Calibrating Non-Metric Cameras Using the Finite-Element Method

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ABSTRACT: A new photogrammetric mathematical model based on the collinearity equation and the finite-element has been investigated in detail. In the new model, comparator coordinates are used directly in the photogrammetric observation equations. An experimental verification of the proposed technique has been carried out. The experimental results support the conclusion that the newly derived mathematical model offers a theoretical and practical alternative to existing models for the self-calibration of non-metric cameras.

INTRODUCTION

THE FINITE-ELEMENT METHOD has been used in many engineering fields during the last two and a half decades. Recently it has been used in digital terrain modeling (Ebner and Reiss, 1978; Ebner *et al.*, 1980; Ebner and Reiss, 1981) and in camera calibration (Munjy, 1982; Munjy, 1986). The current state-of-the-art techniques reported for analytical restitution of non-metric cameras include the direct linear transformation (DLT) (Abdel Aziz and Karara, 1971), the 11-parameter solution (Bopp and Krauss, 1978), analytical self-calibration with either block-invariant additional parameters or a block-variant approach (e.g., Faig, 1975), and combined block-invariant and photo-variant additional parameters. These techniques have a common starting point; compensation of systematic image coordinate errors by analytical models employed directly in the photogrammetric projective equations. The parameters defining the systematic error models are then recovered simultaneously with the projective parameters (position, orientation, focal length, principal point) in a least-squares adjustment leading to the minimization of the quadratic sum of the residuals of measured quantities. Most reported analytic models representing film shrinkage, film unflatness, and radial lens distortion are assumed to be valid throughout the image plane. In the finite-element approach the image plane domain is divided into subdomains or finite elements and then a mathematical model for systematic errors is prescribed over the image plane domain in a piecewise fashion, element by element, thus eliminating the assumption of symmetry (e.g., Munjy, 1982; Munjy, 1986).

REVIEW OF THE FINITE ELEMENT APPROACH IN CAMERA CALIBRATION

It was shown by Munjy (1986) that, by dividing the image domain into triangular elements and assuming that each point on the photograph will have a different focal length, the collinearity condition equations will take the form:

$$x_{ij} - x_{pi} = f_{ij}\,(X'/Z')_{ij} \qquad (1)$$
$$y_{ij} - y_{pi} = f_{ij}\,(Y'/Z')_{ij}$$

where

$$\begin{bmatrix} X' \\ Y' \\ Z' \end{bmatrix} = \mathbf{M}_i \begin{bmatrix} X_j - X_i^c \\ Y_j - Y_i^c \\ Z_j - Z_i^c \end{bmatrix},$$

x_{pi}, y_{pi} = principal point coordinate of the ith photograph,
f_{ij} = focal length at the jth point of the ith photograph,
x_{ij}, y_{ij} = observed photo coordinates of point j on the ith photograph,
X_j, Y_j, Z_j = object space coordinates of the jth point,
X_i^c, Y_i^c, Z_i^c = object space coordinates of the ith exposure station, and
$\mathbf{M}_i$ = unitary orthogonal orientation matrix of the ith photograph.

It was also shown by Munjy (1986) that, by dividing the image domain into rectangular elements, the collinearity equations will take the following form:

$$x_{ij} - x_{pi} = g_i\,(f)\,(X'/Z')_{ij} \qquad (2)$$
$$y_{ij} - y_{pi} = g_i\,(f)\,(Y'/Z')_{ij}$$

where

$$g_i\,(f) = [1 - x/a, x/a] \begin{bmatrix} f_{k,l} & f_{k+1,l} \\ f_{k,l+1} & f_{k+1,l+1} \end{bmatrix} \begin{bmatrix} 1-y/b \\ y/b \end{bmatrix}$$

$f_{k,l}$ = the focal length at node k,l, and
a, b = the dimension of the rectangular element in the x and y directions, respectively.

PHOTOGRAMMETRIC ENGINEERING AND REMOTE SENSING,
Vol. 52, No. 8, August 1986, pp. 1201–1205.

0099-1112/86/5208-1201$02.25/0

CALIBRATION OF CAMERAS WITHOUT FIDUCIAL MARKS

Normally, the photo coordinates in Equations 1 and 2 are obtained by transforming the comparator coordinates using the calibrated data for the fiducial marks. If fiducial marks do not exist, as is the case in most non-metric cameras, Equations 1 and 2 need to be modified. Mathematically, the conformal transformation from the comparator coordinates to photo coordinates can be expressed as

$$\begin{bmatrix} x \\ y \\ f \end{bmatrix}_{ij} = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \overline{x} \\ \overline{y} \\ f \end{bmatrix}_{ij} + \begin{bmatrix} \overline{x}_o \\ \overline{y}_o \\ 0 \end{bmatrix}_i$$

and in matrix notations as

$$\mathbf{SX} = \overline{\mathbf{M}}\,\overline{\mathbf{X}} + \overline{\mathbf{SXO}} \tag{3}$$

where

x_{ij}, y_{ij} = transformed photo coordinates,
$\overline{x}_{ij}, \overline{y}_{ij}$ = measured comparator coordinates,
$\overline{x}_{oi}, \overline{y}_{oi}$ = translation elements between the two coordinate systems, and
θ_i = the angle between the photo coordinate and the comparator coordinate axes of the ith photograph.

Equation 1 can be rewritten as

$$\begin{bmatrix} x \\ y \\ f \end{bmatrix}_{ij} = \mathbf{M}_i \begin{bmatrix} X_j c - X_i \\ Y_j c - Y_i \\ Z_j c - Z_i \end{bmatrix} + \begin{bmatrix} x_P \\ y_P \\ 0 \end{bmatrix}_i$$

or

$$\mathbf{SX} = \mathbf{M}\,\mathbf{XC} + \mathbf{SXP} \tag{4}$$

Substitute for **SX**, Equation 3 in Equation 4. Then

$$\overline{\mathbf{M}}\,\overline{\mathbf{X}} + \overline{\mathbf{SXO}} = \mathbf{M}\,\mathbf{XC} + \mathbf{SXP}. \tag{5}$$

Because $\overline{\mathbf{M}}$ is an orthogonal matrix (i.e., $\overline{\mathbf{M}}^T = \overline{\mathbf{M}}^{-1}$), then Equation 5 can be rewritten as

$$\mathbf{X} = (\overline{\mathbf{M}}^T \mathbf{M})\,\mathbf{XC} + \overline{\mathbf{M}}^T\,\mathbf{SXO} \tag{6}$$

where

$$\mathbf{SXO} = \begin{bmatrix} x_P - x_o \\ y_P - y_o \\ 0 \end{bmatrix}_i = \begin{bmatrix} x'_P \\ y'_P \\ 0 \end{bmatrix}$$

The transformation matrix $\overline{\mathbf{M}}$ has a rotational angles equal to θ around the f-axis. The transformation matrix **M** has rotational angles of w, ϕ, κ between the x,y,f axes and the X,Y,Z axes. The transformation matrix $\mathbf{M}' = \mathbf{M}^T\,\mathbf{M}$ has rotational angles w, ϕ, $\kappa - \theta$ between the x,y,f axes and the X,Y,Z axes. So $\mathbf{M}'$ is a transformation matrix between the object space coordinates (X,Y,Z) and the comparator coordinates (x,y,f). Equation 6 can be rewritten as

$$\begin{aligned} x_{ij} &= f_{ij}\,(X''/Z'')_{ij} + x'_{pi}\cos\theta_i - y'_{pi}\sin\theta_i \\ y_{ij} &= f_{ij}\,(Y''/Z'')_{ij} + x'_{pi}\sin\theta_i + y'_{pi}\cos\theta_i \end{aligned} \tag{7}$$

where

$$\begin{bmatrix} X'' \\ Y'' \\ Z'' \end{bmatrix}_{ij} = \mathbf{M}_i \begin{bmatrix} X_j - X_i^c \\ Y_j - Y_i^c \\ Z_j - Z_i^c \end{bmatrix}$$

Equation 7 was derived on the basis that the image plane was divided into triangular elements. If the image plane were divided into rectangular elements, Equation 7 would have the following form:

$$\begin{aligned} x_{ij} &= g_i\,(f)\,(X''/Z'')_{ij} + x'_{pi}\cos\theta_i - y'_{pi}\sin\theta_i \\ y_{ij} &= g_i\,(f)\,(Y''/Z'')_{ij} + x'_{pi}\sin\theta_i + y'_{pi}\cos\theta_i \end{aligned} \tag{8}$$

Let the symbol $\dot{\mathbf{u}}_i$ denote the exterior orientation parameters, the comparartor rotation angle, and the comparator translation parameters $(X,Y,Z,w,\phi,\kappa,\theta,x'_p,y'_p)$ of the ith photograph, the symbol $\ddot{\mathbf{u}}_i$ denote the focal length at point j (f_{ij}) of the ith photograph if triangular elements were used, and the symbol $\dddot{\mathbf{u}}_j$ denote the object space coordinate (X,Y,Z) of the jth point. For rectangular elements, the symbol $\ddot{\mathbf{u}}_i$ denotes the focal length at node k,l $(f_{k,l})$ of the ith photograph (Munjy, 1986). Note that in the above parameter vectors there are no lens distortion coefficients as in the self-calibration method. Radial lens distortion is accounted for by variable focal length. Equations 7 or 8 are linearized by Taylor's series expansion about the initial approximations $(\dot{\mathbf{u}}^o, \ddot{\mathbf{u}}^o, \dddot{\mathbf{u}}^o)$ for the unknown parameters. Using the least-squares method, the above equations can be solved for the elements of exterior orientation, the comparator rotation and translation, the focal length, and the object space coordinates of the points (ASP, 1980; Brown, 1974; Munjy, 1986).

EXPERIMENTAL VERIFICATION OF THE FINITE ELEMENT APPROACH IN CALIBRATING NON-METRIC CAMERAS

In order to verify the proposed finite element non-metric camera calibration technique and also to assess its practicability, an experiment was conducted. The camera system used in the experiment consisted of two Bronica ETRS non-metric cameras, each with a Zenzanon ETR 150-mm *f*/3.5 lens and a 120 film magazine. The cameras, which were assigned numbers (ONE) and (TWO), were placed 1.00-m apart with their axes parallel to each other. The photography was taken with an exposure time of 1/60 sec at *f*/5.6 and the two cameras were focused at 7.0 m. At *f*/5.6 the depth of field was large enough so that all image points were in clear focus.

CONTROL FIELD

A three-dimensional object space control field was used. This field has 31 points, 20 of the points lying in a plane with the remaining 11 points on four piano wires suspended as plumb lines, each weighted with a heavy plumb bob immersed in an oil bath at one end and fixed to the ceiling by hooks at the other end. Small seed-beads, approximately 3 mm in diameter, are fixed on the wires to serve as target points (Figure 1). The configuration of the control field and the camera system is illustrated in Figure 2.

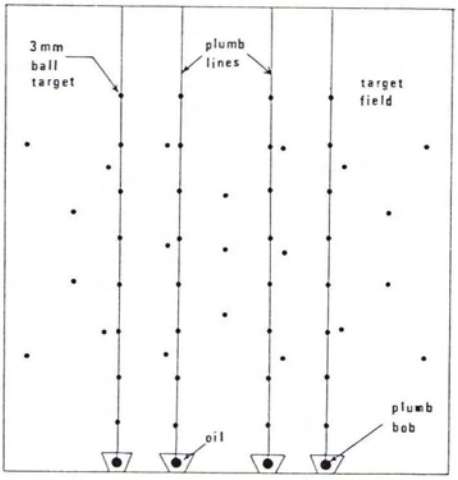

FIG 1. Front view for the control field.

Precise theodolite surveys were carried out to determine the object space coordinates of the control points. A Zeiss TH2 theodolite was used to measure the horizontal and vertical angles to the ends of a baseline, which was accurately measured by a steel tape lying flat on the floor. The mean standard error of X, Y, Z coordinates was 0.084 mm.

IMAGE COORDINATES MEASUREMENTS

Image coordinates were observed on a Kern-MK2 monocomparator. In order to ensure rapid convergence of the camera calibration program, space resection solutions were carried out to obtain reasonably refined preliminary estimates for the values of the exterior orientation elements at each camera station.

RESULTS AND ANALYSIS

Each Bronica ETRS camera lens system was calibrated individually by assuming that all object space coordinates of the points that were imaged in each photograph were free from errors, and by dividing the image plane into triangular elements. The calibration results are listed in Table 1.

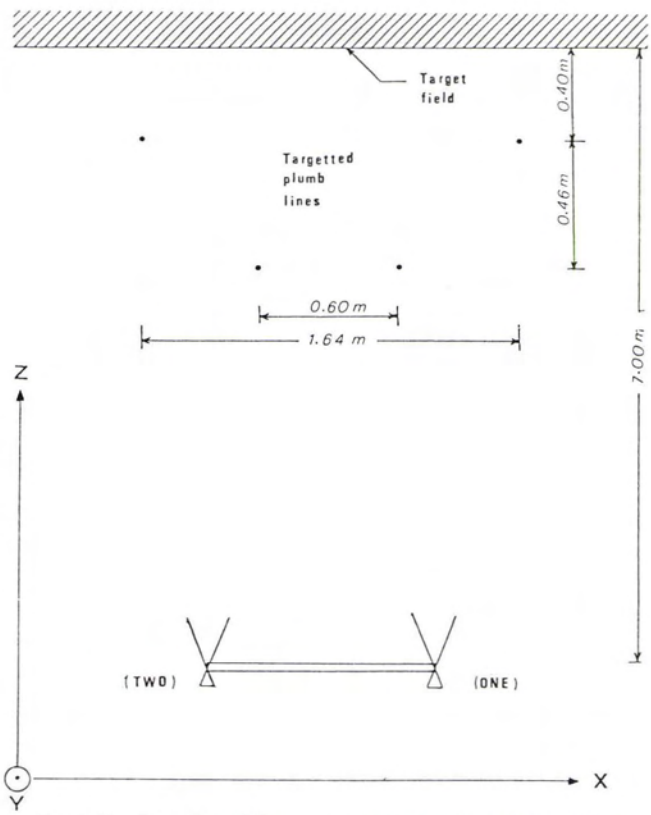

FIG 2. Configuration of the control field and the camera system.

TABLE 1. CALIBRATION RESULTS USING THE FINITE-ELEMENT METHOD.

	Camera System	
	One	Two
Average focal length.	153.255mm	153.245mm
Mean standard error of the focal length.	0.041mm	0.042mm
Mean standard error of the principal point coordinates.	0.017mm	0.019mm
The RMS error of the adjusted machine coordinates.	0.003mm	0.003mm
Lens distortion variation (with respect to the average focal length).	(−0.033 to +0.087mm)	(−0.035 to +0.076mm)

The focal length contour lines in the image plane for camera system ONE and TWO are shown in Figures 3 and 4, respectively. On computing the object space coordinates by intersection, the root-mean-square closure error of the X, Y, Z coordinates was found to be 0.590 mm. This is a closure accuracy of 1/11865 of the photographic distance.

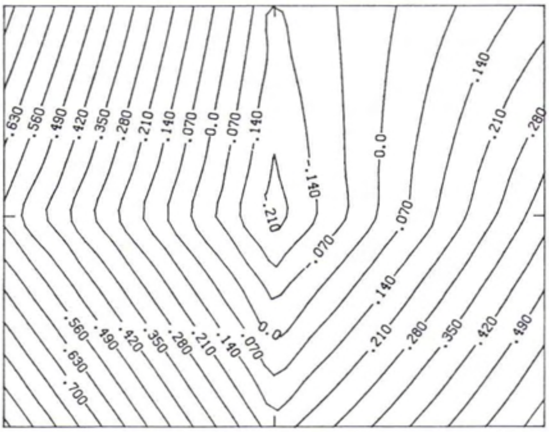

FIG 3. Bronica ETRS (ONE) focal length contour lines. Initial focal length = 153.00 mm. Contour interval = 0.080 mm.

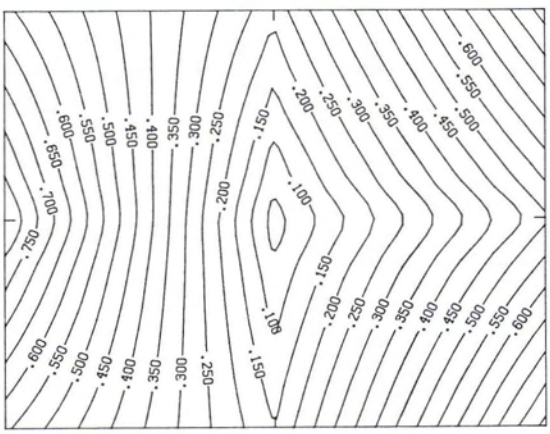

FIG 4. Bronica ETRS (TWO) focal length contour lines. Initial focal length = 153.00 mm. Contour interval = 0.080 mm.

TABLE 2. CALIBRATION RESULTS USING THE SELF-CALIBRATION METHOD WITH AN ODD ORDER POLYNOMIAL.

	Camera System	
	One	Two
The RMS error of the adjusted image coordinates	0.006mm	0.007mm
Radial lens distortion variation	(−0.030mm to 0.082mm)	(−0.035mm to 0.076mm)

TABLE 3. A COMPARISON BETWEEN THE RESULTS OF THE FINITE-ELEMENT METHOD AND THE SELF-CALIBRATION METHOD.

	Camera System	
	One	Two
Lens distortion variation difference between the finite-element method and self-calibration.	(−0.017mm to 0.031mm)	(−0.016mm to 0.016mm)
Average lens distortion difference between the finite-element method and self-calibration.	0.014mm	0.011mm

In addition, the radial lens distortion for each camera was computed using the self-calibration method with an odd power polynomial. The results of this calibration are listed in Table 2. A comparison between the results of the finite-element method and the self-calibration method is listed in Table 3.

CONCLUSIONS

The finite-element approach for non-metric camera calibration developed in the previous sections has been incorporated into the calibration of the Bronica ETRS non-metric camera. Based on the results obtained in this investigation, a number of conclusions can be drawn regarding the use of the finite-element approach in calibrating non-metric cameras:

- The finite-element approach can be successfully applied to the analytical restitution of non-metric imageries. The finite-element approach gives a good representation of the systematic errors in analytical photogrammetry such as lens distortion and film shrinkage.
- The finite-element approach has the additional advantage of reducing the effects of film unflatness which, according to Fraser (1982), is the single most significant factor limiting the attainable accuracy of non-metric image restitution.

REFERENCES

Abdel-Aziz, Y. I., and H. M. Karara, 1971. Direct Linear Transformation from Comparator Coordinates into Object Space in Close Range Photogrammetry. *Proceedings of ASP Symposium on Close Range Photogrammetry*, Urbana, pp. 1–18.

American Society of Photogrammetry, 1980. *Manual of Photogrammetry*, 4th Edition.

Bopp, H., and H. Krauss, 1978. An Orientation and Calibration Method for Non-Topographic Applications, *Photogrammetric Engineering and Remote Sensing*, Vol. 44, pp. 1191–1196.

Brown, D. C., 1974. Close Range Camera Calibration, *Photogrammetric Engineering*, Vol. 38, No. 8, pp. 855–866.

Ebner, H., and P. Reiss, 1978. Height Interpolation by the Method of Finite Elements. Presented paper ASP, Digital Terrain Symposium, St. Louis.

———, 1981. Experiences with Height Interpolation by Finite Elements. ASP-ACSM Fall Technical Meeting, San Francisco—Honolulu.

Ebner, H., B. Hofmann-Wellenhof, P. Reiss, and F. Steidler, 1980. HIFI—A Minicomputer Program Package for Height Interpolation by Finite Elements. Commission IV, ISP—Congress, Hamburg.

Faig, W., 1975. Calibration of Close Range Photogrammetric System: Mathematical Formulation. *Photogrammetric Engineering and Remote Sensing*. Vol. 41, No. 12, pp. 1479–1486.

Fraser, C. S., 1982. On the Use of Non-Metric Cameras in Analytical Close Range Photogrammetry, *Canadian Surveyor*, Vol. 36, No. 3, pp. 259–279.

Munjy, R., 1982. *The Applications of the Finite Element in Photogrammetry*, Ph.D. Dissertation, University of Washington, Seattle, Washington.

———, 1986. Self-Calibration Using the Finite Element Approach, *Photogrammetric Engineering and Remote Sensing*, Vol. 51, No. 3, pp. 411–418.

(Received 20 September 1985; revised and accepted 13 January 1986)

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