Separability of Boreal Forest Species in the Lake Jennette Area, Minnesota

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ABSTRACT: The key parameters in the global circulation and carbon models are the vegetation distribution and the net primary productivity (NPP) on a global scale. Remote sensing offers a practical way of measuring these parameters. Because the boreal forest contains a significant pool of carbon and covers a vast area of the northern latitudes, it is important to the solution of these problems. The Thematic Mapper (TM) with its high spatial resolution permits one to look into relatively uniform tree canopies. In order to exploit the possibility of using TM data to obtain vegetation and NPP maps in the boreal forest, three aircraft flights of the NS001 Thematic Mapper Simulator were made over an area near Ely, Minnesota, in 1983. This paper presents the results of an analysis of these data to separate coniferous trees from deciduous trees. Canopy reflectance models and measured optical properties of the scattering elements have been used to understand this separability and relate the ratio of nadir view reflectances in TM band 4 and TM band 3 to the overstory leaf area index. A map which is proportional to the leaf area index for deciduous species is also produced.

INTRODUCTION

THERE HAS BEEN a steady accumulation of CO_2 in the atmosphere over the past century. Potentially, this may bring changes in the distribution of rain-fall zones, which along with elevated CO_2 levels will affect the primary productivity of the world's natural and cultivated ecosystems. The actual significance of these effects depends on a multitude of uncertain factors, such as the rate of deforestation and the net primary productivity of forest ecosystems (Atjay *et al.*, 1979).

Remote sensing offers a realistic way to attempt to improve the accuracy of vegetation-related factors in the global carbon models (Tucker *et al.*, 1984). Landsat-4/5 Thematic Mapper observations provide data with sufficiently fine spatial resolution to observe relatively uniform tree stands. Classification of forest species is necessary if we are to make maps of biomass or leaf area index because the relationship between canopy reflectance and leaf area index depends on species.

Boreal forests contain a significant proportion of the total carbon pool of the world (Larsen, 1980). The objectives of this study were to use the Thematic Mapper Simulator (TMS) data (1) to determine how well boreal forest species could be identified, (2) to determine which spectral wavelength bands carried maximum information for species identification, and (3) to produce classification and leaf area index maps of the boreal forest study region and to evaluate the classification maps. (Resources were not available for evaluation of the LAI map).

Classification problems encountered using the 79-m MSS data over forested regions have been well documented (Beaubien, 1979; Harris *et al.*, 1978; Bryant *et al.*, 1980; Latty and Hoffer, 1981; Markham and Townshend, 1981). Work using the higher resolution (~30-m) TMS data has been carried out by Latty and Hoffer (1980) over a forested region in South Carolina; Teillet *et al.* (1981) in British Columbia, Canada; and more recently by Nelson *et al.* (1984) in the Baxter State Park, Maine. These studies have also investigated the question of optimum spectral band selection, but did not study the time of year effect.

This paper presents the results from TMS data acquired over a different geographical area of the boreal forest from that used in previous studies using a new analysis technique. A leaf area index map of the deciduous species in the region is also produced.

STUDY AREA

Thematic Mapper Simulator data and coincident aerial photography were acquired near the

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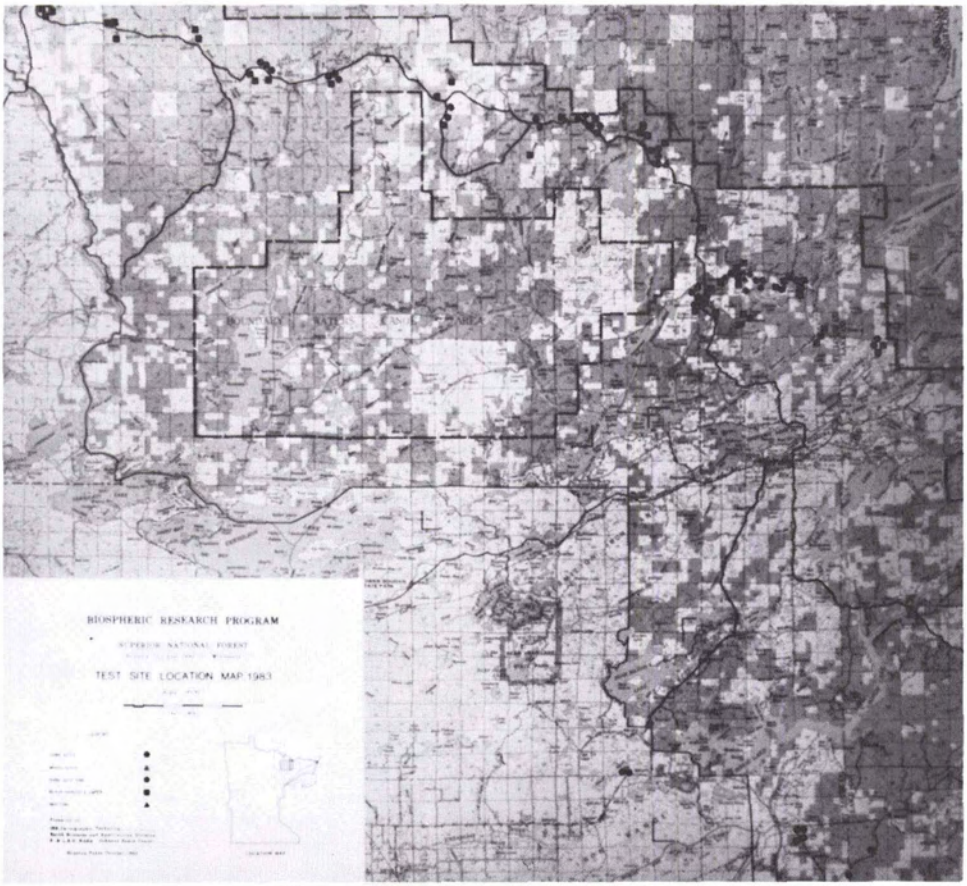

FIG. 1. Geographic map of the study area.

Boundary Waters Canoe Area (BWCA) along the Echo Trail near Ely, Minnesota. The BWCA is the largest (200,000 ha) undisturbed boreal forest in the contiguous United States. It lies in a region of relatively little topographic relief (<200 m) and contains stands with a fairly wide range of variation in leaf area index, biomass, and productivity (Heinselman, 1973). This region contains tree species representative of much of the entire boreal forest. The more abundant species are *Pinus banksiana* lamb. (jack pine), *Picea mariana* (black spruce), *Abies balsamea* L./Mill. (balsam fir), *Populus tremuloides* Michx (quaking aspen), and *Acer rubrum* L. (red maple). This area has been well studied ecologically, and for a few tree species regression equations which relate biomass and productivity to convenience measurements such as diameter at breast height (dbh) and tree height are available. In addition, the U.S. Forest Service has detailed species maps of this region. A geographic map of the area is presented in Figure 1.

DATA ACQUISITION

The eight-band TMS data were acquired on three dates in 1983 using an NS001 scanner mounted on a C-130 aircraft. The scanner, with a 2.5-milliradian instanteneous field of view (IFOV), was flown at approximately 7200 m, giving a nominal spatial resolution at nadir of about 18 metres for all spectral bands (Table 1) (Colwell, 1984). Table 2 gives a summary of the flight information. The aircraft flew essentially E-W and thus the ground was scanned in the N-S direction.

Coincident color infrared aerial photographs at a scale of 1:24,000 were obtained along with scanner data.

TABLE 1.

NS001 Band	TM Band	Wavelength	Sensitivity
1	1	0.45-0.52	Chlorophyll & Carotinoid
2	2	0.53-0.61	Chlorophyll
3	3	0.62-0.69	Chlorophyll
4	4	0.78-0.91	Vegetation Density
5	—	1.15-1.30	Vegetation Density
6	5	1.57-1.78	Moisture Content
7	7	2.10-2.35	Moisture Content

TABLE 2. SYNOPSIS OF NS001 SCANNER FLIGHTS IN BWCA

Day of Year	Flight No.	Sun Angle		Flight Direction (heading)	Observation Condition	Aircraft Altitude (Km)	Ground Speed (Km/hr)	GMT*	
		Zenith	Azimuth					Start	Stop
157	7-1	27	156	106°	Puffy clouds	7.71	426	17:19	17:24
157	7-2	26	165	282°	Puffy clouds	7.81	365	17:35	17:41
194	10-1	33	132	108°	Clear	7.15	426	16:26	16:33
218	9-1	32	169	108°	Clear	7.91	427	17:48	17:55

* Greenwich Mean Time

PROCEDURE

A 3-mile by 3-mile area near Lake Jennette was chosen for this study of the separability of the boreal forest species (Figure 1). This area was chosen because it contained many of the species present in this region, there were a number of pure stands in the area, and U.S. Forest Service stand maps were available. The stand maps show the locations of various stands in the region along with an indication of the density and purity of the stands. A pixel-level ground truth inventory was created for this area using the stand maps and the aerial photography which was acquired in conjunction with the TMS data. Stands were selected for inclusion in the inventory if they were pure according to the stand maps and looked homogeneous on the aerial photography.

Inventories were created to correspond to TMS data from two dates in 1984 (6 June and 6 August). On the 6 June (day 157) acquisition, the deciduous trees were leafed out while the understory was not completely developed. The 6 August (day 218) acquisition represents a mid-season situation with all of the trees and understory completely developed. The ground reference data set contained pure stands of the following species: black spruce (23 percent), red pine (*Pinus resinosa*) (7 percent), jack pine (*Pinus banksiana*) (37 percent), birch (*Betula allegha niesis-paprifera*) (20 percent), and aspen (12 percent).

The results obtained with this data set represent the "best case" situation, because the stands were all chosen to be pure and homogeneous. They represent the inherent separability between the species. This separability would be degraded if the stands were not homogeneous or consisted of a combination of species.

TABLE 3. FISHER SEPARABILITY MEASURE FOR VARIOUS SPECIES

	Fisher Separability Measure for All Bands	
	Day 157	Day 218
Coniferous Species		
Black Spruce/Jack Pine	0.31	0.42
Black Spruce/Red Pine	0.96	0.69
Jack Pine/Red Pine	0.82	0.55
Deciduous Species		
Aspen/Birch	0.59	0.18
Coniferous/Deciduous Species		
Black Spruce/Aspen	0.95	0.79
Black Spruce/Birch	0.98	0.84
Jack Pine/Aspen	0.85	0.74
Jack Pine/Birch	0.98	0.87
Red Pine/Aspen	0.54	0.21
Red Pine/Birch	0.98	0.79

ANALYSIS OF DATA

The TMS data and corresponding ground truth inventories were used to evaluate both the proportion estimation accuracy and the classification accuracy. The evaluation of the ability to determine species proportion for the two dates available was based on the Fisher Information measure (Fisher, 1960). It provides an estimate of the lower bound for the variance of any unbiased estimate of areal extent based on the spectral data. This measure is not dependent on the form of the probability distribution of the spectral observation and does not imply a specific estimation procedure (Appendix A).

TABLE 4. CLASSIFICATION ACCURACY OF VARIOUS SPECIES (NON-INDEPENDENT TRAINING SAMPLE)

	Percent Correctly Classified	
	Using All Bands	Using Band 4
All 5 Species	84.2	67.8
Coniferous/Deciduous	96.8	96.6
Among Coniferous Species	86.4	71.8
Among Deciduous Species	87.0	72.6

TABLE 5. CLASSIFICATION ACCURACY FOR VARIOUS BANDS

TMS Band	Percent Correctly Classified — Non-Independent Data	Percent Correctly Classified — Independent Data
All	84.2	80.0
1	44.0	43.4
2	57.4	54.0
3	53.8	50.4
4	68.6	67.8
5	65.2	61.0
6	60.0	51.4
7	45.0	41.4

Table 3 shows the Fisher separability measure using all of the TMS bands for the two dates for each combination of species. The separability is, in general, quite good between coniferous and deciduous species, with less separability within these categories. The separability between species is consistently better on the earlier date (day 157) than later in the season (day 218). The Fisher separability measure can also be calculated for each of the individual bands. The results of these calculations are that, for day 157, band 4 is the best individual band for separating these species. The separability is generally just as good with band 4 as it is for all bands.

The Fisher separability measure describes the ability to determine the areal extent of a species in the presence of another species. In terms of the use of this type of data for input to ecological models, it is perhaps more important to determine the identity of an individual pixel so that some characteristic of the pixel (such as leaf area index) can be determined.

In order to determine how well pixel classification can be done, a maximum likelihood classification was performed on a sample of five hundred pixels randomly selected from the scene. These pixels were used to train the classifier, and then the classification was performed on the same pixels.

This classification was done for all five classes at one time, for two classes (deciduous and coniferous), and within the deciduous and coniferous classes. Table 4 shows the results of these calculations for day 157. It should be noted that the data produces good identification of deciduous and coniferous species, with less accuracy in identifying species within these classes. The deciduous/coniferous separability was almost as good with only band 4 as it was when all of the bands were used. However, the within-class separability was better when all of the bands were used rather than just band 4.

The classification accuracy for all five classes was determined using an independent training sample. Table 5 shows the results for all bands and for each of the individual bands for day 157. These results are consistent with the results when the classifier was trained on pixels and the same pixels reclassified, except that the overall accuracies are slightly lower. Band 4 provides the most separability and band 1 the least.

Figure 2 gives a smoothed histogram of band 4 (0.76-0.90 μm) measurements for a number of species in the Lake Jennette area. It is clear that a discriminant chosen around 165 counts would provide excellent separability between deciduous and coniferous species. This ability to accurately differentiate conifer and hardwood is consistent with the results of Coggeshall and Hoffer (1973)—MSS and 12 channel aircraft scanner data; Latty and Hoffer (1980)—TMS data; and Stibig (private communication)—SPOT simulated data. Stibig (private communication) has pointed out that not only different species but different age classes of certain species will be spectrally distinguishable.

In order to explain the observed separability based on the physical characteristics of the forest stand, canopy reflectance model calculations were

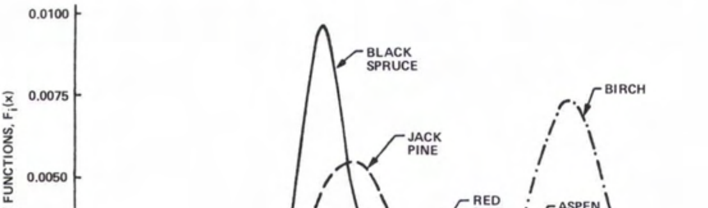

FIG. 2. The density function, $f_i(x)$, for various tree species in the Lake Jennette area for Band 4 (0.76-0.90μm). Note the very clear separation between coniferous and deciduous tree canopies.

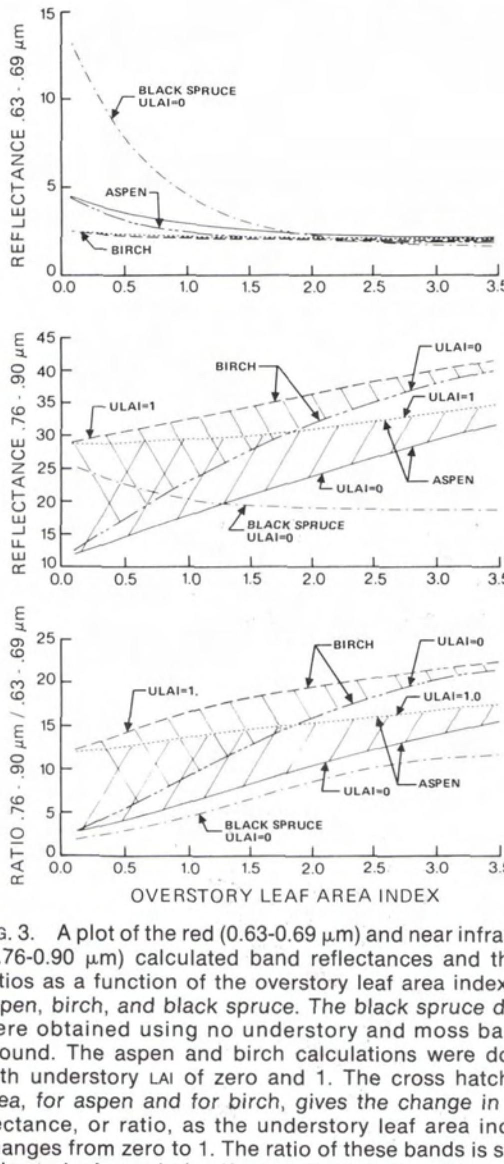

FIG. 3. A plot of the red (0.63-0.69 μm) and near infrared (0.76-0.90 μm) calculated band reflectances and their ratios as a function of the overstory leaf area index of aspen, birch, and black spruce. The black spruce data were obtained using no understory and moss background. The aspen and birch calculations were done with understory LAI of zero and 1. The cross hatched area, for aspen and for birch, gives the change in reflectance, or ratio, as the understory leaf area index changes from zero to 1. The ratio of these bands is sensitive to leaf area index if no understory is present.

made using a modified Suits model (Suits, 1972). This model assumes that the canopy can be stratified into homogenous layers in which the tree components are randomly distributed. The model idealizes the canopy by replacing plant scattering elements (leaves, bark, etc.) by thin horizontal and vertical projections, and assigns these projections the same hemispherical spectral properties as the actual components. Such a model has been used by Fox (1976, 1978) for study of coniferous trees.

The canopy reflectance model was set up with three layers. The top layer consisted of leaves or needles and formed the overstory. The second layer consisted of bark with mean inclination angle of 90°. The third layer was the understory. The lower boundary was brown litter from last fall for aspen and birch canopies and moss for black spruce canopies.

The input values needed to run the model are the optical properties of the components, the amount of each component present (in units of leaf area index), and the mean leaf angle inclination for each component. The hemispherical reflectance and transmittance of scattering elements were measured for leaf and bark samples taken from the actual flight line area on the ground (Johnson and Morris (private communication); Daughtry and Biehl, 1984). The fraction of bark area index to leaf area index for aspen and black spruce was estimated from dimensional measurements (Feiveson, 1984). It varies from about 8 percent to 22 percent with an average value of 15 percent. We used a value of 15 percent which is consistent with the measurements of Hutchinson *et al.*, (1984). The mean leaf inclination angle was estimated to be about 75° for aspen, 60° for black spruce (Little, 1984), and 65° for birch. Based on selected sites along the Echo Trail, the overstory leaf area index of aspen lies between 0.8 and 3.5 and for black spruce between about 1 and 5 (Feiveson, 1984).

Based on a more detailed analysis of helicopter acquired data, it was pointed out (Badhwar *et al.*, 1984) that, for deciduous canopies of aspen on day 157 (6 June 1983), the overstory is fairly well developed and the understory has not developed. Thus, on this date the understory need not be considered. In calculating the reflectance from coniferous canopies, the transmittance of scattering elements was taken to be 14 percent in the 0.76-0.90 μm band and 1 percent in the visible bands. This is lower than the measured needle transmittance. However, following the work of Norman and Jarvis (1979), it appears that shoots act as the scattering elements. These values are, however, consistent with the work of Fox (1978).

Figure 3 shows the results of the model calculation. This is a plot of the calculated band 3 and band 4 reflectances and the ratio of TMS 4/TMS 3 versus the leaf area index of the overstory of various tree species. The aspen calculations represent the range from no understory to an understory with leaf area index (ULAI) of 1 (the understory was assumed to be hazelnut, the common understory species in this area). For aspen, at a given overstory leaf area index, there is a marked increase in 0.76-0.90 μm band reflectance and decrease in 0.63-0.69 μm as the understory leaf area index is increased. Thus, the ratio of the reflectances in these bands shows considerable sensitivity to the overstory leaf area index when no understory is present but virtually no sensitivity when the understory leaf area index is 1. The results for birch are for the case of no understory and an understory of hazelnut of leaf area index 1. These results are essentially similar to that for aspen. However, birch is slightly brighter in the infrared reflectance. The behavior of the black spruce reflectance in band 4 is nearly the reverse of what is seen for aspen and birch. This is due to low transmission, thus reducing the reflectance. Based on 30 test sites of aspen and black spruce (Feivison, 1984), the average overstory leaf area index is 2.21 for aspen and 2.84 for black spruce.

The radiance, L, as measured at flight altitude, is given by $L = ETR/\pi + L_p$, where E is the total irradiance at the target of reflectance R, T is the nadir transmission, and L_p is the atmospheric path radiance. An estimate of the path radiance, L_p, was obtained from data taken over clear lakes that dot this scene. The corrected path radiance (L-L_p) can be converted to TMS counts using the NS001 radiometeric calibration (Millard, private communication). If L is in $\mu W/cm^2\ \mu m$ sr, the calibration is ($A \equiv E/\pi$).

$$\text{TMS4 counts} = 68.11 A_4 R_4 T_4 + 12;$$
$$\text{TMS3 counts} = 68.02\ A_3 R_3 T_3 + 12.1$$

These can be calculated using a good atmospheric model and knowledge of conditions of the atmosphere. From Figure 2, using the average overstory LAI above (Figure 3), we see the model calculations are in good agreement with observations for 6 June, that is, no understory data.

Thus the canopy reflectance models provide a reasonable explanation of the observed separability and provide a calibration to convert reflectance to leaf area index if the tree species is known.

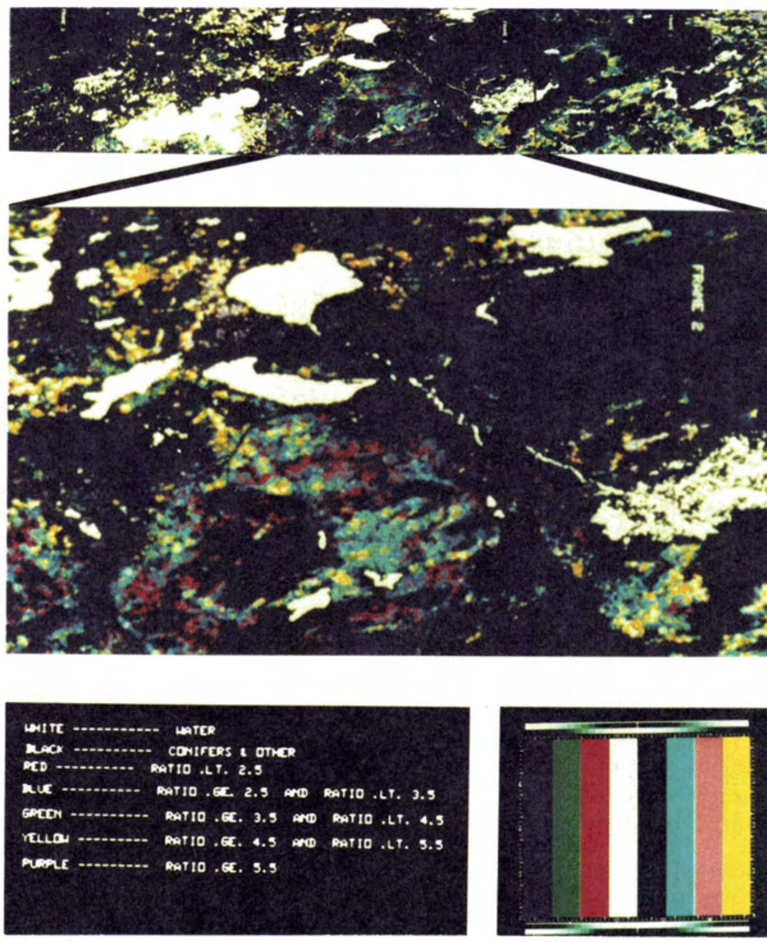

PLATE 1. A map of the ratio of path radiance corrected TMS ratio of Band 4/Band 3. The white areas are water or cloud pixel and the black areas cover coniferous forest pixel. The deciduous area are divided into five colors—Red are low leaf area index canopies and purple the highest leaf area index values.

Pixel-Level Estimates of Leaf Area Index

The third part of this study was the actual production of a map of leaf area index for deciduous stands based on the TMS data. In order to create this map, it was first necessary to determine which pixels corresponded to deciduous species. We have shown in this paper that this separation can be accomplished with high accuracy using band 4 of the TMS data. The next step is the transformation of the pixel level spectral data into a variable which is sensitive to the leaf area index. Previous work (Badhwar *et al.*, 1984) has shown that the ratio of band 4 reflectance to band 3 reflectance is sensitive to the leaf area index for aspen stands early in the season. Because the separation of birch and aspen is not perfect, we assumed that the relationship between the reflectance ratio and the leaf area index is the same for both the deciduous species (Fig. 3). This ratio can be used as a variable for estimating leaf area index. A small error in LAI would be introduced due to this assumption.

The TMS flight line chosen for this study was obtained on 6 June. Only data taken with a view angle between $\pm 20°$ of nadir was used. This was done to avoid scan-angle-dependent variations and the need for sophisticated atmospheric models to remove the path radiance component of the data.

The entire flight line was classified into species categories using the inventoried pixels as training for the classifier. Only pixels which were classified as aspen or birch were processed further. The data were corrected for path radiance by assuming that the clear lakes in the region had essentially zero reflectance. The TMS band values for these lakes then provided a measure of the path radiance. By subtracting the path radiance term, a variable which was proportional to the actual reflectance was obtained. The ratio of the corrected band counts was calculated. The range of the ratio was divided into 5 parts and each was assigned a color. Plate 1 shows the resulting image. Because this ratio is almost a linear function of the leaf area index, the image represents ranges of LAI. Preliminary analysis of selected sites over which ground estimates of LAI have been made indicates the product to be qualitatively correct.

CONCLUSIONS

We have shown that pure stands of deciduous and coniferous trees in the boreal forest can be well separated using TMS nadir view data on clear days, with band 4 providing the best separability. Within class separability, however, is limited when only single date data are considered. Best separation is provided in early June, before the understory has fully developed. Fortunately, this is also the time when nadir view data has the maximum sensitivity to leaf area index. These results based on a single year of data in a small geographic area are encouraging, but other areas should be tested before global surveys can be made using this technique. The combined model calculation and observation, however, are encouraging.

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APPENDIX A

Consider the two-class case. Suppose that x is a random variable, for example, the spectral response in a TMS band (or vector, spectral response of several TMS bands) having the overall probability density function $f(x,p) = pf_1(x) + (1-p)f_2(x)$, a mixture of two densities f_1 and f_2. The Fisher Information regarding the mixing proportion, p, contained in a single observation, x, is (E stands for expectation value)

$$
\begin{aligned}
I_x(p) &= \mathrm{E}\left(\frac{\partial \log f(x,p)}{\partial p}\right)^2 \\
&= \mathrm{E}\left(\frac{f_1(x) - f_2(x)}{f(x,p)}\right)^2 \\
&= \int_{-\infty}^{\infty}\left(\frac{f_1(x) - f_2(x)}{f(x,p)}\right)^2 f(x,p)dx \\
&= -\frac{1}{p(1-p)}\int_{-\infty}^{\infty}\frac{(f(x,p) - f_1(x))\,(f(x,p) - f_2(x))}{f(x,p)}\,dx \\
&= \frac{1}{p(1-p)}\left(1 - \int_{-\infty}^{\infty}\frac{f_1(x)\,f_2(x)}{f(x,p)}\,dx\right)
\end{aligned}
$$

The Fisher Information regarding p contained in a random sample of N observations $\{x_1, \ldots, x_n\}$ is $NI_x(p)$. The reciprocol of the Fisher Information is the lower bound on the variance of any unbiased estimator of p. If the spectral data for N samples are used to determine the proportion of class 1 in the population, the lower bound (L.B.) on the variance of any unbiased proportion estimate $\hat{p}$ is given by

$$
\text{L.B.}\{\text{Var}_p(\hat{p})\} = \frac{1}{N}\,\frac{p(1-p)}{1 - \int \frac{f_1(x)\,f_2(x)}{f(x,p)}\,dx} = \frac{1}{NI_x(p)}
$$

where $I_x(p)$ is the Fisher Information measure regarding the mixing proportion p contained in a single observation x.

If the samples were all correctly labeled using ground truth (GT), the problem of estimating p reduces to a Bernoulli trial and the variance of the sample mean $\bar{x}$ (uniformly minimum variance of the unbiased estimator of p) would be

$$
\text{Var}_{\text{GT}}(\bar{x}) = \frac{1}{N}\,p(1-p)
$$

(Note that, in this Bernoulli trial case, the sample mean x achieves the lower bound on the variance.) The separability measure used for this study is the ratio of the lower bound on the variance for correct labeling to the lower bound on the variance for spectral labeling.

$$S = \frac{\mathrm{Var}_{GT}(\bar{x})}{\mathrm{L.B.}\{\mathrm{Var}_p(\hat{p})\}} = p(1 - p)I_x(p)$$
$$= 1 - \int \frac{f_1(x)\, f_2(x)}{f(x,p)}\, dx$$

If the probability density functions for the two classes are completely disjoint, the integral is zero and the separability measure S becomes 1. In other words, there is complete information (separability) concerning the classes in the spectral data. At the other extreme, when the two density functions completely overlap ($f_1(x) = f_2(x)$), the integral becomes 1 and the separability measure S is zero. This implies that there is no information in the spectral data for separating the classes. This measure has been previously used to study the separability of corn and soybeans using Thematic Mapper data (Pitts *et al.*, 1984).

Unlike many frequently-used separability measures such as Mahalanobis distance, the Fisher measure does not rely on the normality assumption. For each pair of species, the density functions $f_1(x)$ and $f_2(x)$ were respectively estimated as mixtures of multivariate normal densities and $f(x) = p\,f_1(x) + (1 - p)\,f_2(x)$ where p is estimated from the pixel counts in the ground truth inventory. In statistics, many theorems are proved and criteria developed based on the assumption that the underlying distributions are normal. In practice, however, those densities are rarely truly normal. Nevertheless, the violation of the normality assumption is ignored. This normal mixture approximation allows one to better describe density functions that are multimodal and/or skewed.

Forum

The International Society for Photogrammetry and Remote Sensing—75 Years Old, or 75 Years Young

Konecny has recently provided a valuable summary of developments in photogrammetry (*PE&RS*, July 1985, pp. 919-933). The compression of facts is a difficult undertaking and it is impossible, in the space available, to convey every facet of technological history. It would be easy to criticize any brief history and that is not the intention of this note. Nevertheless, some facts are incontrovertible and it serves no useful purpose to publish incorrect information.

Konecny mentions the work of the unrelated British Thompsons. On page 926, he states that Vivian Thompson's Stereo-plotter* was designed in 1908. Atkinson (1980) has pointed out that the first Vivian Thompson Stereo-plotter was designed, built, and used in 1907 and, in fact, this activity resulted in the design of a second instrument, the Stereo-planigraph, in January 1908.

In discussing analytical photogrammetry, Konecny (page 930) relegates E. H. Thompson's contribution to the late 1950s. Doyle *et al.* (1966) have stated that ". . . credit for the first operational system of analytical aerotriangulation goes to the British Ordnance Survey. Begun in 1947 . . . ," this system was based on theories and practice worked out by E. H. Thompson from 1936 onwards and used in conjunction with the Cambridge Stereocomparator during the Second World War (Proctor, 1979). Ordnance Survey production work in analytical aerial triangulation was already based on six Cambridge Stereocomparators by March 1949 (Lawrence, 1949).

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—*K. B. Atkinson*

* Figure 16 on page 926 does not refer to this instrument but to the Autostereograph, as von Orel's first instrument was called.