

# Photogrammetric Measurement of Microwave Antennae

S. F. El-Hakim

Photogrammetric Research Section, National Research Council of Canada,  
Ottawa, Ontario K1A 0R6, Canada

**ABSTRACT:** Large microwave antennae are required to be manufactured with a very high accuracy in order to be able to receive clearly waves of certain length and frequency. Photogrammetric techniques are more suitable for this application than any direct measuring method due to the shape of the object and the nature of its material which changes in size with temperature. The program GEBAT has been employed. It allows for spatial distances, height differences, and other available external information to be used in the adjustment. It also applies a full self-calibration approach. The program evaluates the results by computing the variance-covariance matrices and the error ellipsoids of the adjusted points. It also checks for blunders by the "data snooping" approach. The accuracy achieved in this application was better than the required accuracy.

## INTRODUCTION

CLOSE-RANGE PHOTOGRAMMETRY, with its flexibility, very large scale, and well defined targets, provides accuracies in all three coordinates that are often difficult to achieve by direct measuring techniques. Many theoretical investigations and practical applications have demonstrated this fact (e.g., Brown, 1980; Faig, 1981; Faig and El-Hakim, 1982; Fraser, 1982; Granshaw, 1980; Kenefick, 1977). For a certain project, all the necessary parameters such as camera locations and orientation, control point distribution, and shape and size of targets can be designed and precisely applied to provide the geometrically strongest solution possible. However, in some projects it is impossible or very difficult to apply these ideal designs, particularly when it comes to placing the cameras at the planned locations or establishing control points with the desired accuracy. Therefore, successful close-range photogrammetric systems should be flexible enough to adapt to any situation rather than to follow conventional techniques based on aerial triangulation experience (El-Hakim, 1982; Granshaw, 1980). In particular, the following features should provide a great deal of flexibility, accuracy, and reliability in a close-range system:

- The camera-station location and orientation elements may be treated either as unknown, known, or partially known parameters depending on the possibility of measuring them. If any one of these parameters can be measured with sufficient accuracy, it is of advantage to include it in the solution and reduce the control point requirement.
- It is often the case that lack of intervisibility, tight time schedule, or the nature of the project site make it difficult to establish control points with the required accuracy. However, it is always possible

to measure some distances or height differences, which may not necessarily form a geometrically strong network or even a continuous network. In these cases, rather than utilizing the traditional two-step approach of separately adjusting terrestrial measurements to obtain control points for the photogrammetric adjustment, it is of great advantage to be able to utilize the available measurements directly with the photogrammetric data in a single adjustment.

- In many cases the use of non-metric cameras, and/or a different camera for each exposure station, is desirable. Therefore, the system must be capable of assigning different calibration parameters for each camera exposure and of applying the self-calibration approach.
- Because high accuracy is usually required, and because it is often too costly or even impossible to establish check points to evaluate the accuracy, it is essential that the system be "reliable" and that the accuracy assessment be realistic. It should have the capability of detecting gross errors using a statistically sound approach, such as "data snooping" (e.g., Grün, 1978). In addition, some studies (El-Hakim, 1981) show that the use of error ellipsoids provides accuracy figures equivalent to those obtained by check point discrepancies. On the other hand, figures computed from quantities such as scaled residuals follow another pattern and usually provide a higher, misleading accuracy.

The close-range version of the program GEBAT (El-Hakim, 1982) has all the features described above. It has been used in a project that required such flexible features.

## REVIEW OF PROGRAM GEBAT-V

This program is designed for close-range applications where either a metric or non-metric camera, or a number of different cameras, may be employed.

Derived from the general GEBAT system, the geodetic observations are restricted to spatial distances and height differences, while self-calibration is arranged in a photo-variant mode (i.e., each photograph has its own calibration parameters: calibrated principal distance, principal point coordinates, plus eight parameters of a harmonic function). The "V" designates the photo-"variant" mode. Gross error detection is applied using the data snooping approach. The program computes the redundancy number for each observation and applies a statistical test as described in (El-Hakim, 1981). Finally, the variance-covariance matrix and the error ellipsoid for each adjusted object point are computed.

The critical value for the data snooping test and the confidence level for the error ellipsoid form part of the input to be provided by the user. Furthermore, any information available about the camera-station parameters as well as ground coordinates of any point, not necessarily a control point, can be utilized in the program. Suitable weights must be introduced for such information. The program allows for the weighting of each station parameter or even of fixing it, and for the weighting of each coordinate of each object point.

The photogrammetric mathematical model is the self-calibration bundle adjustment model

$$\begin{aligned} x_A - x_0 + dV_x &= \\ -f \frac{(X_A - X_c)m_{11} + (Y_A - Y_c)m_{12} + (Z_A - Z_c)m_{13}}{(X_A - X_c)m_{31} + (Y_A - Y_c)m_{32} + (Z_A - Z_c)m_{33}} \\ y_A - y_0 + dV_y &= \\ -f \frac{(X_A - X_c)m_{21} + (Y_A - Y_c)m_{22} + (Z_A - Z_c)m_{23}}{(X_A - X_c)m_{31} + (Y_A - Y_c)m_{32} + (Z_A - Z_c)m_{33}} \end{aligned}$$

where  $m_{ij}$  ( $i, j = 1, 2, 3$ ) are the elements of rotation matrix  $\mathbf{R}^T$ ,

$X_A, Y_A, Z_A$  are the object coordinates of point A,  
 $X_c, Y_c, Z_c$  are the projection center coordinates,  
 $x_A, y_A$  are the image coordinates of point A,  
 $x_0, y_0$  are the principal point coordinates, and  
 $f$  is the principal distance.

Correction terms  $dV_x$  and  $dV_y$  are given by

$$dV_x = (x_A - x_0) \cdot T \quad (2)$$

and

$$dV_y = (y_A - y_0) \cdot T$$

with  $T$  being the harmonic function,

$$\begin{aligned} T &= a_{00} + a_{11} \cos \lambda + b_{11} \sin \lambda + a_{20}r \\ &+ a_{22}r \cos 2\lambda + b_{22}r \sin 2\lambda \\ &+ a_{31}r^2 \cos \lambda + b_{31}r^2 \sin \lambda. \end{aligned} \quad (3)$$

and

$$r = \sqrt{(x_A - x_0)^2 + (y_A - y_0)^2}, \quad \lambda = \arctan \frac{y_A - y_0}{x_A - x_0}.$$

The observation equations for the terrestrial measurements are as follows: for slope distance,  $S_{ij}$ ,

$$V_{S_{ij}} = C_1(dX_j - dX_i) + C_2(dY_j - dY_i) + C_3(dZ_j - dZ_i) + (S_c - S_0) \quad (4)$$

where  $C_1 = (X_j - X_i)/S = \Delta X/S$

$$C_2 = (Y_j - Y_i)/S = \Delta Y/S$$

$$C_3 = (Z_j - Z_i)/S = \Delta Z/S.$$

$S_c$  is computed as  $(\Delta X)^2 + (\Delta Y)^2 + (\Delta Z)^2)^{1/2}$  from approximate coordinates of points  $i$  and  $j$ , and  $S_0$  is the observed value. For height difference,  $h_{ij}$ ,

$$V_{h_{ij}} = (Z_j - Z_i) - \Delta h_0 \quad (5)$$

where  $\Delta h_0$  is the observed value.

For details and solution algorithms, see El-Hakim (1982).

### MEASUREMENTS OF MICROWAVE ANTENNA SUBASSEMBLIES

In order to receive clearly certain wavelengths, large microwave antennae must be manufactured with very high accuracy. The antenna considered in this paper, which has a diameter of more than 30 metres and is a paraboloid in shape, consists of a number of subassemblies, each approximately 2.7 m by 1.5 m in size, welded together in a delicate manufacturing process. These subassemblies have to be closely monitored to ensure high accuracy.

Photogrammetry has been considered as an alternative to the direct measurement technique that was being used by the manufacturer and was regarded as unsatisfactory by the purchaser. Keeping in mind the objective of achieving better than 0.15 mm accuracy, the project has been planned and executed as follows:

- **Camera.** A precision terrestrial camera with minimum distortion and stable parameters was selected. A Wild P-31 with maximum lens distortion of  $\pm 4 \mu\text{m}$  and wide angle lens ( $f = 100 \text{ mm}$ ) was available. This camera has four fiducial marks and a center cross. Glass plates of dimensions 102 by 127 mm, of which 83 by 117 mm represents the usable format, were used. The camera was calibrated in-house shortly before the project. The main objective of the calibration was to determine the camera's focal length and principal point coordinates.
- **Targets and Scale.** The subassemblies were targeted at 110 points in locations selected by the manufacturer. Figure 1 shows one of these panels and the target shape and locations. To achieve proper contrast to the grey aluminum, a white ring was centered around 1-mm clear circle (Figure 2). The exact target point is represented by a cross inside the clear circle. Using a Zeiss PSK stereo-comparator, a standard error of  $3 \mu\text{m}$  for the measurements could easily be achieved for these targets. An average scale of 1:25 (2.5 m from the center of the panel) was considered sufficient providing that systematic errors were controlled.
- **Control.** Due to an insufficient number of terres-

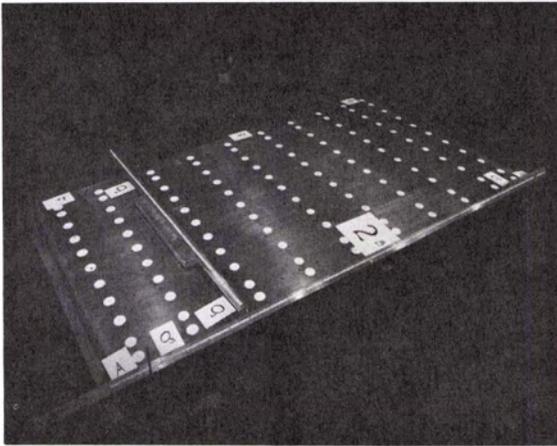


FIG. 1. One of the projects photographs.

trial observations, no point could be coordinated with sufficient reliability to be used as a control point for such a high accuracy project. Because the particular coordinate system was of no interest, one of the points at one end of the network was chosen as the origin of the coordinate system, while the X-coordinate of another point at the opposite end was given an arbitrary value of zero, to define the azimuth. These three coordinates plus three height control points represented the only fixed control values. Distances and height differences between six precisely established points were measured, and two perpendicular measuring tapes were placed flat on the panel surface to be adjusted simultaneously with the photogrammetric observations. Figure 3 shows the distribution of the various controls and observations (see also Figure 1).

- **Photography.** Due to its improved geometrical strength, highly convergent photography was employed. Four convergent photographs were taken from the corners of the panel, as shown in Figure 4. A platform was used to raise the exposure station

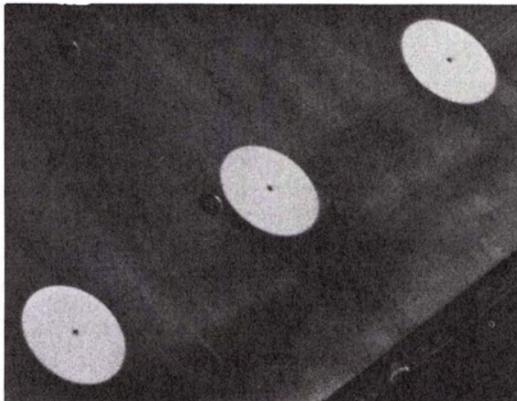
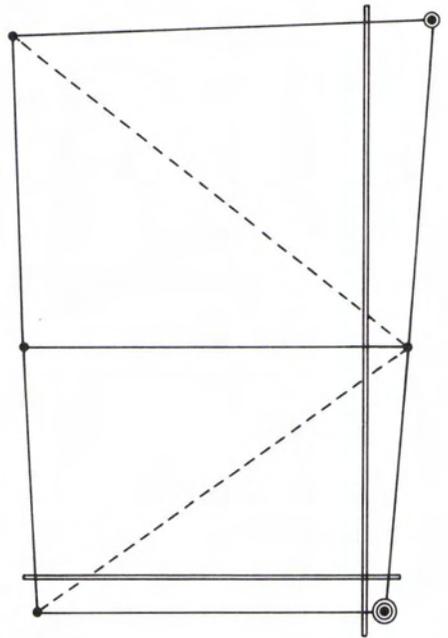


FIG. 2. Targets.



- free control
- ⊙ x-fixed control
- ⊗ x,y-fixed control
- highly accurate distance
- - - less accurate distance
- == tape

FIG. 3. Terrestrial observations and control.

to about 3 metres above ground. The photo stations were placed in such a way that all the points appear in every photograph. However, some points, due to the wide range of scale variation within the photograph, were precisely measurable from only three photographs. The plate negatives were mea-

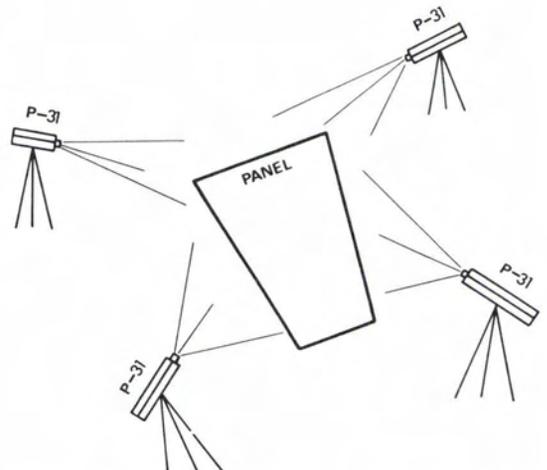


FIG. 4. Camera-station arrangement.

TABLE 1. ACCURACY FIGURES

Standard Errors	Without Self-calibration	With Self-calibration
—of unit wt. $\hat{\sigma}_0$	6 $\mu\text{m}$	4.5 $\mu\text{m}$
—of adjusted coordinates: $\hat{\sigma}(x)$	0.07 mm	0.05 mm
$\hat{\sigma}(y)$	0.07 mm	0.05 mm
$\hat{\sigma}(z)$	0.07 mm	0.05 mm
—error ellipsoid axes: $e(x)$	0.16 mm	0.12 mm
$e(y)$	0.17 mm	0.13 mm
$e(z)$	0.20 mm	0.15 mm

sured directly without converting them into diapositives in order to reduce the processing operations and thus the chance of additional systematic errors.

## RESULTS AND CONCLUDING REMARKS

Image coordinates of points measured on the four photographs, distances, and height differences were adjusted simultaneously with GEBAT-V. Applying the data snooping resulted in the rejection of an average of eight observations out of an average of 1050 image coordinates (about 0.75 percent) for each panel. This rejection was based on the test

$$\omega_i = v_i/\sigma_v = v_i/(\sigma_0\sqrt{q_i}) \geq K \quad (6)$$

where  $\omega_i$  is the standardized residual,  $v_i$  is the residual,  $q_i$  is the diagonal element  $i$  of matrix  $Q_{vv}$  (the weight cofactor matrix of the residuals), and  $K$  is a critical value for a specific confidence level. For these data,  $K$  has been chosen to be 3.29.

The adjustment was carried out with self-calibration. The focal length has been given a large weight because the calibrated focal length was used. Table 1 displays the results for one of the panels. The accuracy is evaluated based on the axes of the error ellipsoid at 95 percent confidence level. Other figures are given for comparison purpose. These are the standard error of unit weight  $\hat{\sigma}_0$  and the average standard deviations of the adjusted coordinates. The case of a basic adjustment without self-calibration is also shown. For this panel the accuracy is

0.12 mm (X), 0.13 mm (Y), and 0.15 mm (Z),

which is within the requirements. Self-calibration improved the results by about 25 percent, although the absolute value of the improvement is only 1.5  $\mu\text{m}$  in image coordinates.

To obtain the panel deformations, or its deviation from the ideal surface, a best fitting paraboloid was determined from the adjusted coordinates of the targets by the method of least squares. The paraboloid surface equation is

$$X^2 + Y^2 = 4CZ \quad (7)$$

where  $C$  is the focal length of the paraboloid. The

coordinates of the center of the surface, the orientation angles with X, Y, Z coordinate axes, and C are the unknowns. The RMS of the residuals at the targeted points did not exceed 0.10 mm for the tested panels.

In conclusion, it has been demonstrated that a flexible close-range photogrammetric system can achieve a high accuracy with minimum control utilizing available terrestrial observations. This reduces effort, time, and cost compared to a direct measurement technique or a traditional photogrammetric approach.

## ACKNOWLEDGMENT

The author wishes to thank Al Way-Nee, of his affiliation, for his help with the field work.

## REFERENCES

- Brown, D. C., 1980. *Application of Close-Range Photogrammetry to Measurements of Structures in Orbit*, GSI Technical Report No. 80-012, GSI, Melbourne, Fla.
- El-Hakim, S. F., 1981a. "An Evaluation of the Different Criteria to Express Photogrammetric Accuracy," ASP Fall Technical Meeting, San Francisco, Calif.
- , 1981b. A Practical Study of Gross-Error Detection in Bundle Adjustment, *The Canadian Surveyor*, Vol. 35, No. 4.
- , 1982. *The General Bundle Adjustment Triangulation (GEBAT) System—Theory and Application*, Photogrammetric Research, Division of Physics Report P-PR53 National Research Council of Canada.
- , 1984. Precision Photogrammetry for Microwave Antennae Manufacturing, International Archives of Photogrammetry and Remote Sensing, Vol. 25, Part A5, pp. 232-239.
- Faig, W., 1981. Close-Range Precision Photogrammetry for Industrial Purposes, *Photogrammetria*, Vol. 36, pp. 183-191.
- Faig, W., and S. F. El-Hakim, 1982. "The Use of Distances as Object Space Control In Close-Range Photogrammetry," Symposium of Commission V, ISPRS,

- on Precision and Speed in Close Range Photogrammetry, York, England.
- Fraser, C. S., 1982. Optimization of Precision in Close-Range Photogrammetry, *Photogrammetric Engineering and Remote Sensing*, Vol. 48, No. 4, pp. 561-570.
- Granshaw, S. I., 1980. Bundle Adjustment Methods in Engineering Photogrammetry, *Photogrammetric Record*, 10(56), pp. 181-207.
- Grün, A., 1978. "Accuracy, Reliability and Statistics in Close-Range Photogrammetry," ISPRS Symposium, Commission V, Stockholm.
- Kenefick, J. F., 1977. Applications of Photogrammetry in Shipbuilding, *Photogrammetric Engineering and Remote Sensing*, Vol. 43, No. 9, pp. 1169-1175.

(Received 31 March 1984; accepted 11 March 1985; revised 6 May 1985)

## CALL FOR PAPERS

### International Symposium on Photogrammetric and Remote Sensing Systems for Data Processing and Analysis

Sheraton Inner Harbor, Baltimore, Maryland  
26-30 May 1986

The Inter-Congress Symposium of Commission II of the International Society for Photogrammetry and Remote Sensing has as its theme "Photogrammetric and Remote Sensing Systems for Data Processing and Analysis." Technical program will consist of papers on instrumentation in the following areas:

- Analytical Instruments
- Photogrammetric Digital Image Processing Systems
- Systems for Reception, Recording, Preprocessing, Archiving, and Dissemination of Remotely Sensed Data
- Integrated Photogrammetric Systems
- Systems for Analysis of Remotely Sensed Data
- Systems and Instrumentation for Synthetic Aperture Radar Processing
- Optical Photographic Instruments and Methods for Remote Sensing

Abstracts of papers should be submitted to the Commission President or Secretary by *30 November 1985*. A one-page abstract of less than 250 words, in English, should include the paper's title and the author's name, affiliation, and address. Emphasis should be on advances and new developments. Authors of papers accepted for presentation will be notified by 15 March 1986. Camera-ready copies of the accepted papers must be received by 30 April 1985. Special form paper will be provided to authors of accepted papers. Papers submitted on time and in the proper format will be published in formal Symposium proceedings, available 31 July 1986.

Please send abstracts to

Lawrence W. Fritz  
President, Comm. II, ISPRS  
NOAA/NOS N/CG21  
6001 Executive Blvd.  
Rockville, MD 20852, USA

or

William P. Bishop  
Secretary, Comm. II, ISPRS  
NOAA/NESDIS Ex 1  
FOB 4, Room 2069  
Washington, DC 20233, USA