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A Parallel Case of Photogrammetry and its Application in Narrow Transits

The technique, for the two cases where the camera axes lie in the camera base line, resulted in increased accuracy of coordinate determination.

Abstract: *The application of photogrammetric techniques, where space is insufficient to apply normal stereophotogrammetric procedures, is investigated. Two parallel cases of photogrammetry, where photographs are taken from exposure stations along the camera axis, are introduced. Theoretical error analysis and practical application show that the use of the proposed technique appreciably increases the accuracy of coordinate determination.*

Introduction

One of the main problems that is often encountered in the recording of indigenous architecture or archaeological monuments is the surveying of narrow places such as tunnels, corridors, galleries, arcades, skyways, or subterranean passages. In such cases, although the normal stereophotogrammetric case can still be used, there is usually insufficient space across the structure's axis for a base-line of the length necessary to precisely determine the coordinates in three dimensions. This article presents a solution to this problem by which the accuracy of coordinate determination may be increased appreciably.

Generally, two images from two different exposure stations are required in order to determine coordinates in three-dimensional space. The angle subtended between the camera axis and the base could take any value between 0° and 180°. In the two cases of the two extreme values of this angle, the camera axis coincides with the base-line, thus offering a number of applications in photogrammetry; in particular, the surveying of narrow longitudinal structures where there is not enough space for the normal stereophotogrammetric base.

Coordinate determination

Figure 1 shows the proposed parallel case of photogrammetry. Figures 1a and 1b show the two extreme cases when the camera axes coincide with the base line.

For case 1 (Figure 1a) there is a possibility of forming a strip containing any number of successive photographs. For case 2 (Figure 1b) only objects situated between the two exposure stations can be photographed. Hence, case 2 offers limited possibilities of practical applications. It is easily realized that the coordinate determination along the camera axis is not possible. It is also evident that for these cases no stereoscopic vision is possible, and coordinates can only be determined analytically.

Most conveniently, a coordinate system is chosen with the origin at the camera station closer to the object (Figure 1a). The directions of the X, Y, and Z axes are as shown in the figure. The coordinates of an arbitrarily chosen object point are derived from Figure 1a.

In this figure, point n is photographed from the two exposure stations o_1 and o_2 separated by the base b_2 along the Y axis. The point images n' and n'' have the coordinates x', z' and x'', z'', respec-

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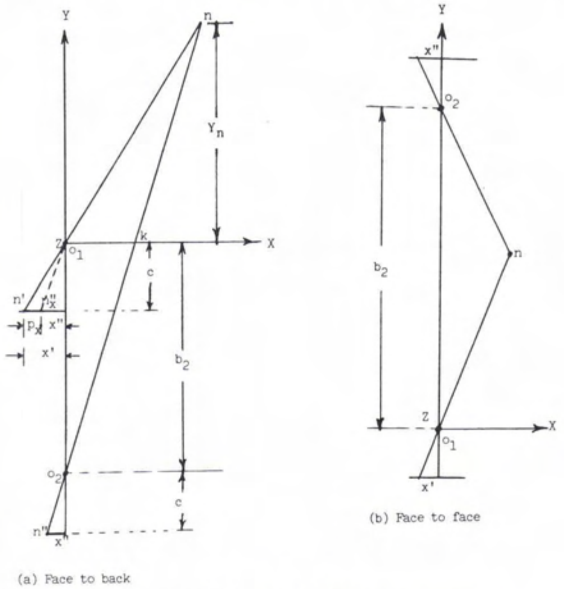

FIG. 1. Parallel case of photogrammetry.

tively. The line $o_1n''_x$ is drawn parallel to no_2 so that the distance $n'n''_x$ is equal to the parallax p_x. The line no_2 intersects the X-axis in point k.

The two triangles nko_1 and $o_1n''_xn'$ are similar; hence

$$Y_n = c\,\frac{ko_1}{p_x}$$

where c is the camera constant.
But

$$ko_1 = b_2\,\frac{x''}{c}\,.$$

Hence, by substitution, the Y coordinate can be written in the form

$$Y_x = b_2\,\frac{x''}{p_x} \text{ or } Y_z = b_2\,\frac{z''}{p_z} \tag{1}$$

where Y_x and Y_z are the Y-coordinates calculated from the x- and z-image coordinates, respectively; p_z is the z-parallax ($p_z = z' - z''$); and p_x is the x-parallax ($p_x = x' - x''$).

Similarly, the X and Z coordinates can be found from

$$X = Y\,\frac{x'}{c} \tag{2}$$

$$Z = Y\,\frac{z'}{c} \tag{3}$$

It is noticed that the Y coordinates can be determined either from the x' and x'' or z' and z'' image coordinates. The same equations can be applied in the case of using the second procedure (face to face), but x'' and z'' should have negative signs.

ERROR ANALYSIS

The fundamental error expressions can be derived from Equations 1, 2, and 3. Assuming small errors db_2, dx', dx'', and dc in the determination of b_2, x', x'', and c, respectively, the differential form is obtained for the Y and X coordinates of any point n to be

$$dY_x = \frac{Y^2Y_0}{b_2cX}dx' - \frac{YY_0^2}{b_2cX}dx'' + \frac{Y}{b_2}db_2 \tag{4}$$

and

$$dX = \frac{Y^2}{b_2c}dx' - \frac{Y_0^2}{b_2c}dx'' + \frac{X}{b_2}db_2 - \frac{X}{c}dc \tag{5}$$

where $Y_0 = Y + b_2$.

In the present technique, the X and Z coordinates are functions of the dimensions of the cross section of the passage, which are usually constant. Hence, the error propagation for depth determination along the Y axis is of prime importance.

The true errors of the photo-coordinates dx' and dx'' and of the base measurement db_2 are actually not known, but generally an estimate of the standard deviation of these values can be made by studying the different sources of errors. Then the variance $m^2_{Y_x}$ of the Y coordinates can be obtained by the law of error propagation; i.e.,

$$m^2_{Y_x} = \frac{Y^4Y_0^2}{b_2^2c^2X^2}m^2_{x'} + \frac{Y^2Y_0^4}{b_2^2c^2X^2}m^2_{x''} + \frac{Y^2}{b_2^2}m^2_b. \tag{6}$$

Similarly, if z' and z'' image coordinates are used,

$$m^2_{Y_z} = \frac{Y^4Y_0^2}{b_2^2c^2Z^2}m^2_{z'} + \frac{Y^2Y_0^4}{b_2^2c^2Z^2}m^2_{z''} + \frac{Y^2}{b_2^2}m^2_b \tag{7}$$

The variances of the image coordinates are contributed by errors in measuring image coordinates and by errors of the elements of inner and outer orientation. The differential formulas for the influence of these errors on the image coordinates are known from literature. For the first photograph, dc represents the error of the camera constant c and dx'_i is the error of the determination of the position of the principal point from the fiducial marks in the photograph; dx'_r and dz'_r are the translations of the principal point due to rotations of the negative about the z' and x' axes in the photograph; db_{X_1} and db_{Y_1} are the translations of the first camera in the X and Y directions; and $d\phi_1$, $d\omega_1$, and $d\kappa_1$ are small rotations about the Z, X, and Y axes, respectively, through the perspective center.

The differential formula for the influence of the assumed errors of inner and outer orientation on the image coordinate x' is then

$$dx' = -dx'_t - \frac{x'^2}{c^2}\,dx'_r - \frac{x'z'}{c^2}\,dz'_r + \frac{x'}{c}\,dc - \frac{c}{Y}dbX_1$$
$$+ \frac{x'}{Y}dbY_1 - \left(1 + \frac{x'^2}{c^2}\right)cd\phi_1 - \frac{x'z'}{c}d\omega_1 + z'd\kappa_1 \tag{8}$$

A similar equation can be written for the error dx'' in the image coordinate of the second photograph; i.e.,

$$dx'' = -dx''_t - \frac{x''^2}{c^2}\,dx''_r - \frac{x''z''}{c^2}\ dz''_r$$
$$+ \frac{x''}{c}\,dc - \frac{c}{Y_0}\,dbX_2 + \frac{x''}{Y_0}dbY_2$$
$$- \left(1 + \frac{x''^2}{c^2}\right)cd\phi_2 - \frac{x''z''}{c}\ d\omega_2 + z''d\kappa_2. \tag{9}$$

Using the relations

$$x' = \frac{X}{Y}c, \tag{10}$$

$$z' = \frac{Z}{Y}c, \tag{11}$$

$$x'' = \frac{X}{Y_0}c, \text{ and} \tag{12}$$

$$z'' = \frac{Z}{Y_0}c, \tag{13}$$

Equations 8 and 9 become

$$dx' = -dx'_t - \frac{X^2}{Y^2}\,dx'_r - \frac{XZ}{Y^2}\,dz'_r + \frac{X}{Y}\,dc - \frac{c}{Y}dbX_1$$
$$+ \frac{Xc}{Y^2}dbY_1 - \left(1 + \frac{X^2}{Y^2}\right)cd\phi_1 - \frac{XZ}{Y^2}\,cd\omega + \frac{Z}{Y}cd\kappa_1 \tag{14}$$

and

$$dx'' = -dx''_t - \frac{X^2}{Y_0^2}\,dx''_r - \frac{XZ}{Y_0^2}\,dz''_r + \frac{X}{Y_0}\,dc - \frac{c}{Y_0}dbX_2$$
$$+ \frac{Xc}{Y_0^2}dbY_2 - \left(1 + \frac{X^2}{Y_0^2}\right)cd\phi_2 - \frac{XZ}{Y_0^2}cd\omega_2 + \frac{Z}{Y_0}\,cd\kappa_2 \tag{15}$$

The true errors of the elements of inner and outer orientation cannot be known, but usually their standard errors may be estimated statistically. For example, it is possible in terrestrial photogrammetry to determine the standard error of the measurements of the elements of orientation from the repeated measurements of these elements. Accordingly, it can be assumed that the measurements of the different elements of orientation are free from mutual correlation.

Assume that the standard errors of the elements of orientation are equal at both stations. Then the variance of the image coordinates can be obtained from Equations 14 and 15 as

$$m_{x'}^2 = m_{x_t}^2 + \frac{X^4}{Y^4}\,m_{x_r}^2 + \frac{X^2Z^2}{Y^4}\ m_{z_r}^2$$
$$+ \frac{X^2}{Y^2}\,m_c^2 + \frac{c^2}{Y^2}\,m_{bX}^2\,\frac{X^2c^2}{Y^4}\,m_{bY}^2$$
$$+ \left(1 + \frac{X^2}{Y^2}\right)^2 c^2m_\phi^2 + \frac{X^2Z^2}{Y^4}\ c^2m_\omega^2 +$$
$$+ \frac{Z^2}{Y^2}\ c^2m_\kappa^2 \tag{16}$$

and

$$m_{x''}^2 - m_{x_t}^2 + \frac{X^4}{Y_0^4}\,m_{x_r}^2 + \frac{X^2Z^2}{Y_0^4}\ m_{z_r}^2$$
$$+ \frac{X^2}{Y_0^2}\,m_c^2 + \frac{c^2}{Y_0^2}\,m_{bX}^2 + \frac{X^2c^2}{Y_0^4}\ m_{bY}^2$$
$$+ \left(1 + \frac{X^2}{Y_0^2}\right) c^2m_\phi^2 + \frac{X^2Z^2}{Y_0^4}\ c^2m_\omega^2 +$$
$$+ \frac{Z^2}{Y_0^2}\ c^2m_\kappa^2 \tag{17}$$

In order to investigate the influence of errors in image coordinates upon coordinate determination, Equations 16 and 17 are evaluated at the average conditions of the specific photogrammetric application under consideration. From past experience, the elements of inner orientation of a metric camera can be determined with a standard error of a few micrometres. For further investigation, a reasonable value of ± 5 μm is taken as a standard error for each of the elements of inner orientation, and the camera constant is taken as 16 cm.

The base length in this kind of application is not very long, usually not exceeding a few metres. Accordingly, it can be measured with a steel tape to an accuracy of ± 1 mm, i.e., $m_{bX} = m_{bY} = \pm 1$ mm. The rotation about the Z axis is done by using the telescope of the theodolite, if a phototheodolite is used, or by an auxiliary telescope, if a terrestrial camera is used. Experiments indicated that an average standard error m_ϕ of ± 5 seconds of arc can be obtained by a normal operator. The adjustment of the rotation about the X and Y axes may be done using spirit levels with an average sensitivity of 10 seconds of arc. Therefore, a reasonable assumption of ± 10 seconds of arc for m_ω and m_κ is acceptable for further investigations.

Now consider an average case of a corridor 4-m wide and 3-m high. If the camera axis is close to a side wall of the corridor and the base was chosen to be 10 m, an object on the top corner of the wall at a distance from the first camera station of 10 m will have the coordinates $X = 4$m, $Y = 10$m, $Z = 1.5$ m, and $Y_0 = Y + b = 20$ m.

Substituting these values in Equations 16 and 17, the standard errors of image coordinates are calculated to be

$$m_{x'} = 12\ \mu\text{m and } m_{x''} = 9\ \mu\text{m}$$

It should be noted that the only orientation elements that have a major influence on the accuracy determination of image coordinates are dx_t and db_x. The influence of the other factors is negligible. Fortunately, in the parallel case of photogrammetry b_x vanishes; hence, db_x can be minimized by accurate plumbing of the camera above the ground stations.

The accuracy of measuring the image coordinates should be added in order to obtain the total influence upon coordinate determination. If the measurements are conducted with a standard deviation $m_{x'(\text{measure})}$ of $\pm 10\ \mu$m, the total $m_{x'}$ and $m_{x''}$ will be equal to $\pm 15\ \mu$m and $\pm 12\ \mu$m, respectively. These values are used in Equations 6 and 7 to compute the variance of the Y coordinates.

It should be mentioned that the computed standard error represents the estimated absolute accuracy of the Y coordinate determination. Much better accuracy can be obtained if only the coordinate differences are to be determined. The influence of the errors of the inner orientation in that case will be negligible.

PRACTICAL APPLICATION

The present technique was used to survey the 4.5-m wide air conditioning tunnel at the University of Petroleum and Minerals (UPM) in Dhahran. The pipelines occupy the middle of the tunnel and run along its centerline. Hence, it was possible to occupy only the two 0.5-m wide sidewalks along and adjacent to the walls of the tunnel. The possibility of using the normal stereophotogrammetric case was investigated by calculating the variance of the Y determination. In such a case the Y coordinate is determined from

$$Y = \frac{cb_1}{p_x} \tag{18}$$

where $p_x = x' - x''$ is the x-parallax.

Assuming small errors db_1, dp_x, and dc in the resolution of b_1, p_x, and c, respectively, the differential form is obtained from Equation 18; i.e.,

$$dY = \frac{Y}{b_1}db_1 + \frac{Y}{c}dc - \frac{Y^2}{b_1c}dp_x. \tag{19}$$

Furthermore, the variance of the Y determination is obtained from Equation 19, assuming independent determination of b_1, c, and the image coordinates

$$m_Y^2 = \frac{Y^2}{b_1^2}m_{b_1}^2 + \frac{Y^2}{c^2}m_c^2 + \frac{Y^4}{b_1^2c^2}m_{p_x}^2 \tag{20}$$

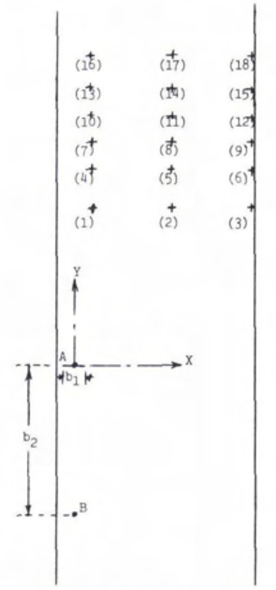

FIG. 2. Arrangement of the test area (plan view).

where

$$m_{p_x}^2 = m_{x'}^2 + m_{x''}^2$$

With consideration to the conditions under discussion, the variance of Y is evaluated at $b_1 = 0.5$ m, $X = 4$ m, $Y = 10$ m, and $c = 16$ cm, and with standard deviations $m_{x'} = m_{x''} = 15\ \mu$m and $m_c = 5\ \mu$m. Equation 20 yields

$$m_Y = \pm 33.2\ \text{mm}.$$

To apply the proposed arrangement, Equation 6 is evaluated for the same point, i.e., at $Y = 10$ m and $X = 5$ m, and for the same camera constant. In this case, the base b_2 is chosen to be 10 m; hence, $m_{x''}$ is calculated to be 12 μm. Using these values, m_{Y_x} is calculated from Equation 6 to be only ± 5.5 mm, with an improvement of about six times from the previously obtained value of ± 33.2 mm.

In order to investigate the practical accuracy of the proposed technique, 18 targets, distributed on the roof and the walls of a 10-m long section of the above mentioned UPM tunnel, were surveyed with three methods as shown in Figure 2. In the first one, direct measurements were conducted using calibrated steel tapes and one-second theodolites. The results of this method are taken as a reference for the two photogrammetric techniques. In the second method the normal stereophotogrammetric case was employed at station A, 10-m away from the first target in the section, so that $b_1 = 0.5$ m as shown in the figure. In the third method the proposed parallel case of photogrammetry was used at

TABLE 1. ACCURACY OF COORDINATE DETERMINATION OF THE TEST AREA

Target	Coordinates: Direct Measurements (m) X	Z	Y	Standard Deviation of Parallel Case (mm) m_x	m_z	m_y	Y-Photo Determination: Normal case Y_1	Parallel case Y_2	$Y_1 - Y$ (mm)	$Y_2 - Y$ (mm)
1	0.518	2.542	10.254	3.0	3.2	6.9	10.266	10.261	+12	+7
2	2.506	2.539	10.258	4.6	2.1	7.1	10.291	10.246	+33	−12
3	4.521	0.043	10.252	3.4	2.4	5.2	10.210	10.256	−42	+4
4	0.501	2.523	12.315	2.8	2.5	8.1	12.284	12.305	+31	−10
5	2.524	2.540	12.318	4.2	2.5	8.1	12.276	12.304	+42	−14
6	4.535	0.161	12.320	2.6	2.7	6.0	12.370	12.314	−50	−6
7	0.510	2.518	14.208	3.1	3.9	8.2	14.243	14.215	+35	+7
8	2.528	2.500	14.208	3.2	3.7	7.4	14.181	14.196	−27	−12
9	4.541	0.037	14.212	3.1	2.9	5.8	14.156	14.218	−56	+6
10	0.511	2.517	16.076	3.2	3.0	7.6	16.007	16.091	−69	+15
11	2.545	2.501	16.071	3.7	3.6	8.7	16.134	16.091	+63	+20
12	4.535	0.034	16.080	5.5	4.0	8.4	16.034	16.070	−46	−10
13	0.546	2.523	18.005	6.5	8.0	10.8	18.051	18.990	+46	−15
14	2.530	2.524	18.001	7.1	7.4	11.6	18.060	18.009	+59	+8
15	4.523	0.105	18.016	8.2	6.0	11.8	18.064	18.026	+68	+10
16	0.538	2.511	20.054	4.2	5.8	14.6	19.969	20.045	−85	−9
17	2.542	2.529	20.059	3.2	5.6	13.2	19.970	20.046	−89	−13
18	4.530	0.022	20.074	7.4	4.3	12.7	20.150	20.085	+76	+11
									m = 55	m = 11

stations A and B along the tunnel so that $b_2 = 10$ m. The Wild Terrestrial Camera P-31 with $c = 100$ mm was used in the two photogrammetric methods, and the Y coordinates were computed to be compared with the direct method. Because the Y coordinates in the parallel case can be determined either from the x' and x'' or z' and z'' image coordinates, it was decided, according to Equations 6 and 7, that the more reliable value may be obtained from the larger of these values. A large disagreement in Y values indicates a mistake in measurement or computation. The study was based on computing the standard deviation of the photogrammetric determination of the 18 targets. The coordinate system is chosen as shown in the figure and its origin is at point A. Table 1 shows the obtained results. The location of the different targets is given in the table by the three-dimensional coordinates X, Z, and Y of each target.

Comparing the average standard deviation, it is obvious that there is an appreciable improvement in the accuracy of the Y determination. It is also noted that the accuracy in the parallel case depends on the values of X or Z, which is not the case with the normal case. Therefore, it is recommended that Equations 6, 7, and 20 be used to analyze the expected accuracy for each specific situation before deciding which case is to be applied.

In the new technique, it is obvious that the coordinates cannot be computed for objects situated along the camera axis. In most cases this should not present any difficulty, as the photographed objects are not usually located along the core of the narrow transits.

Conventional methods that use intersections from two theodolite stations may be used if the number of the points to be surveyed is small and if time and space allow. If the object points are numerous or when long tunnels are to be surveyed by succeeding photographs, the use of such point-by-point techniques is very time consuming and economically unfeasible.

CONCLUSION

A parallel case of photogrammetry, where the camera axes coincide with the base, is proposed when there is not enough space to establish a stereoscopic base for the normal case of photogrammetry. Practical experiments and rigorous error analysis show that the accuracy may increase tenfold when applying the proposed technique.

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References

Hallert, B., 1960. *Photogrammetry*, McGraw-Hill Book Co., Inc., New York.

Hottier, P. H., 1976. Accuracy of Close-Range Analytical Restitutions: Practical Experiments and Prediction, *Photogrammetric Engineering and Remote Sensing*, Vol. 42, No. 3, pp. 345-375.

Karara, H. M., 1967. Universal Stereometric Systems, *Photogrammetric Engineering*, Vol. 33, No. 11.

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