

# Calibrating Stereo Plotter Encoders

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**ABSTRACT:** A stereo plotting instrument was not performing accurately, due to a problem in the digital measuring system. A description of how the problem was diagnosed and corrected is given.

## INTRODUCTION

AN IMPORTANT PART of quality control in photogrammetric production is a complete knowledge of a stereo plotting instrument's performance characteristics. This was vividly demonstrated in our office recently. For some time our Wild B8S had not been performing as accurately as should be expected in semi-analytical aerotriangulation or digital terrain modeling. Whenever a three-dimensional coordinate transformation was computed using measurements from this instrument, the model would not fit its ground control points properly, even when the control was known to be reliable. The same model could be scaled and leveled to the same control for compilation with no problem. There was apparently some kind of a problem in the three-dimensional digital measurements.

## THE MEASURING SYSTEM

The measuring system on the B8S is known as the "tri-axis locator." The  $x$  and  $y$  axes are digitized using a pair of rack and pinion gear systems. Rotary shaft encoders are attached to the pinion gears, translating the planimetric movement of the tracing stand into electronic signals for the digitizer. The  $z$  axis is digitized using a third rotary encoder which is driven through a gear train from the  $z$  crank on the tracing stand. In the time before our problem was first detected, one faulty encoder had been replaced. Later, the old digitizer unit was retired and replaced with a new system from a different manufacturer. Either of these events had the potential for causing the kind of trouble we were experiencing.

## LOOKING FOR THE PROBLEM

Two questions quickly came to mind. Do all three encoders measure at the same scale? Are all three axes really perpendicular to one another? At first glance, these seemed like simple questions to answer. Closer consideration proved otherwise.

The scale question could be resolved in two ways. The physical size of a "digit," as measured by each individual encoder, could be determined by comparing each encoder against a known value. Or, because the true size of a "digit" is of little consequence, the encoders could simply be compared to one another. Relative scale corrections could then be applied if necessary.

The relative test seemed like the easiest one. All we had to do was test each encoder on the same axis by moving through the same distance and observing any difference in the measurement. The first test measurement was made using the  $x$  encoder on its own axis. With the  $y$  and  $z$  axes locked, the tracing stand was moved so that the left half of the floating mark traveled precisely from one tick mark to another on the diapositive carrier. The measurement was recorded. After a few minutes with a screwdriver and a hex wrench, the  $z$  encoder was installed on the  $x$  axis and moved across the same distance. It was then obvious that a direct comparison of the encoders in this manner could not be done. Because of the difference in gear ratios between the  $z$  spindle and the  $x$ - $y$  rack and pinion system, the  $z$  encoder is clearly designed to deliver a substantially different count per revolution than the  $x$  and  $y$  encoders. The alternate approach of comparing each encoder against an established standard was dictated.

## CALIBRATING THE ENCODERS

Calibrating the  $z$  encoder was easy. The digital readings corresponding to nine elevations on one of the glass compilation scales were recorded. The scale factor was computed by fitting the line

$$H = S z + z_0$$

to the data, using the method of least squares, where

$H$  = elevation from the glass scale (millimetres),  
 $S$  =  $z$  encoder scale factor (millimetres/digit),  
 $z$  = observed digital elevation (digits), and  
 $z_0$  = initial index (millimetres).

Using this method, the  $z$  scale factor was found to be 12.500 millimetres/digit.

Calibrating the  $x$  and  $y$  encoders was essentially the same as calibrating a comparator by measuring a precision grid plate and using an affine coordinate transformation to determine the scale factors and non-perpendicularity of the axes. All of the required resources were available. The question was where to put the grid plate? If it was placed on one of the diapositive carriers, there would be no way to insure an exact 1:1 conformal projection into the model space, where the measurements are made. Because our calibration grid is on a one quarter inch thick glass plate, it could not be placed in the model space, under the tracing stand. Even if a grid could be placed under the tracing stand, the only way to measure it would be to point at the intersections with the drawing pencil. This, of course, cannot be done with sufficient precision for instrument calibration.

The problem was solved by creating and measuring new points and calibrating them later instead of measuring pre-calibrated ones. A piece of 24- by 30-inch clear film was fastened to the granite slab under the tracing stand. At the center of the model space, the corners of a 9- by 9-inch area were marked in ink. Inside this square, 25 small circles were inked at approximately 50-millimetre intervals in a 5 by 5 array, as shown in Figure 1.

Next, a steel point was placed in the pencil chuck of the tracing stand. It was moved to each of the circles, dropped, and tapped gently into the clear film, making a mark almost invisible to the naked eye. The  $x$  and  $y$  tri-axis locator coordinates were recorded as each point was marked. When all 25

points were finished, the piece of film was removed from the instrument and the 9- by 9-inch area was cut out from the rest of the sheet. We now had our own "calibration plate" with measurements, except that the calibrated positions of the points were unknown.

To obtain "true" positions for the steel point marks, the clear film "plate" was measured on our Kern MK2 monocomparator. In the comparator, the marks appear as very well defined black circles, approximately 75 micrometres in diameter, and are quite easy to measure. The glass precision grid plate was used to cover the film on the comparator stage. This allowed the film marks to appear as arbitrarily located unknown points relative to the grid plate. The orientation of the film with respect to the grid was of no real concern except that the points were arranged so as not to coincide so closely that they interfered with each other. Measurements were then made of the 25 steel point marks and the 25 intersections of the grid plate without disturbing this arbitrary orientation.

A two-dimensional affine coordinate transformation, controlled by the grid plate intersections, was made to compute the coordinates of the film marks in the coordinate system of the grid plate, as shown in Figure 2. This transformation would remove most of the systematic instrumental error inherent in the comparator measurements and provide reliable (though arbitrary) positions with which to calibrate the tri-axis locator.

The final step was to compute the parameters of a second two-dimensional affine coordinate transformation, this one being between the tri-axis locator coordinates of the steel point marks and their newly computed corresponding values in the grid plate coordinate system. The results of this transformation are shown in Figure 3. No unknown points were actually transformed. The desired information was the transformation parameters themselves, from which could be extracted the scale factor for each axis and the non-perpendicularity between them.

## THE RESULTS

The scale factors for the tri-axis locator  $x$  and  $y$  axes were found to be 12.700 and 12.702 millimetres/digit respectively (a "digit" turns out to be nominally one-half inch). The axes were also found to be 59.2 seconds out of perpendicular. Looking back to the calibration of the  $z$  axis, which had a scale factor of 12.500 millimetres/digit, it can be seen immediately why we were having trouble with three-dimensional conformal coordinate transformations. The model measurements were not the same scale horizontally as vertically. We were essentially trying to fit rectangles into squares! The slight difference in the  $x$  and  $y$  scales, and the non-perpendicularity between them, also contribute to



FIG. 1. The geometry of the marks made with the steel point on the B8S.

TWO DIMENSIONAL AFFINE COORDINATE TRANSFORMATION

$$X1 = AX2 + BY2 + C$$

$$Y1 = DX2 + EY2 + F$$

COMPUTE JENA GRID PLATE SYSTEM COORDINATES FOR B8 MARKS

TRANSFORMATION PARAMETERS

A= .999929623 (SX= .999993007)  
 B= .011191034 (SY= .999978669)  
 C= -117.846539270 ( E= -14.0 SECONDS)  
 D= -.011258985  
 E= .999916046  
 F= -116.507525374

STD ERROR UNIT WEIGHT= .003 ITERATIONS= 3

RESIDUALS ON CONTROL POINTS

POINT	VX2	VY2	VX1	VY1
1	-.001	-.001	.000	.000
2	-.003	-.001	.000	.000
3	-.007	.001	.000	.000
4	.000	.003	.000	.000
5	.007	-.003	.000	.000
6	-.005	.003	.000	.000
7	-.001	.003	.000	.000
8	-.007	-.002	.000	.000
9	-.001	-.003	.000	.000
10	.001	-.005	.000	.000
11	.001	-.002	.000	.000
12	.001	-.005	.000	.000
13	-.004	-.001	.000	.000
14	.000	.000	.000	.000
15	.000	.000	.000	.000
16	.000	-.003	.000	.000
17	.006	.005	.000	.000
18	-.002	-.001	.000	.000
19	-.001	-.002	.000	.000
20	.000	-.001	.000	.000
21	.001	.000	.000	.000
22	-.002	-.002	.000	.000
23	-.002	-.003	.000	.000
24	-.001	.000	.000	.000
25	.003	.001	.000	.000

SYSTEM 1 COORDINATES FOR TRANSFORMED POINTS

PLATE	POINT	X	Y
1130	801	-97.679	-98.689
1130	802	-49.656	-99.749
1130	803	-.953	-101.269
1130	804	49.596	-102.766
1130	805	97.910	-104.087
1130	806	-99.450	-51.019
1130	807	-48.017	-50.400
1130	808	1.068	-51.803
1130	809	51.977	-53.056
1130	810	100.124	-51.954
1130	811	-99.651	-1.602
1130	812	-50.137	-2.438
1130	813	-.250	-1.693
1130	814	49.289	-2.836
1130	815	100.069	-4.282
1130	816	-94.639	45.715
1130	817	-49.453	47.338
1130	818	2.117	46.876
1130	819	52.198	46.852
1130	820	102.277	46.284
1130	821	-98.965	96.314
1130	822	-49.205	97.292
1130	823	-1.114	97.268
1130	824	51.369	95.813
1130	825	100.823	96.592

NUMBER OF POINTS TRANSFORMED= 25

TWO DIMENSIONAL AFFINE COORDINATE TRANSFORMATION

$$X1 = AX2 + BY2 + C$$

$$Y1 = DX2 + EY2 + F$$

TRANSFORM B8 MEASUREMENTS TO JENA GRID PLATE SYSTEM

TRANSFORMATION PARAMETERS

A= 12.700044103 (SX= 12.700082809)  
 B= .027714040 (SY= 12.702028129)  
 C= -2625.077266139 ( E= -59.2 SECONDS)  
 D= -.031355021  
 E= 12.701997895  
 F= -5081.092972783

STD ERROR UNIT WEIGHT= .002 ITERATIONS= 3

RESIDUALS ON CONTROL POINTS

POINT	VX2	VY2	VX1	VY1
801	-.003	.000	.000	.000
802	-.003	-.003	.000	.000
803	.003	-.002	.000	.000
804	.001	.002	.000	.000
805	.003	.002	.000	.000
806	.000	.001	.000	.000
807	-.001	.001	.000	.000
808	-.003	.000	.000	.000
809	-.001	-.001	.000	.000
810	-.002	-.002	.000	.000
811	.004	.002	.000	.000
812	.001	.002	.000	.000
813	.002	.001	.000	.000
814	-.001	-.001	.000	.000
815	.001	.002	.000	.000
816	.003	.000	.000	.000
817	.002	.000	.000	.000
818	-.001	-.001	.000	.000
819	.001	.000	.000	.000
820	-.003	-.001	.000	.000
821	-.001	.000	.000	.000
822	-.003	-.001	.000	.000
823	-.002	.000	.000	.000
824	-.001	-.001	.000	.000
825	.003	.001	.000	.000

SYSTEM 1 COORDINATES FOR TRANSFORMED POINTS

PLATE	POINT	X	Y
1130	801	-97.679	-98.689
1130	802	-49.656	-99.749
1130	803	-.953	-101.269
1130	804	49.596	-102.766
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1130	821	-98.965	96.314
1130	822	-49.205	97.292
1130	823	-1.114	97.268
1130	824	51.369	95.813
1130	825	100.823	96.592

NUMBER OF POINTS TRANSFORMED= 0

FIG. 3. Transformation of the B8S tri-axis locator measurements of the steel point marks to the grid plate coordinate system. The scale factors of the x and y axes of the tri-axis locator were determined by this transformation.

horizontally and vertically. The root-mean-square error of the vertical control point residuals was cut roughly in half. The instrument was once again performing in its expected accuracy range.

For routine applications, the problem is now corrected by entering the 12.500/12.700 vertical scale factor when initializing the digitizer. For higher accuracy applications, software will be written to apply corrections not only to the vertical scale, but also for the difference in horizontal scales and the non-perpendicularity of the x and y axes. This may prove to be a better solution than making physical adjustments to the instrument.

Detecting and correcting this problem did more than just return the instrument to its proper performance. It also relieved the frustration level of the operator, who knew first hand that the measurements were better than their statistics indicated. This made the correction effort even more worthwhile.

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FIG. 2. Transformation of the comparator measurements of the steel point marks to the grid plate coordinate system. Points 1 to 25 are the grid plate intersections. Points 801 to 825 are the steel point marks.

the problem, but are negligible when compared to the difference in the vertical scale.

CORRECTING THE PROBLEM

Now that the systematic errors have been quantified, they can be corrected mathematically. As a test, some data that had been measured recently was retrieved from storage. The z coordinates were multiplied by the factor 12.500/12.700 and the three-dimensional coordinate transformations were re-computed. The improvement was dramatic, both