

The Reliability of Block Triangulation

Theoretical studies on the reliability of photogrammetric blocks give clear guidelines for project planning, leading to coordinates of high quality.

INTRODUCTION

DURING THE LAST ten years aerial triangulation has become a powerful tool for point determination. The main reason is the rigorous application of adjustment theory, which enabled a simultaneous orientation of images and thus increased the accuracy by an order of magnitude. The refinement of the mathematical model based on this development, with the aim to compensate systematic errors, lead to a further increase in the accuracy by a factor of two to three. Today one can reach a precision of the adjusted coordinates of 2 to 3 μm , measured at the photoscale, if the full potential is used to advantage. This result has been confirmed by various controlled tests.

It is at the same time pleasing and amazing that

"cleaned" a block, there does not exist an objective and commonly accepted criterion about when to stop the process of elimination of possibly erroneous observations. Thus, undetected gross errors may still remain which it is hoped do not deteriorate the result too much. This immediately leads to the reliability of adjusted coordinates being the intrinsic problem of point determination.

Two tasks have to be solved:

- One needs methods for the detection and elimination of gross errors. Automatic procedures have to take into consideration the different types of gross errors and, thus, have to be able also to handle large gross errors. They therefore cannot be reduced to the application of a statistical test. The development of efficient strategies seems to con-

ABSTRACT: The theory of reliability treats the ability to detect gross errors by using statistical tests and the sensitivity of the result with respect to non-detectable gross errors. The theory developed by W. Baarda, Delft, for use in geodetic networks is outlined. In an extensive investigation it was applied to photogrammetric point determination. The study results in clear recommendations for project planning, consisting in the appropriate choice of the block parameters as overlap, and in the distribution of control and tie points, which leads to a homogeneous precision as well as to a high reliability of block triangulation.

these accuracies have also been achieved in normal application, as the theoretical studies on the precision of block triangulation were based on very simplified assumptions about the stochastic properties of the image or model coordinates. It is true that the discrepancies between theoretical predictions and empirical results have pushed forward the development of methods for compensation of systematic errors and have lead to a deeper insight into the powerful tool of self calibration. But the effects of unmodeled errors, especially gross errors, on the adjustment of photogrammetric blocks had not been studied thoroughly until a few years ago.

However, each block adjustment has to cope with a certain percentage of gross errors, which in general are found and eliminated by an analysis of the residuals. But, as known to everybody who has once

verge to a three-step procedure treating large, medium-sized, and small gross errors by pre-error detection, automatic weighting, and statistical procedures, respectively (*cf.* the papers presented at the Commission III Symposium of ISP, 1982, in Helsinki and at the ISP Congress in Rio de Janeiro, 1984).

- The detectability of gross errors and the influence of non-detectable gross errors on the result of the block adjustment, i.e., the reliability according to Baarda (1967, 1968, 1973, 1976), has to be investigated with respect to the project planning. Here the type of test for detecting very small gross errors will have a large influence and will require a statistical description of reliability.

This paper is supposed to motivate and describe the concept of reliability. Based on the result of comprehensive studies, recommendations for the

project planning are given, which complete the known methods for improving the precision of photogrammetric coordinates.

THE CONCEPT OF RELIABILITY

The theory of reliability is part of a concept for evaluating the quality of adjustment results, which was developed by W. Baarda (1967, 1968, 1973, 1976) for use in geodetic networks. The notion of quality, according to Baarda, includes precision and reliability. Figure 1 shows the interrelations between the different parts of the theory.

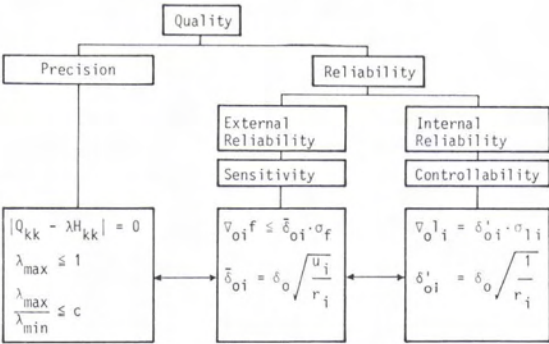


FIG. 1. Evaluation of quality according to W. Baarda (1967, 1968, 1973, 1976).

The evaluation of precision consists in comparing the covariance matrix Q_{kk} of the adjusted coordinates with a given matrix H_{kk} , also called criterion matrix. It is required that the error ellipsoid described by Q_{kk} lies inside the error ellipsoid described by H_{kk} and that it is as similar to H_{kk} as possible. This results in a check as to whether a required measure of precision is reached.

We are only concerned here with reliability. Baarda distinguishes internal and external reliability. *Internal reliability* is the controllability of the observations, described by lower bounds for gross errors, which can just be detected with a given probability. The effect of non-detectable gross errors on the result is described by factors for the standard errors of the coordinates, which indicate by what amount the coordinates may be deteriorated in the worst case. These factors describe the *external reliability* or the sensitivity of the unknown coordinates. Obviously, there is a close connection to the evaluation of precision.

Remark: Baarda himself used the notion "accuracy" for describing precision and reliability. As in photogrammetric applications, the notion accuracy is too much associated with the specification of estimated standard deviations; quality seems to be a more proper term for also describing non-detectable errors in the mathematical model and their effects.

Before a mathematical definition of reliability is given, three examples show that the conditions for

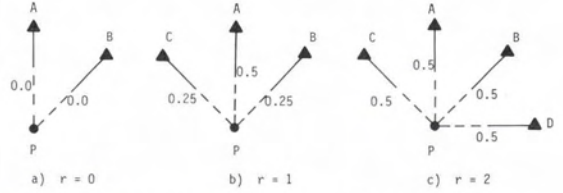


FIG. 2. Forward intersections with different redundancy, good geometry, and redundancy numbers. In all directions of Figure 2c, errors are locatable.

good reliability are closely related to the local geometry. Figure 2 shows network diagrams of forward intersections, which differ by one observation each. They are to demonstrate the influence of additional observations on the strength of the geometry. In Figure 2a the observations are not controllable, i.e., they are necessary for the determination of the coordinates. In Figure 2b each observation is controllable, i.e., it is not necessary for the determination of the coordinates. On the other hand, errors in the observation are not locatable or identifiable, i.e., each is necessary for the control of the others. In Figure 2c, finally, errors in the observations are locatable, i.e., each is not necessary for the control of the others. Any further observation would be superfluous for the detection of a single gross error. The increase in the number of rays goes along with an increase in the strength of the geometry, which may be described by the sequence of the common terms: determination-control-location or identification.

Obviously the network design in Figure 2c is chosen appropriately to reach a reliable determination of the coordinates. This is different in Figures 3 and 4.

In Figure 3a the determination of point P is weak; therefore, the direction D in Figure 3b is not controllable. The directions A and B control each other. As a consequence, gross errors in directions C and D in Figure 3c and in all directions of Figure 4 practically cannot be located. Any additional direction in Figure 4c cutting the existing directions with an angle of about 45° enables a location of a gross error in any of the directions.

As these configurations may occur also in net-

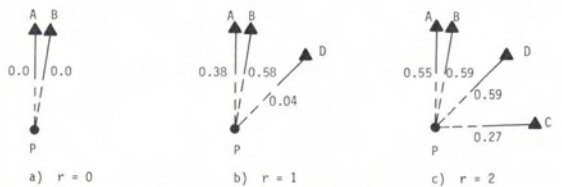


FIG. 3. Forward intersections with different redundancy, poor geometry, and redundancy numbers. Errors in directions C and D in Figure 3c not locatable.

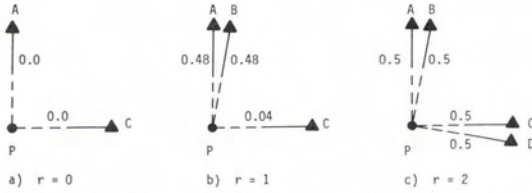


FIG. 4. Cf. Figure 3, but errors in directions A, B, C, and D in Figure 4c are not locatable.

works with high redundancy, it is obvious that the local geometry is decisive for the reliability of the point determination. Such situations also arise in bundle blocks with 20 percent sidelap where the points in the middle of the strips are measured only in three images. The x-coordinates (x parallel to direction of flight) are controllable here, but possibly wrong image rays cannot be identified (cf. geometry of Figure 2b).

This discussion of the geometric properties of a network design can be rendered precise using the information that adjustment theory and mathematical statistics offer.

Notation: Scalars and vectors are written in small letters, matrices in capital letters, A' is the transpose of A , stochastic values are underscored, \hat{x} and $\hat{\nabla}l$ are estimates for x and ∇l , respectively.

Let the block adjustment be given by the linearized error equations

$$l + \underline{v} = A \hat{x} + a_0 \quad P_{11} \quad (1)$$

with the vector $l = (l_i)$ containing the observations l_i , the corresponding vector $\underline{v} = (v_i)$ of the residuals, the error equation or design matrix A with the error equation vectors a'_i , the estimated vector \hat{x} of the unknown parameters (coordinates, transformation parameters, and, possibly, additional parameters), and a constant vector a_0 , resulting from the linearization. The weight matrix P_{ll} , being the inverse of the weight-coefficient matrix Q_{ll} , is supposed to be known.

From the solution $\hat{x} = (A' P_{ll} A)^{-1} A' P_{ll} (l - a_0)$ and the weight coefficient matrix

$$Q_{vv} = Q_{ll} - A (A' P_{ll} A)^{-1} A' \quad (2)$$

of the residuals \underline{v} (which are used for error detection), the direct relationship between residuals and observations can be derived:

$$\underline{v} = - Q_{vv} P_{ll} (l - a_0). \quad (3)$$

The matrix $Q_{vv} P_{ll}$ is idempotent, i.e., $(Q_{vv} P_{ll})^2 = Q_{vv} P_{ll}$. Using the eigenvalue decomposition of $Q_{vv} P_{ll}$, it can be shown that the rank and the trace of $Q_{vv} P_{ll}$ are equal to the redundancy $r = n - u$ of the system: i.e.,

$$\text{rk} (Q_{vv} P_{ll}) = \text{tr} (Q_{vv} P_{ll}) = \sum_{i=1}^n (Q_{vv} P_{ll})_{ii} = r \quad (4)$$

The diagonal elements $(Q_{vv} P_{ll})_{ii}$ obviously show the distribution of the redundancy onto the observations. The redundancy number

$$r_i = \overset{\text{def}}{(Q_{vv} P_{ll})_{ii}} \quad (5)$$

is the contribution of l_i to the total redundancy r (cf. Förstner, 1979).

The redundancy numbers range from 0 to 1. Observations with $r_i = 1$ are fully controllable, whereas observations with $r_i = 0$ cannot be checked at all. The average value, the relative redundancy $\bar{r} = r/n$, for photogrammetric blocks is about 0.2 to 0.5. An average value of 0.5 already indicates a rather stable block. Single redundancy numbers, however, easily reach values below 0.1, indicating a very weak local geometry.

Using the r_i , we are now able to calculate the influence ∇v_i of a single gross error ∇l_i in the observation l_i on the corresponding residual v_i : i.e.,

$$\nabla v_i = - r_i \nabla l_i \quad (6)$$

(∇ designates "gross error" or, more generally, a deviation from the assumed mathematical model).

Equation 6 shows that only a small part (from about 50 percent down to less than 10 percent) of the original gross error ∇l_i is revealed in the residual v_i . However, only in case the diagonal element $(Q_{vv} P_{ll})_{ii} = r_i$ of $Q_{vv} P_{ll}$ (cf. equation 2) is larger than all other elements of the i -th column, a gross error in l_i will influence v_i more than the other residuals, i.e., only in that case can ∇l_i be expected to be locatable using the largest residual as indicator.

Equation 6, furthermore, can be used in practical error detection procedures. With the knowledge of the local geometry, i.e., with r_i , one can obtain an estimate $\hat{\nabla}l_i$ for the size of the original gross error ∇l_i : i.e.,

$$\hat{\nabla}l_i = - v_i / r_i. \quad (7)$$

This simplifies the evaluation of the residuals. It can be shown that the value $\hat{\nabla}l_i$ equals the residual in an adjustment without l_i , i.e., in an adjustment with $p_i = 0$. The spatial distribution of the redundancy onto the block can be described using the redundancy numbers and gives a first insight into the controllability of the observations. Figures 2 to 4 show the values r_i , which demonstrate that the visual evaluation of the design quality is confirmed by the numerical values (cf. also Kavouras, 1982).

Remark: The redundancy numbers r_i do not give any information about the ability to identify or locate gross errors. In general, however, gross errors in an observation are locatable if l_i is not necessary for the control of the other observations. In this case all correlation coefficients ρ_{ij} of v_i and v_j ($j \neq i$) are $\neq \pm 1$.

Testing the observations based on the residuals has to consider the different precision of the residuals. With the standard deviation

$$\sigma_{v_i} = \sigma_0 \sqrt{Q_{v_i v_i}}^L = \sigma_{l_i} \sqrt{r_i} \quad (8)$$

(^L provided $\mathbf{P} = \text{diag}(p_i)$)

of the i -th residual v_i , we obtain the *standardized residual*

$$\underline{w}_i = \frac{-v_i}{\sigma_{v_i}} = \frac{\widehat{\nabla l_i}}{\sigma_{\widehat{\nabla l_i}}} = \frac{-v_i \sqrt{p_i}}{\sigma_0 \sqrt{r_i}} = \frac{\widehat{\nabla l_i} \sqrt{r_i}}{\sigma_{l_i}} \sim N(0,1) \quad (9)$$

which is used as the test statistic. It is at the same time a test of the estimated size $\widehat{\nabla l_i}$ of the gross error (cf. Equation 7)! Here σ_0 is assumed to be known. Moreover, if it is assumed that the observations are normally distributed, then the test statistic \underline{w}_i also is normally distributed with expectation 0 and variance 1.

Remark: If there is no *a priori* information about the precision of the observations and, thus, if σ_0 is not known, then instead of \underline{w}_i , the test statistic

$$\overline{w}_i = \frac{-v_i}{\hat{\sigma}_{0i} \sqrt{Q_{v_i v_i}}} = \frac{-v_i \sqrt{p_i}}{\hat{\sigma}_{0i} \sqrt{r_i}} \sim t(r-1) \quad (10)$$

can be used, which follows a student distribution with $r - 1$ degrees of freedom. The variance

$$\hat{\sigma}_{0i}^2 = \frac{[\underline{v} \mathbf{P} \underline{v}] - \underline{v}_i^2 p_i / r_i}{r - 1} \quad (11)$$

is an estimate for σ_0^2 and is identical to $\hat{\sigma}_0^2$ in an adjustment without observation l_i (cf. Förstner, 1980, Equation 12). This test statistic \overline{w}_i is functionally dependent on the one given by Pope (1975), which is τ -distributed (cf. Grün, 1982, Equation 22b). \overline{w}_i , however, is as well suited as \underline{w}_i for the following derivations, as in both cases the non-central distribution is known.

The *test of the observations*, the “*data-snooping*” proposed by Baarda, now consists in comparing the absolute value $|\underline{w}_i|$ of the test statistic with the *critical value* k , which depends on the present significance level $S = 1 - \alpha_0$. If the standardized residual exceeds the critical value k , the corresponding ob-

servation l_i is suspected to be erroneous. In a practical procedure one of course will only check the observations with the largest test statistics and take into consideration the interrelation between the residuals, as a single large gross error in general will lead to many test statistics which exceed the critical value. As can be seen from Equation 9, a large \underline{w}_i also can be caused by a wrong weight. Therefore, \underline{w}_i not necessarily must be rejected, if only being a little larger than the critical value k .

The normal distribution—at least in principal—allows deviations of any size from the mean value. The probability that the test statistic exceeds the critical value, if the observations are not erroneous, and therefore leads to an *erroneous decision of type I*, is the significance number α_0 , which usually is chosen small (e.g., 5 percent, 1 percent, or 0.1 percent).

On the other hand, gross errors may stay undetected. The power β_i of the test, i.e., the probability of detecting a gross error, and the probability for this erroneous decision, an *error of type II*, depends on the size ∇l_i . The gross error ∇l_i changes the test statistic \underline{w}_i by $\delta_i = \nabla w_i$, i.e., ∇l_i shifts the probability density function of \underline{w}_i by δ_i , thus leading to a non-central distribution (cf. Figure 5) with non-centrality parameter δ_i .

Because the size of gross errors is unknown, Baarda proposed requiring a minimum probability β_0 to detect a gross error, i.e., starting with a minimum power β_0 of the test and determining the lower bound $\nabla_0 l_i$ for a gross error in the observation l_i which can be detected with a probability $\beta > \beta_0$. As can be seen from Figure 5, a given lower bound β_0 leads to a lower bound $\delta_0 = \nabla_0 w_i$ for the non-centrality parameter. We will use $\delta_0 = 4$, for simplicity, which corresponds to a significance level $1 - \alpha_0 = 99$ percent and a probability $\beta_0 = 93$ percent for error detection. Table 1 shows the dependency of β_0 on the critical value k for a given $\delta_0 = 4$. In accordance with experience, gross errors can be found more easily, i.e., β increases as smaller critical values k are chosen.

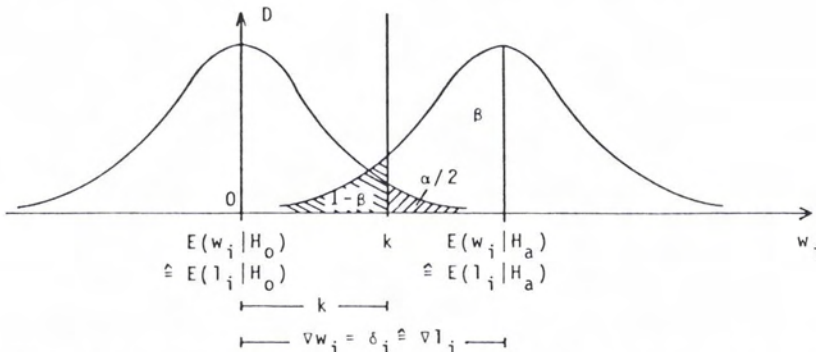


FIG. 5. Probabilities α and $1 - \beta$ of wrong decisions of type I and II when using data-snooping.

TABLE 1. SIGNIFICANCE LEVEL $S = 1 - \alpha_0$, CRITICAL VALUE k , AND POWER β_0 OF TEST FOR GIVEN NON-CENTRALITY PARAMETER $\delta_0 = 4$

$S = 1 - \alpha_0$	k	β
99.9%	3.29	76%
99.7%	3.00	84%
99.0%	2.56	93%
95.0%	1.96	98%

From Equation 9 now the lower bound $\nabla_0 l_i$ for gross errors, which are detectable with a probability $> \beta_0$, can be derived:

$$\nabla_0 l_i = \delta_{0i} \cdot \sigma_{li}; \quad \delta_{0i}' = \delta_0 \sqrt{\frac{1}{r_i}} \tag{12}$$

The value δ_{0i} is the factor for σ_{li} giving a minimum size $\nabla_0 l_i$ of a just detectable gross error in l_i . The lower bounds $\nabla_0 l_i$ or the factors δ_{0i} designate the *controllability* of the observations or the *internal reliability* according to Baarda. They essentially depend on the redundancy numbers r_i , i.e., on the local geometry. Redundancy numbers between 0.1 and 0.5 lead to lower bounds for detectable gross errors between $\nabla_0 l_i = 6 \sigma_{li}$ and $\nabla_0 l_i = 13 \sigma_{li}$ or to controllability factors between $\delta_{0i}' = 6$ and $\delta_{0i}' = 13$. Thus, gross errors much larger than the three-fold standard deviation may stay undetected and may falsify the result.

Remark: A similar line of thought leads to the notion of locatability or identifiability, describing the ability to correctly locate or identify gross errors. Requirements for a high locatability analogously lead to requirements for the correlation coefficients of the residuals. This aspect will not be treated here (cf. Förstner, 1983).

All not-detected gross errors contaminate the result of the adjustment. The influence $\nabla_0 x_j$ of a gross error of size $\nabla_0 l_i$ on the unknown \hat{x} can directly be obtained using $\hat{x} = (A' P_{ll} A)^{-1} A' P_{ll} (l - a_0)$:

$$\nabla_0 x_j = ((A' P_{ll} A)^{-1} A' P_{ll})_{ji} \nabla_0 l_i \tag{13}$$

The values $\nabla_0 x_j$ give conspicuous insight into the sensitivity of the result and may be useful in small systems. There are, however, several reasons to use a different measure:

- The calculation of all $n \times u$ values $\nabla_0 x_j$ is prohibitive in large blocks.
- In most cases the influence of non-detectable gross errors on the orientation parameters or even on additional parameters is of no interest.
- In free blocks, without any control points, the influence values $\nabla_0 x_j$ depend on the coordinate system.

Therefore, Baarda proposed using the standardized length $\bar{\delta}_{0i}$ of the vector $\nabla_0 k$, a subvector of $\nabla_0 x$, containing the influence on the coordinates \underline{k} of the new points

$$\bar{\delta}_{0i} = \|\nabla_0 k\| = \sqrt{\nabla_0 k' Q_{kk}^{-1} \nabla_0 k} / \sigma_0. \tag{14}$$

It is a measure for the total deformation of the block, caused by a gross error $\nabla_0 l_i$ in observation l_i . This seemingly abstract measure for the deformation gives at the same time an upper bound $\nabla_0 f$ for the influence of gross errors $\nabla l_i < \nabla_0 l_i$ on an arbitrary function $f = f(\underline{k})$ of the coordinates \underline{k} .

Using Cauchy-Schwarz's identity, it can be shown that, with the standard deviation σ_f of f , the influence is bounded: i.e.,

$$\nabla_0 f \leq \bar{\delta}_{0i} \cdot \sigma_f \tag{15}$$

For the special functions $f = x_j, f = v_j, f = z_j$ one obtains

$$\begin{aligned} \nabla_0 x_j &\leq \bar{\delta}_{0i} \cdot \sigma_{x_j}, & \nabla_0 f \nabla_0 y_j &\leq \bar{\delta}_{0i} \cdot \sigma_{y_j}, \\ & & \text{and } \nabla_0 z_j &\leq \bar{\delta}_{0i} \cdot \sigma_{z_j} \end{aligned} \tag{16}$$

Thus, the coordinates x, y , and z are not contaminated by more than $\bar{\delta}_{0i}$ times their standard deviation. The factors $\bar{\delta}_{0i}$ describe the *sensitivity* of the result or the *external reliability* according to Baarda. A practical formula for calculating the values δ_{0i} is given by Klein/Förstner in Seminar (1981) (cf. also Schwarz *et al.*, 1982).

THE THEORETICAL RELIABILITY OF PHOTOGRAMMETRIC BLOCKS

The previous section has provided three measures for describing the reliability of an adjustment, which are based on the theory of Baarda (1967, 1968, 1976):

- The *redundancy number* r_i (cf. Equation 5) is the contribution of observation l_i to the total redundancy r , as $\Sigma r_i = r$. They enable the calculation of the influence ∇v_i of a gross error ∇l_i in observation l_i on the corresponding residual v_i : $\nabla v_i = -r_i \nabla l_i$ (cf. Equation (6)).
- The *controllability factor* δ_{0i}' (cf. Equation 12) is the factor for the standard deviation σ_{li} giving a lower bound $\nabla_0 l_i = \delta_{0i}' \sigma_{li}$ for just detectable gross errors (cf. Equation 12) in l_i . Gross errors in observation l_i which are smaller than the δ_{0i}' -fold standard deviation σ_{li} , cannot be detected by a statistical test, e.g., the "data-snooping"-test (Equation 9).

Both r_i and δ_{0i}' describe the *internal reliability* according to Baarda.

- The *sensitivity factor* $\bar{\delta}_{0i}$ (cf. Equation 14) is the factor for the standard deviation σ_0 of an arbitrary unknown x giving an upper bound $\nabla_0 x \leq \bar{\delta}_{0i} \cdot \sigma_0$ for the influence of nondetectable gross errors. Thus, the adjusted coordinates x, y , and z are not contaminated by more than $\bar{\delta}_{0i}$ times their standard deviation, if a statistical test has been applied.

The values $\bar{\delta}_{0i}$ describe the *external reliability* according to Baarda.

Table 2 gives an indication how to evaluate the different reliability measures.

On the basis of the reliability theory described in the previous section, several photogrammetric blocks were investigated at the Institute of Photo-

TABLE 2. ON THE EVALUATION OF RELIABILITY MEASURES: r_i , REDUNDANCY NUMBER; δ'_{0i} , CONTROLLABILITY FACTOR; $\bar{\delta}_{0i}$, SENSITIVITY FACTOR

	Good	Acceptable	Bad	Not Acceptable
r_i	$r_i > 0.5$	$0.1 \leq r_i < 0.5$	$0.04 < r_i < 0.1$	$r_i \leq 0.04$
δ'_{0i}	$\delta'_{0i} < 6$	$6 \leq \delta'_{0i} < 12$	$12 \leq \delta'_{0i} < 20$	$\delta'_{0i} \geq 20$
$\bar{\delta}_{0i}$	$\bar{\delta}_{0i} < 4$	$4 \leq \bar{\delta}_{0i} < 10$	$10 \leq \bar{\delta}_{0i} < 20$	$\bar{\delta}_{0i} \geq 20$

grammetry, Stuttgart, in order to obtain information about the dependency of the internal and external reliability on different project parameters. These were, in particular, especially

- the control point distribution,
- the degree of overlap,
- the density and the distribution of the, tie points, and
- the size of the blocks.

The investigation used simulated regular square shaped blocks with bundles and independent models, single blocks (designated with S) with 20 percent sidelap and double blocks (designated with D) with either 60 percent sidelap or consisting of two single blocks flown crosswise. The blocks with independent models use four or six single or twin points per model. The bundle blocks have nine single or twin points and 25 single points per image. The number of points per unit is added to the S or D to describe the block concerned. The horizontal control points are situated at the perimeter of the blocks (cf. Figure 6). The vertical control varies for single and double blocks. The height of single blocks is stabilized by chains, while the vertical control points in double blocks form a regular grid. The control point interval varies from 2 to 20 baselengths b . The precision of all observations, including the coordinates of the control points, is assumed to be equal, with one exception: the x - and y -coordinates of the projection centers in independent model blocks are assumed to have double the standard deviation.

Before investigating the influence of the different block parameters on the reliability, two examples are examined. Figures 7 to 9 show two representative blocks with independent models and with bundles, respectively. For symmetry reasons only

one-fourth of each of the blocks is plotted. The redundancy numbers r_i , the controllability factors δ'_{0i} , and the sensitivity factors $\bar{\delta}_{0i}$ are shown in each block. The values are given separately for the planimetry and height of independent model blocks and for the x - and y -coordinates of the bundle block.

The model blocks $S8$ with sparse and dense control ($i = 2b$ and $i = 6b$) (cf. Figures 7 and 8) have four twin points in the corner of each model. The reliability figures are identical for the two points of a group and are, therefore, only given once. The figures suggest the interior parts, the border parts, and the control points be considered separately.

The redundancy numbers r_i in the interior of the blocks are about 0.5. This proves the block to be very stable. Gross errors larger than $\delta'_{0i} \cdot \sigma_{li} = 5.6 \cdot \sigma_{li}$ can be detected with the data snooping. Undetectable gross errors, however, falsify the coordinates of the new points only up to 3 times their standard deviation ($\bar{\delta}_{0i} \leq 3$). The reliability is fully acceptable.

This is different at the border parts of the blocks, especially at the borders with the short model sides. They are determined as being not very reliable, with sensitivity factors $\bar{\delta}_{0i}$ of around 5. The influence of the control points on the reliability is negligible.

The coordinates of the control points, which are introduced as observations, are the most poorly controlled. Already with moderate control point distance $i = 6b$, only 7 or 12 percent of the size of the gross errors show up in the residuals ($r_i = 0.07$ or 0.12, respectively). Gross errors must be larger than 6 or 15 (!) times the standard deviation σ_{xy} or σ_z of the model coordinates to be detectable.

Bundle blocks reveal a similar reliability structure. In the images of block $S18$ shown in Figure 9 double points are measured at the nine standard positions. Again, the values suggest that the interior and the border parts be considered separately. This

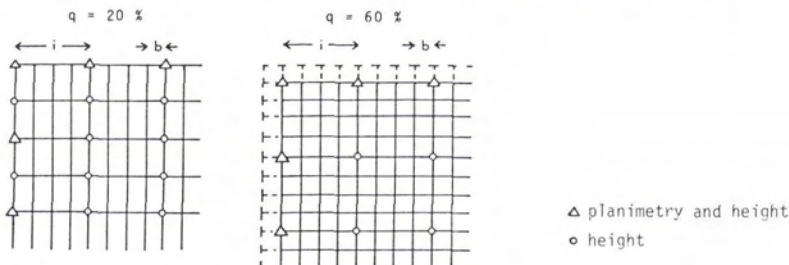


FIG. 6. Control point distribution.

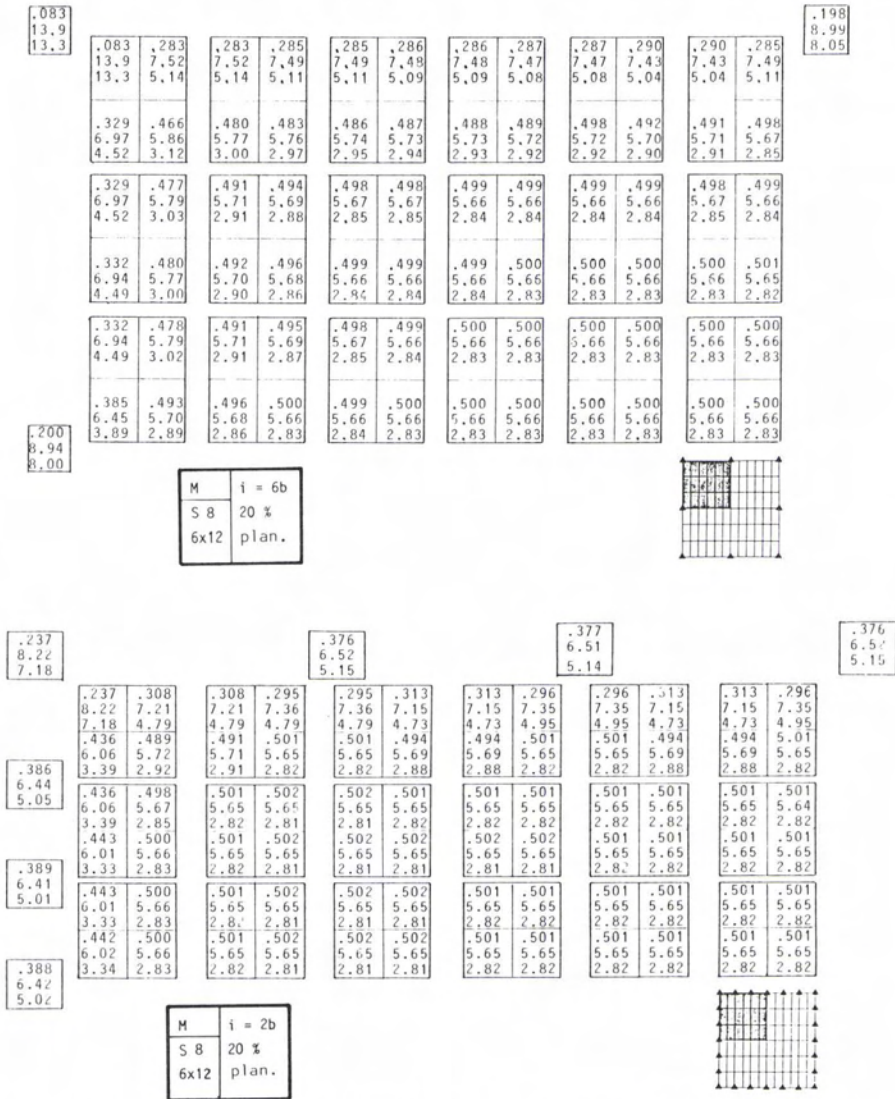


FIG. 7. Redundancy numbers r_i , and controllability and sensitivity factors δ_{0i} and $\bar{\delta}_{0i}$ of planimetric coordinates of a block with $6 \times 12 = 72$ independent models with sparse and dense control; four pairs of tie points in the corners of each model; values for x and y identical.

statement is not influenced by the points in the middle of the strips, which occur in all images. At these points only $1/6$ of gross errors in the x -coordinates are revealed in the residuals ($r_i = 1/6$), which moreover cannot be located (cf. the earlier section on reliability).

At the border parts some observations are not controllable at all (δ_{0i} and $\bar{\delta}_{0i} = \infty$). These points would be single points in an adjustment with independent models and in a previous analytical relative orientation would be controllable only in the y -direction. The points in the overlap zone of adjacent strips, however, are well determined, with sensitivity factors $\bar{\delta}_{0i}$ below 3.

The control points are as weakly controllable as in model blocks, discussed above. Here also the height control points are less controllable than the horizontal control points, with factors $\bar{\delta}_{0i}$ of 14.6 or 11.0 versus 13.0 or 8.0 at the corners or the borders, respectively.

A direct comparison of the reliability of bundle and independent model blocks is not possible, as the controllability and sensitivity factors refer to different types of observations and as the structure of the precision of the new points is different.

Blocks with single tie points (not shown) are the least reliable. Here the controllability factors are larger for the interior and the border parts by factors

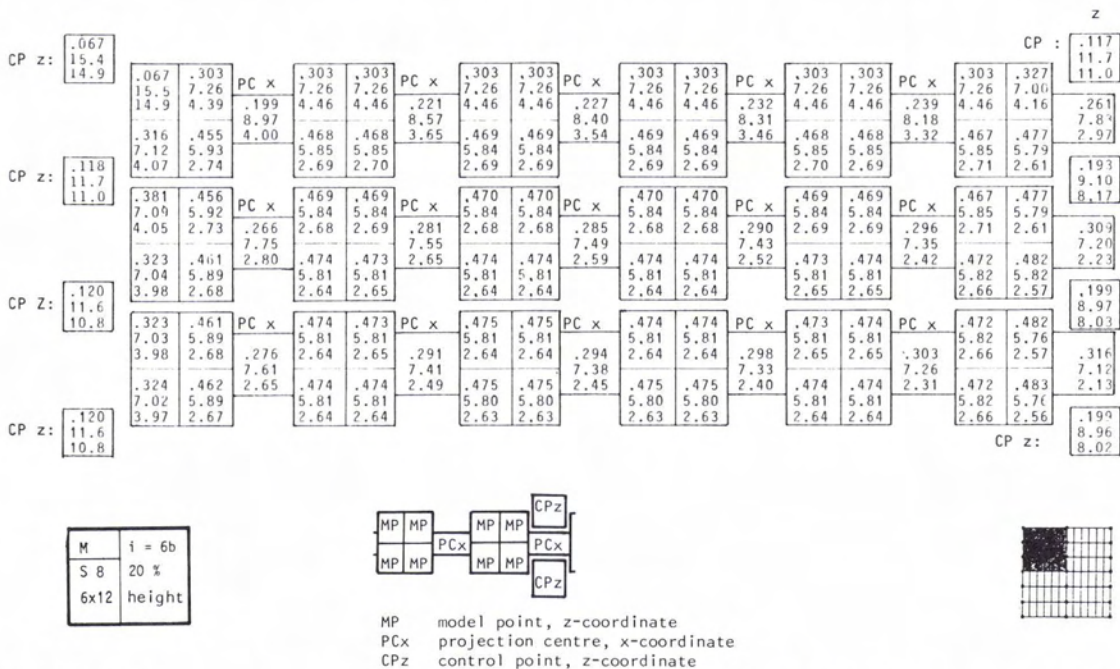


FIG. 8. Redundancy numbers r_i , and controllability and sensitivity factors δ'_{0i} and $\bar{\delta}_{0i}$ of height coordinates and of x-coordinates of projection centers of a block with $6 \times 12 = 72$ models, control point interval $i = 6b$, four pairs of tie points in the corners and projection centers (x, y, and z) per model.

Comment: The values for the y- and z-coordinates of the projection centers are nearly independent of the location within the block: i.e., $y: r_i = 0.44, \delta'_{0i} = 6.0, \bar{\delta}_{0i} = 1.6; z: r_i = 0.40, \delta'_{0i} = 6.3, \bar{\delta}_{0i} = 0.3$.

of 1.2 to 1.5 and 2, respectively. In blocks with independent models with only four tie points, the lower bounds for detectable gross errors reach values of $22\sigma_1$ at the borders of the blocks. The sensitivity factors $\bar{\delta}_{0i}$ of blocks with single tie points are larger by factors of about 1.1 to 1.5 and 1.5 to 3.0 for the interior and the border parts, respectively. The measurement of double tie points thus leads to a rather high reliability. Moreover, in case a tie point has to be eliminated, the connection is not lost and still can be controlled.

The two examples give a first impression about the reliability of photogrammetric coordinates, but provide no information about the dependency on the block parameters. The main result, however, is the high homogeneity of the values in the interior of the blocks. This indicates that the values are independent of the block size and also of the shape of the block. It can be expected that the reliability will be affected by an increase in the tie point density or the overlap and that the controllability of the control points will also depend on the control point interval i/b .

Figure 10 summarizes the main results of the investigation with regard to the reliability of the photogrammetric tie points. Here the maximum controllability factors δ'_{0i} and the maximum sensitivity factors $\bar{\delta}_{0i}$ are given for the three areas of interest, the values for the corners being given separately.

The controllability of model coordinates obviously is increased when using more tie points, especially at the corner and the border parts. Changing from four tie points (S4) to double points (S8) already leads to an acceptable reliability of the coordinates ($\bar{\delta}_{0i} \leq 5$). Double blocks cannot really be made more reliable by increasing the tie point density.

The situation is different for bundle blocks. Using more tie points (S9 \rightarrow S18) does not increase the reliability, due to the already mentioned points in the middle of the strips, where additional points do not change the weak local geometry. The reliability of bundle blocks can be improved only by using higher coverage, i.e., 60 percent sidelap or two single blocks flown crosswise. Figure 11 shows that in all cases sensitivity values of $\bar{\delta}_{0i} < 4$ are achieved, even with single tie points.

The controllability of horizontal and vertical control points is given in Figures 12 and 13 for single blocks. Double blocks will usually be applied in special cases in which high precision is demanded and the reliability of the control points will (hopefully) be guaranteed by geodetic means. Table 3 also contains approximations for the values of control points in the other areas of the blocks. The sensitivity values, which are not shown, can easily be obtained from $\bar{\delta}_{0i}^2 = \bar{\delta}'_{0i}{}^2 - \delta_{0i}^2$ and in most cases $\bar{\delta}_{0i} \approx \delta'_{0i}$ as $\delta_0 \ll \delta'_{0i}$. The dependency of the reliability on the control point interval i/b is different for horizontal

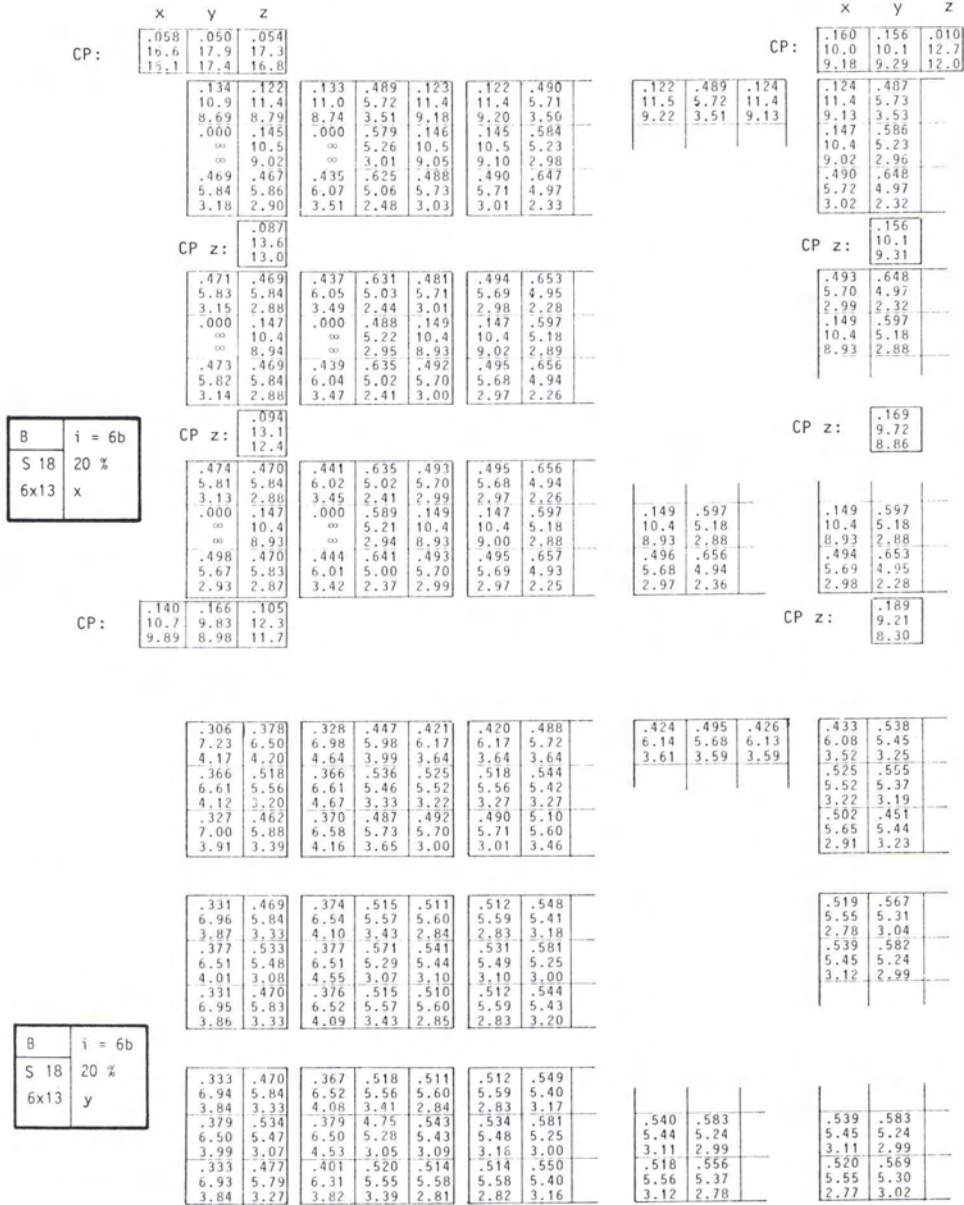


FIG. 9. Redundancy numbers r_i and controllability and sensitivity factors δ'_{0i} and $\bar{\delta}_{0i}$ of x- and y-coordinates (top and bottom figure, respectively) of a bundle block with $6 \times 13 = 84$ images; point interval $i = 6b$; nine pairs of tie points at the standard positions.

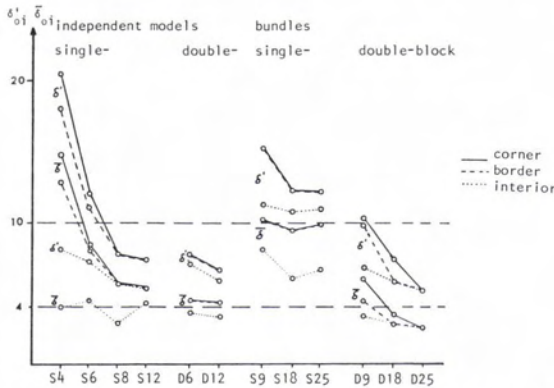


FIG. 10. Maximum values δ'_{oi} and δ_{oi} of controllability and sensitivity of photogrammetric blocks (from Förstner, 1980).

and vertical control points. The factors δ'_{oi} of horizontal control points increase approximately proportionally to the control point interval, whereas the controllability of vertical control points increases only with the square root of the interval i/b . This can be explained by the different control point patterns: single horizontal control points at the perimeter versus chains of vertical control points across the block. The absolute values are rather high ($\delta'_{oi} > 10$) even for small distances i/b .

CONCLUSIONS

Photogrammetric point determination can reach a high reliability. Summarizing, this is the conclusion which can be drawn from this investigation. The stable geometry of photogrammetric blocks is the reason for the good experiences gained in practical application. The results, however, show the weak areas in photogrammetric blocks: the geodetic control, the perimeter of the blocks, and the points with only a few rays in bundle blocks.

The main result for project planning is the independence of the reliability on the block size and the block form and the moderate influence of the different block parameters on each other. This could allow a separate discussion, especially of photogrammetric and geodetic observations. But, also, two different types of application have to be distinguished: the determination of pass points for a subsequent mapping and the densification of highly accurate point fields.

Block triangulation as a *basis for mapping* needs only 20 percent sidelap. Because only pass points in the edges of the images are needed, also the bundle (only with self calibration) method can be used. The points in the middle of the strips than are no longer anymore used after the adjustment. Double points are recommended in any case. This increases the reliability, while not requiring much additional effort for targeting or point transfer and for measuring. But what is more important, this

remedy also simplifies the error detection procedure, because the elimination of points does not weaken the connection. In blocks with independent models four pairs of tie points are sufficient. If, however, self calibration is applied, tie points in the middle of the strips are also necessary in order to guarantee the determinability of the additional parameters. Here single points suffice.

There are several possibilities to strengthen the border areas of the blocks:

- Increasing of the tie point density at the perimeter of the block. Especially in independent model blocks this is a very effective action.
- Increasing of the block size by one strip or two base lengths in the strip direction, in order to keep the area of interest within the interior of the block, i.e., one base length at the perimeter is not used for mapping.
- Bordering the block by a strip, which strengthens the perimeter. This is a variant of the previous remedy against the weak geometry (cf. Ackermann, 1966).

In all cases a high reliability can be obtained with sensitivity factors $\delta_{oi} \leq 3$, which guarantee the quality of the result.

In contrast to mapping applications, aerotriangulation for purposes of *photogrammetric network densification* requires at least a four-fold overlap. With regard to reliability, 60 percent sidelap and cross flights are equivalent; cross flights, however, have advantages in compensating systematic errors. Because of the high overlap, each point can be measured in at least four images, which guarantees a location of gross errors. Thus, no points are lost by the elimination of a single observation. Here the increase of the block by one base length is best for strengthening the border. The low sensitivity values $\delta_{oi} \leq 4$ achieved then make photogrammetric point determination comparable to geodetic densification, if not superior.

The reliability of the *control points* will always cause problems. Even for small control point intervals only controllability values around $\delta'_{oi} \approx 10$ are reached. Thus, it appears impossible to check the coordinates of the ground control during the adjustment. This check has to be done and documented by the geodesist.

Then only the targeting has to be kept under control. Groups of control points are excellent for this purpose, too. The distribution of the points within the groups may consider the following recommendations:

- The points should be determined as independently as possible and therefore may be laid wide apart.
- The points should belong to at least two models or three images in order to be able to distinguish photogrammetric and geodetic errors. Thus, one should avoid using a point in the corner of the block as a control point, but rather use some tie points at the border possibly together with a point more inside the block.

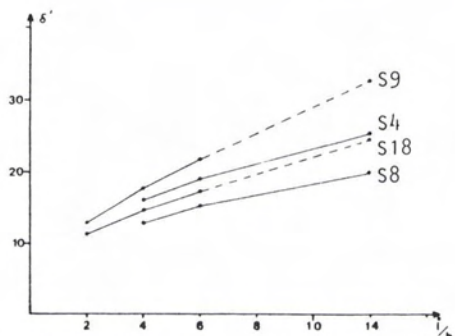


FIG. 11. Controllability factors δ'_{oi} of horizontal control points, single blocks, corners.*

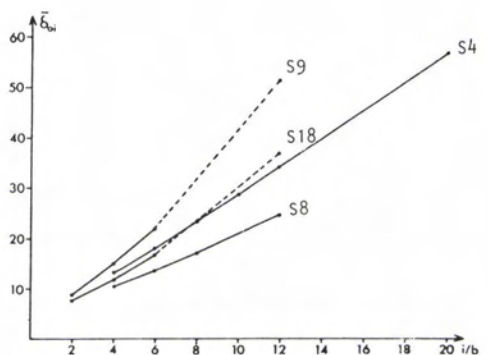


FIG. 12. Controllability factors δ_{oi} of vertical control points, single blocks, corners.*

* The end points of the dashed lines result from blocks with only four horizontal control points and two chains of vertical control points.

CP:		x	y	z	CP:		x	y	z
		.336	.336	.181			.547	.548	.300
		6.90	6.90	9.40			5.41	5.40	7.31
		5.62	5.62	8.50			3.61	3.63	6.11
		.000	.087	.087			.087	.346	.092
		"	"	13.6			13.6	6.80	13.2
		"	"	9.11			9.10	4.22	8.78
		.199	.377	.350			.307	.605	.388
		8.98	6.51	6.76			6.43	5.14	6.42
		6.89	4.31	4.47			4.06	2.36	4.06
		.172	.473	.301			.366	.580	.363
		9.64	5.81	7.29			6.61	5.25	6.64
		5.82	2.95	3.54			2.74	2.04	2.77
		.199	.435	.301			.345	.576	.346
		8.98	6.07	7.10			6.81	5.27	6.80
		5.22	3.31	3.55			2.98	2.08	2.97
		.337	.515	.475			.516	.681	.516
		6.89	5.57	5.80			5.57	4.85	5.57
		4.62	3.05	3.24			2.90	1.77	2.90
		.237	.505	.350			.391	.596	.391
		8.22	5.63	6.76			6.40	5.18	6.40
		4.51	2.68	2.92			2.44	1.91	2.44
		.223	.481	.347			.382	.587	.385
		8.46	5.77	6.79			6.48	5.22	6.44
		4.74	2.89	2.96			2.55	1.98	2.51
		.377	.584	.515			.553	.681	.558
		6.52	5.24	5.57			5.38	4.85	5.36
		4.18	2.52	2.91			2.60	1.78	2.56
		.235	.486	.350			.385	.590	.388
		8.25	5.74	6.76			6.45	5.21	6.42
		4.54	2.84	2.92			2.52	1.96	2.48
		.240	.509	.350			.397	.592	.399
		8.16	5.61	6.69			6.34	5.20	6.33
		4.44	2.65	2.83			2.37	1.94	2.35
		.373	.524	.493			.505	.677	.524
		6.55	5.52	5.70			5.63	4.86	5.52
		4.27	2.98	3.09			2.99	1.80	2.83
		.000	.126	.126			.126	.504	.128
		"	"	11.3			11.3	5.63	11.2
		"	"	9.05			9.05	3.39	8.94
		.348	.542	.501			.519	.725	.519
		6.78	5.44	5.65			5.55	4.70	5.55
		5.01	3.29	3.54			3.40	2.00	3.40
		.417	.429	.553			.584	.733	.584
		6.19	5.04	5.38			5.23	4.67	5.24
		3.67	2.45	2.50			2.25	1.70	2.25
		.348	.571	.473			.497	.713	.497
		6.78	5.29	5.81			5.68	4.74	5.68
		4.40	2.87	3.16			2.96	1.85	2.96
		.500	.649	.639			.660	.784	.660
		5.66	4.97	5.00			4.92	4.52	4.92
		3.55	2.53	2.52			2.38	1.59	2.38
		.448	.660	.575			.597	.741	.597
		5.98	4.92	5.27			5.18	4.65	5.18
		3.39	2.23	2.32			2.14	1.64	2.14
		.443	.633	.575			.582	.736	.594
		6.01	5.03	5.28			5.24	4.66	5.19
		3.43	2.43	2.32			2.27	1.68	2.17
		.519	.703	.655			.680	.783	.685
		5.55	4.77	4.94			4.85	4.52	4.83
		3.40	2.16	2.41			2.24	1.60	2.21
		.448	.636	.576			.584	.738	.595
		5.98	5.02	5.27			5.24	4.66	5.19
		3.38	2.41	2.31			2.25	1.67	2.16
		.450	.664	.579			.598	.738	.603
		5.97	4.91	5.26			5.17	4.66	5.15
		3.37	2.21	2.29			2.14	1.66	2.10
		.518	.645	.648			.645	.782	.664
		5.56	4.95	4.97			4.98	4.52	4.91
		3.40	2.49	2.46			2.48	1.60	2.35
		.602	.595	.364			.602	.595	.364
		5.16	5.19	6.63			3.25	3.30	5.28
		.126	.504	.128			.126	.512	.126
		11.3	5.63	11.3			11.3	5.59	11.3
		9.03	3.39	8.94			9.03	3.32	8.94
		.519	.725	.519			.562	.719	.562
		5.34	4.72	5.34			5.34	4.72	5.34
		3.07	2.05	3.07			3.07	2.05	3.07
		.584	.733	.584			.586	.734	.586
		5.23	4.67	5.24			5.23	4.67	5.24
		2.23	1.69	2.23			2.23	1.69	2.23
		.540	.694	.540			.540	.694	.540
		5.44	4.80	5.44			5.44	4.80	5.44
		2.60	1.99	2.60			2.60	1.99	2.60
		.662	.786	.662			.662	.786	.662
		4.92	4.51	4.92			4.92	4.51	4.92
		2.37	1.57	2.37			2.37	1.57	2.37
		.605	.740	.605			.605	.740	.605
		5.14	4.65	5.14			5.14	4.65	5.14
		2.08	1.65	2.08			2.08	1.65	2.08
		.537	.546	.537			.537	.546	.537
		3.72	3.72	3.72			3.72	3.72	3.72
		.606	.742	.606			.606	.742	.606
		5.14	4.64	5.14			5.14	4.64	5.14
		2.07	1.64	2.07			2.07	1.64	2.07
		.668	.789	.668			.668	.789	.668
		4.89	4.50	4.89			4.89	4.50	4.89
		2.32	1.55	2.32			2.32	1.55	2.32

FIG. 13. Redundancy numbers r_i , and controllability and sensitivity factors δ'_{oi} and $\bar{\delta}_{oi}$ for the x -coordinates of the image points and for the x -, y -, and z -coordinates of the control points in a bundle block with 49 images; sidelap 60 percent, control point interval $i = 2$ base lengths, nine tie points and nine pairs of tie points per image. The values for the y -coordinates are achieved by mirroring the values at the main diagonal.

TABLE 3. CONTROLLABILITY FACTORS δ'_{0i} OF HORIZONTAL AND VERTICAL CONTROL POINTS IN DEPENDENCY OF THE CONTROL POINT DISTANCE i IN BLOCKS WITH 20 PERCENT SIDELAP ($\delta_0 = 4$).

Block Type	Tie Points	Position of CP within Block	$(\delta'_{0i})^2$ Horiz. CP	$(\delta'_{0i})^2$ Vert. CP
Independent models	4/model	corner	$64 + 8 (i/b)^2$	$50 + 51 (i/b)$
		border	$56 + 1.9 (i/b)^2$	$37 + 27 (i/b)$
		interior		$19 + 14 (i/b)$
Bundles	2×4 /model	corner	$64 + 4 (i/b)^2$	$58 + 30 (i/b)$
		border	$40 + 1.2 (i/b)^2$	$38 + 16 (i/b)$
		interior		$24 + 10 (i/b)$
	2×9 /image	corner	$42 + 12.8 (i/b)^2$	$80 (i/b)$
		border	$30 + 3.7 (i/b)^2$	$42 (i/b)$
		interior		$21 (i/b)$
Bundles	2×9 /image	corner	$48 + 6.9 (i/b)^2$	$42 + 42 (i/b)$
		border	$30 + 2.1 (i/b)^2$	$22 + 22 (i/b)$
		interior		$11 + 11 (i/b)$

- At least three points should be targeted, in order to be able to control the targeting by checking the similarity of triangles, thus to be independent of the local scale. If remeasurements are not possible or not wanted, one should use at least four points. Then a single gross error can be located without additional information. In case the distances between the points within a group are small, the controllability is higher (due to the smaller influence of the local scale). Then one needs one point less, i.e., two or three points at least, provided the points are rather independent.

When using groups of points, one must realize that a joint shift of the group is more difficult to detect than if one would use a single point, due to the higher weight of the group.

The joint effect of bordering, using double tie points and groups of control points, on the reliability is shown in Figure 14, which summarizes the result of this investigation.

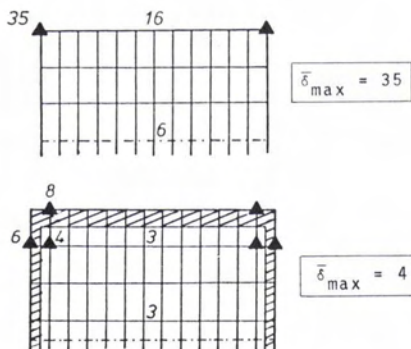


FIG. 14. Optimization of the reliability of a planimetric block with independent models, four tie points per model (a) given block, external reliability at control points, at border and interior of block; (b) optimized block, external reliability at control points, border and interior of block

- using groups of control points
- using pairs of tie points
- not using the hatched parts of the block

Thus, compared to the expedients which are necessary to reach a high precision, only a few additional remedies have to be considered in order to guarantee a high quality of aerial triangulation.

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