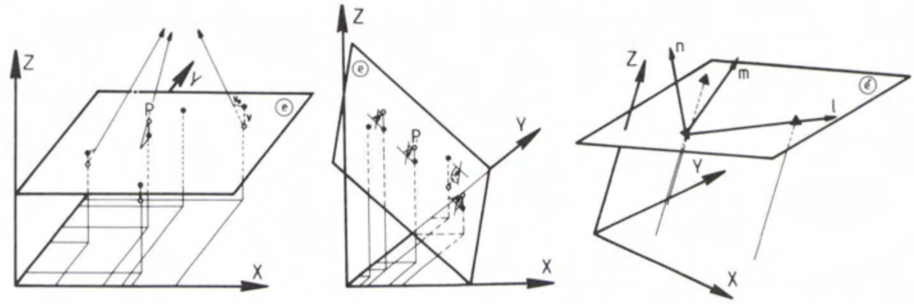


FIG. 2. The three means of defining the object plane, e, in the SORA-PR program.

3). This transformation obtains a deformed grid in the image. The image coordinates Ξ' of the corner points of the deformed grid are stored on magnetic tape and can be used for controlling and monitoring the transformation in the OR1.

The photographic transformation is now carried out as follows:

The original photograph (projection II) is oriented on the OR1 picture carrier by means of three orientation points (two points for determining the translations $\Delta\xi$ and $\Delta\eta$ and plane-rotation κ, and one point for mirror-reverse check). Following this, a linear interpolation is carried out by the process computer of the OR1 (a Data General NOVA 3) in the direction η for all grid points Ξ'. The photograph is now divided into profiles in the η direction, and the linear elements, s', obtained from the interpolation (Figure 1) are scanned optically.

The linear element, s', is rotated, displaced, and enlarged/reduced to correspond with its respective s, and is then photographically projected onto the film drum through a slit mask. The length of this slit mask is the same as the mesh-width of the grid $\mathbf{X}$ at the scale of the orthophoto.

Definition of the Object Plane, e

The object plane, e, is defined in the three-dimensional coordinate system $\mathbf{X}$ by the equation (Bronstein and Semendjajew, 1966)

$$(e)\colon \mathbf{E} \cdot \mathbf{X} + e_4 = 0 \tag{4}$$

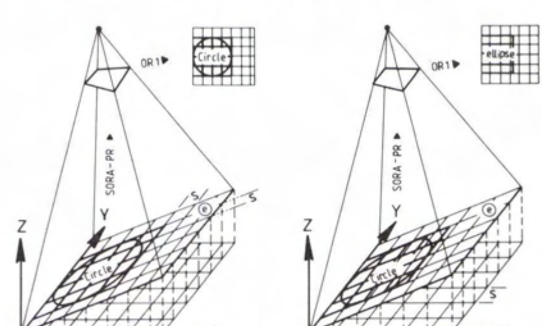


FIG. 3. Basic principle of transformation with the OR1/SORA-PR system. (a) perspective transformation. (b) combined perspective and affine transformation.

where $\mathbf{E} = [e_1\ e_2\ e_3]$, e_4 are the parameters of the object plane.

The SORA-PR program allows two ways of defining the object plane (Figure 2). In the first case, the plane e is parallel to the XY plane at a distance e_4 from it. The value of e_4 is computed by the program as the arithmetic mean of all the Z coordinates of specially coded points which are given in the input program. In this case, the plane parameters are

$$e_1 = e_2 = 0,\ e_3 = -1,\ e_4 = \Sigma\, Z_i/K. \tag{5}$$

The distance $v_i^0 = Z_0 - e_4$ gives a good indication whether the approximation of the object surface by the plane e is adequate. In addition, the program also computes and prints out the values of their projections on the orthophoto (Figure 2). In the second case, the object plane is inclined relative to the XY plane. The parameters (Equation 4) are computed by the program, based on the coordinates $\mathbf{X}$ of specially coded points which are given in the input program. If more than three such points are entered, the parameters are determined by adjustment, where the sum of the squares of the distances v_i of these points from the plane e is a minimum.

A first approximation of e is derived from three of these points defining the greatest area. A coordinate system is then defined on this and shown on Figure 2. The coordinates $\mathbf{L} = [l\ m\ n]^T$ of the points of this plane are related to the coordinates $\mathbf{X}$ by the following spatial transformation:

$$\mathbf{L} = \mathbf{R}' \cdot \mathbf{X} + \mathbf{L}_0 \tag{6}$$

where $\mathbf{R}'$ $(\mathbf{r}_1'\ \mathbf{r}_2'\ \mathbf{r}_3')$ is a rotation matrix with the elements $\mathbf{r}'_{ij}$ $(i,j = 1,2,3)$ and $\mathbf{L}_0$ $[l_0\ m_0\ n_0]^T$ are the coordinates of the origin of $\mathbf{X}$ in the system $\mathbf{L}$.

By means of Equation 5, every point used to determine e is transformed from the $\mathbf{X}$ to the $\mathbf{L}$ system.

The plane e can now be described by the equation

$$\mathbf{D} \cdot \mathbf{L} + d_4 = 0 \tag{7}$$

where $\mathbf{D} = [d_1\ d_2\ d_3]^T$ and d_4 are the new parameters of the plane.

The error equation

$$V = d_1 \cdot l + d_2 \cdot m + d_4 - n \tag{8}$$

then represents with sufficient accuracy the adjustment of the plane for determining the new parameters.

Finally, the parameters d_i, $i = 1,2,3,4$ are transformed to e_i, by combining Equation 6 with Equation 7: i.e.,

$$\mathbf{D} \cdot (\mathbf{R}' \cdot \mathbf{X} + \mathbf{L}_0) + d_4 = 0$$

Hence, the parameters of the plane e are:

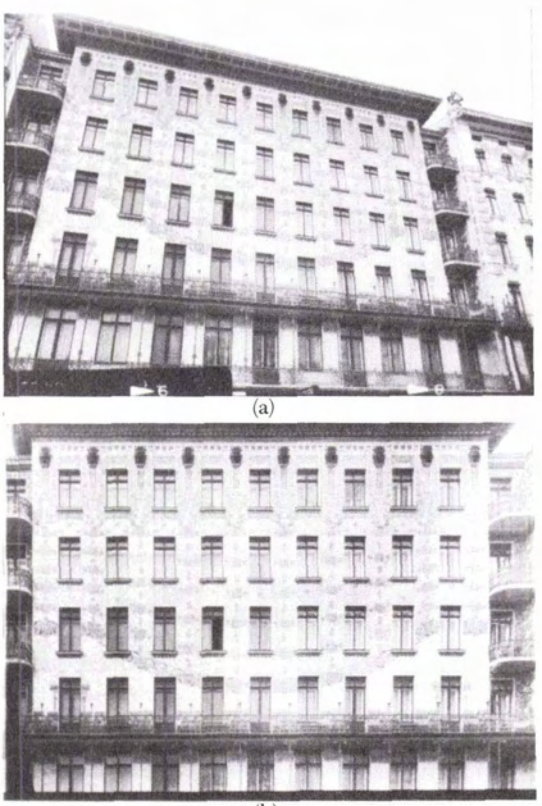
$$\begin{aligned} e_1 &= \mathbf{D} \cdot \mathbf{r}_1' \\ e_2 &= \mathbf{D} \cdot \mathbf{r}_2' \\ e_3 &= \mathbf{D} \cdot \mathbf{r}_3' \\ e_4 &= \mathbf{D} \cdot \mathbf{L}_0 + d_4 \end{aligned} \tag{9}$$

After the adjustment, as in the first case, the distances v_i^0 and their projections v_i in the rectified end product are computed and printed out.

Types of Transformations

With the ORI/SORA-PR system, the following types of transformation can be carried out (Figure 3):

- *Perspective transformation.* In this case, the square grid is extended on plane e. Following computation of the outer and possibly also of the inner orientation (use of amateur camera) of the photograph, these grid points are transformed from plane e into the image by means of a central projection (Equation 3). The result is a set of coordinates with which the distorted image (projection II) is to be transformed in the OR1 into a new photograph corresponding to the square grid, i.e., plane e = projection I. A circle in the object plane is therefore reproduced in the rectified image as a circle. This situation arises most frequently in practice in architectural and industrial photogrammetry, and in archaeology.
- *Combined perspective and affine transformation.* The second type of transformation with the ORI/SORA-PR system is the so-called combined perspective and affine rectification. This case always occurs in aerial photogrammetry and frequently also in terrestrial photogrammetry. Here, what is required is not a rectification in an inclined plane, but the orthogonal projection of the object plane e into the XY plane. This transformation comprises a perspective rectification and an affine rectification. In this case, the square grid is first defined on the XY plane. It is then projected parallel to the Z axis onto the plane e. From here, the same procedure is carried out as that described above for the perspective transformation. In this, a circle in the object plane will be reproduced in the rectified image as an ellipse.



(a)

(b)

FIG. 4. (a) Building elevation photographed with the Wild P31 terrestrial camera. (b) Perspective transformation with the OR1/SORA-PR system, original scale 1:50.

The ORI/SORA-PR system can be used for both simple or affine enlargement. The SORA-PR program provides a number of possibilities for defining the transformation range. The limits can be defined in either the **X** or in the **Ξ** system.

Further details of the structure of the SORA-PR program, the transformation possibilities it provides, and the definition of the transformation range are given in Vozikis and Loitsch (1980).

Practical Examples

The ORI/SORA-PR system has to date been used successfully in practice for carrying out a variety of projects world-wide.

Four representative examples are given below, two for each type of transformation. Further examples from practice and experiences made are given in Vozikis (1979).

Figure 4a shows an example taken from architectural photogrammetry. This is a photograph taken with a Wild P31 of the elevation of a Art nouveau building at 'Naschmarkt' in Vienna. For the definition of the plane e, several points in the plane of the elevation were measured with a Wild T16 Theodolite in a local coordinate system which did not lie exactly in plane e. In addition to these points, several control points were also available in front of the plane of the elevation, for determining the outer orientation. Figure 4b shows the perspective transformation of this photograph, using the ORI/SORA-PR system, carried out at the Institute of Photogrammetry, Technical University Vienna at original scale 1:50.

Figure 5 shows an example taken from aerial archaeology. These very oblique photographs were taken from a light aircraft. This is a case where archaeologists have taken advantage of the fact that, at certain times of the year, underground structures can be clearly recognized from the air. Following the combined perspective and affine rectification of the pictures, the coordinate grid can be marked on the rectified photographs. As a result, each rectified photograph can be correctly positioned in a previously plotted coordinate grid, and after the entire area has been rectified and matched, the archaeologist can begin to plot the archaeological information by simply tracing the various features and details. Figure 5a shows one of the many very oblique photographs taken with a Hasselblad 500CM (Vozikis, 1982), made available by the Vienna City Museum. The photograph shows part of the Roman settlement by Carnuntum, to the east of Vienna. The points used to define the plane *e* and the control points for spatial resection were taken from an orthophoto map at a scale of 1:2000, which had been produced for another purpose some time earlier.

Another example of a combined perspective and affine transformation carried out with the OR1/SORA-PR system is shown in Figure 6. Figure 6a shows a metric photograph taken near Casa Grande, Arizona with an RC10 aerial camera with an Aviogon II Lens. The photograph and the control point information was made available by the National Ocean Survey, Rockville, Maryland. The control points used for the space resection as well for the plan definition are good signalized artificial points. Figure 6b shows an area of the photograph after a combined perspective and affine transformation with the system OR1/SORA-PR at an original scale of 1:25 000, carried out at Wild Heerbrugg.

Transformation Accuracy and Limits

Because the OR1 applies a bilinear transformation (Equation 1), it can be theoretically correct only for the grid intersection points. There will be errors at other points, whose magnitude will depend on the line elements, *s*, at the orthophoto scale and on the tilt of the photograph. This so-called 'interpolation' error can be studied with the aid of the cross-ratio of perspective projection. The centers of the sides of the grid are computed first by bilinear transformation (Equation 1) and then by means of the cross-ratio. Vozikis (1979) gives a detailed review of this source of error.

Another source of error is the fact that the photographed surface never coinsides exactly with the defined object plane, *e*. As a result of this, so-called projection errors occur.

If we use the method presented for the production of a map of flat, horizontal terrain from vertical aerial photographs, the projection errors, Δr, in any direction in the image plane can be calculated with the aid of its components $\Delta\xi = \Delta h \cdot \xi/H$, and $\Delta n = \Delta \cdot \eta/H$: i.e.,

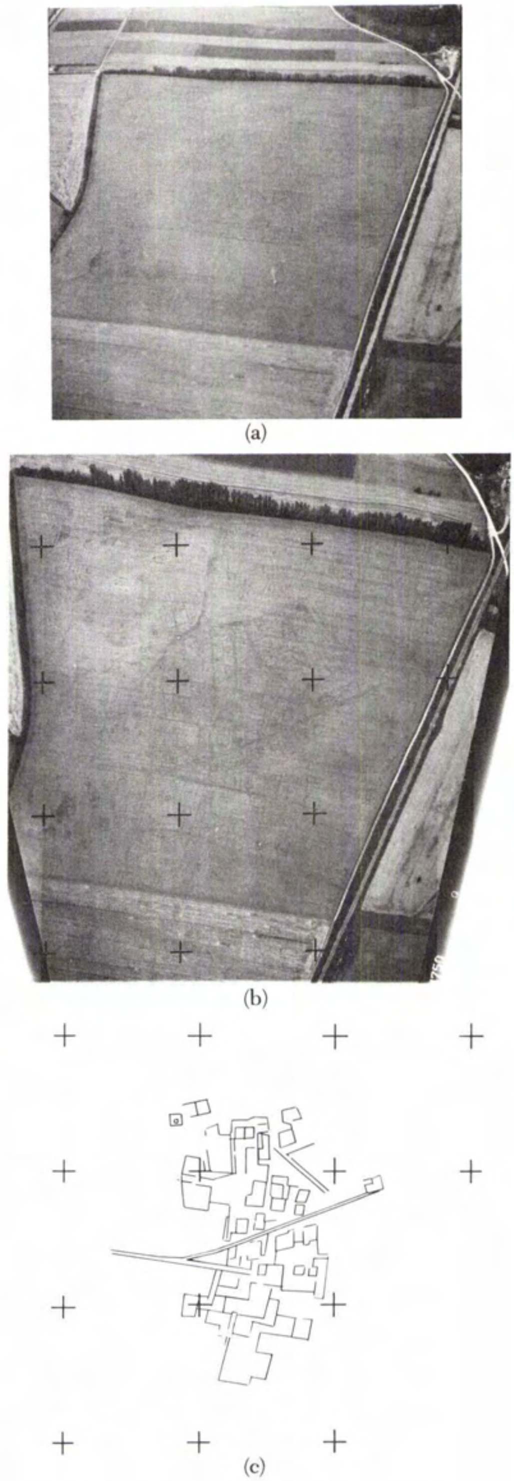
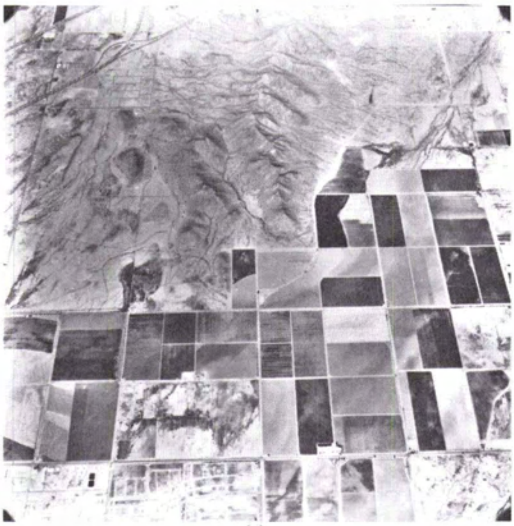
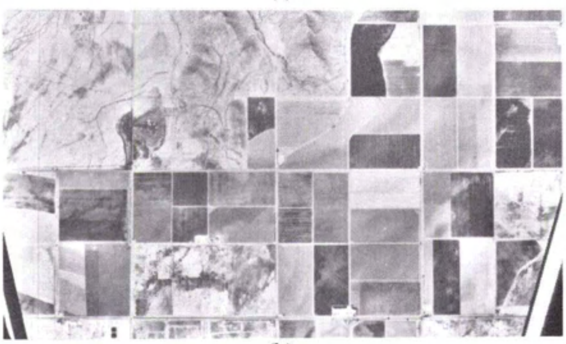


FIG. 5. (a) Oblique photograph taken with an amateur camera from a light aircraft. (b) Combined perspective and affine transformation with the OR1/SORA-PR system, with superimposed coordinate grid. (c) Tracing of some archaeological details.



(a)



(b)

FIG. 6. (a) Aerial photograph taken with an RC10 near Casa Grande, Arizona. (b) Combined perspective and affine transformation at original scale 1:25000.

$$\Delta r = \sqrt{(\Delta\xi)^2 + (\Delta\eta)^2} \qquad (10)$$

where Δh is the distance between an object point, P, and the intersection point, P', of a straight line through P being parallel to the Z-axis with the adjusted object plane e.

If, for simplicity's sake, we assume $Z_0 = H$, $X_0 = Y_0 = 0$, $\omega = \kappa = 0$, $\rho = -\nu$, and $\xi_0 = \eta_0 = 0$, the basic Equation 3 becomes

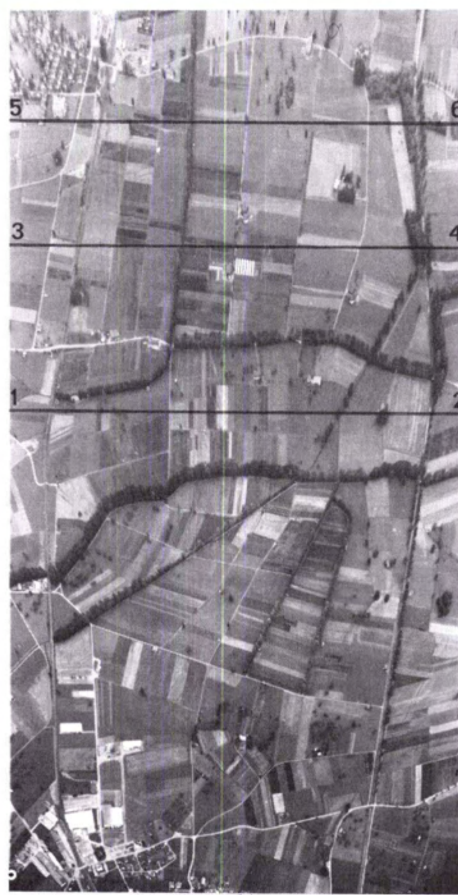
$$\xi = c\,\frac{X \cdot \cos(\nu) - H \cdot \sin(\nu)}{X \cdot \sin(\nu) + H \cdot \cos(\nu)}$$

$$n = c\,\frac{Y}{X \cdot \sin(\nu) + H \cdot \cos(\nu)} \qquad (11)$$

where H is the flying heigth, ν is the nadir angle, and c is the camera constant.

If the method for perspective transformation or combined perpsective and affine transformation from oblique photographs is used, the projection error, Δr, can be calculated as follows:

(a)



(b)

FIG. 7. (a) Extremely titled aerial photograph taken with RC10 near Heerbrugg, Switzerland. (b) The same photograph after its rectification with the ORI/SORA-PR system (original scale 1:10 000). The basic magnification in the four areas formed by the lines 12, 34, and 56 has been changed manually during scanning.

The X, Y coordinates of a point given on a tilted photograph through its image coordinates ξ, η can be calculated by means of the following equations (Wolf, 1974):

$$X = \frac{H - h}{c/\cos(\nu) - n \cdot \sin(\nu)} \cdot \xi$$
$$Y = \frac{H - h}{c/\cos(\nu) - n \cdot \sin(\nu)} \cdot \eta \cdot \cos(\nu) \qquad (12)$$

where ν, H, and c are as defined above, and h is the height of the point.

If we solve the second part of Equation 12 for n and we set this value to the first part, we have the image coordinates of this point as function of its ground coordinates: i.e.,

$$\xi = \frac{X \cdot c}{[(H - h) \cdot \cos(\nu) + Y \cdot \sin(\nu)]}$$
$$n = \frac{Y \cdot c/\cos(\nu)}{(H - h) \cdot \cos(\nu) + Y \cdot \sin(\nu)} \qquad (13)$$

The image coordinates ξ_1, η_1 of an object point which does not lay exactly on the object plane and has the coordinates $X_1 = X$, $Y_1 = Y$, $h_1 = h + \Delta h$, can be determined by means of Equation 13. For $\Delta\xi = \xi_1 - \xi$ and $\Delta n = \eta_1 - \eta$ we can define now the value of Δr according to Equation 10.

Apart from the sources of error described above, the physical limits of the OR1 also limit the system described. These are given in detail in Stewardson (1976), and the relationship to the tilt angle of the photographs, the camera distance, and the position of the image area to be transformed are investigated in Vozikis (1979). The limits which can be reached in such a transformation are the maximum permissible rotation of a line element s' and the maximum permissible enlargement/reduction factor F.

In order to multiply S' by F, a basic magnification V_g is set manually in the OR1 and the residual magnification is continuously taken over by the differential zoom.

If during scanning a limit is reached, the alarm is actuated and transformation stops automatically. If the decision is made to profile in the direction in which the scale differences are greatest, there is a risk that it will not be possible to continue work after the first profile, because the differential zoom reaches its limit. It is possible to overcome this problem by profiling at a right angle to this direction and changing the basic magnification manually while profiling is being carried out. This way of operation is of course not for every day work, but can be applied in extremely critical cases. Figure 7 shows an example of a combined perspective and affine transformation carried out in this way. The control points used for the object plane definition and the space resection were taken from a topographic map of Switzerland, scale 1:50 000. The whole photograph has been scanned in a single operation, but with different basic magnifications set manually for the four areas defined by the lines 12, 34, and 56. Had scanning been carried out in an east-west direction rather than north-south, work would have stopped during the first profile scan somewhere near the line 34.

Acknowledgments

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