# Parallelism of the Stereometric Camera Base to the Datum Plane in Close-Range Photogrammetry

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ABSTRACT: For a facade elevation drawing compiled using an analog plotter, the stereometric camera base should be parallel to the object plane at the time of exposure. The displacements in the single and stereo-image due to rotation of the object plane are discussed. In order to make the focal plane parallel to the datum plane, an auxiliary apparatus was developed. The apparatus consists mainly of a double-pentagonal prism and a base bar with two miniature rods set up at the ends of the base bar. An application of this apparatus at a test field and the obtained accuracy are presented.

#### INTRODUCTION

**T**HE NORMAL CASE of terrestrial photogrammetry is realized most conveniently by using stereometric cameras in which the base is fixed.

The advantage of such an arrangement is that the relative orientation between the two cameras is fixed. The resulting photographs can be measured quite precisely using a parallax bar, or by employing a stereoscopic plotting instrument. However, scale and displacement errors are generated when the focal planes of the cameras are not parallel to the datum plane. The Wild Autograph A40 and the Zeiss Terragraph are both designed to plot from photographs taken in the normal case. The plate holders have no rotational motions, although equal elevation or depression angles at the two ends of the base can be accomodated.

An auxiliary apparatus was developed and new displacement equations are presented in this study. A useful working document can be produced using this auxiliary apparatus.

## ERRORS DUE TO A $\varphi$ ROTATION OF THE OBJECT PLANE

### Errors in a Single Image Due to a $\varphi$ Rotation of the Object Plane

The displacements in an image due to the position of the focal plane are dependent upon the rotation of the object plane with respect to the datum plane, as can be seen in Figure 1. The following relations can be derived from this figure (these equations were developed by Hallert (1960) to account for elevation differences of the ground):

$$\Delta R = \frac{\Delta Y \cdot R_c}{Y} \tag{1}$$

$$\Delta r = \frac{\Delta R \cdot c}{\gamma} \tag{2}$$

and accordingly

because

$$\Delta r = \frac{\Delta Y \cdot R_c \cdot c}{Y^2} \tag{3}$$

$$R_c = \frac{Y \cdot r_c}{c} \tag{4}$$

$$\Delta r = \frac{\Delta Y \cdot r_c}{Y} \tag{5}$$

In addition to these relations, an equation of expected  $\phi$  rotation for a single image was offered by R.Meyer (1970). Based on Figure 2, this expected error can be calculated with the following equations:

$$d\phi = \frac{0.5 \cdot c \cdot \rho}{e' \cdot z'} \tag{6}$$

where

$$e' = x'_1 - x'_2$$
 and  
 $z' = (z'_1 - z'_2)/2.$ 

This expected error has no affect in the analytical precise survey of buildings, etc. (M.O.Altan, 1980).

Errors in a Stereo pair of Images Due to a  $\varphi$  Rotation of the Object Plane

In the analytical approach or in analog plotting, measurements of an image point are normally made in the image coordinate system.  $\Delta X$ ,  $\Delta Y$ , and  $\Delta Z$  displacements in the object can be expressed as a

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FIG. 1. Effects of object plane rotation.



FIG. 2. The geometrical relation between object and image (R. Meyer, 1970).

function of the  $\phi$  rotation, that is, of the angle between the object plane and the datum plane.

 $X_p$ ,  $Y_p$ , and  $Z_p$  are the object-space coordinates

of point P (Figure 3). These can be calculated in the following manner:

$$X_{p} = \frac{b}{p_{p}} \mathbf{x}'_{p}; Y_{p} = \frac{b}{p_{p}} c; Z_{p} = \frac{b}{p_{p}} \mathbf{z}'_{p}$$
(7)

where

$$p_p = (x_p - x'_p)$$

The object space coordinates of point P' can also be calculated according to the normal case of stereophotogrammetry as follows:

$$X_{p'} = \frac{b}{p_{p'}} x'_{p'} ; Y_{p'} = \frac{b}{p_{p'}} c; Z_{p'} = \frac{b}{p_{p'}} z'_{p'}$$
(8)

P is placed on the object plane, and its rotated position on the datum plane is P'. The datum plane is parallel to the focal planes and, consequently, to the base line.

A  $\phi$  rotation will cause displacements in all three dimensions. These are expressed as

$$\Delta X = X_p \cdot (\sec \phi - 1) \tag{9}$$

$$\Delta Y = X_p \cdot \tan \phi \tag{10}$$

$$\Delta Z = \frac{z_p}{c} \cdot X_p \cdot \tan\phi \tag{11}$$

A graphical output is plotted on a two dimensional plane. A displacement of  $\Delta Y$  does not affect the

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FIG. 3. The displacements of an object point due to rotation of the object plane with respect to the datum plane. (a) in the x-y plane. (b) in the y-z plane.

graphical output along the *Y*-axis; only  $\Delta X$  and  $\Delta Z$  displacements will be generated on the graphical output. These two displacements from the true position of point *P* will be seen as a reduced length in the drawing. The maximum standard of accuracy in these two dimensions can be expressed with the drawing scale and the standard 0.2 mm line width, that is:

$$\max \Delta X \text{ (or } \Delta Z) = 0.2 \cdot m_p \tag{12}$$

From Equations 9 and 12, a maximum limit of the plane rotation can be calculated in the following manner:

$$\max \phi = \cos^{-1} \frac{X_p}{X_p + \max \Delta X}$$
(13)

#### AN AUXILIARY APPARATUS FOR THE PARALLEL ORIENTATION OF THE STEREOMETRIC CAMERA BASE

If the focal plane is parallel to the datum plane, analog plotting can be done within the minimum error limits from a pair of photographs which are obtained in the normal case configuration.



FIG. 4. Geometrical relations between base line, focal plane, datum plane, and normal plane.



FIG. 5. Auxiliary apparatus for the parallel orientation of the stereometric camera base.

In stereometric cameras which are manufactured according to the normal case of photogrammetry, the base axis must be parallel to the focal plane. Therefore, the base axis can be used instead of the focal plane for alignment.

Assume that q is the normal plane that is perpendicular to the datum plane and passes through the base axis; 00' is the intersection line of the normal plane and the datum plane (Figure 4). In practical applications, if the 00' intersection line and the base axis are parallel to each other, it is assumed the focal plane is also parallel to the datum plane.

An auxiliary apparatus was developed in order to establish the base axis of the stereometric camera and to align it parallel to the datum plane. This apparatus consists mainly of a double-pentagonal prism and a base bar (Figure 5). In addition, two miniature rods  $J_1$  and  $J_2$ , which are parallel to each other and normal to the base bar, are attached to the ends of the base bar. The double-pentagonal prism is set up between the miniature rods by attaching it to the middle of base bar. When the rods are on the base line, the two reflections of the min-

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iature rods will appear as one rod in the prism to the eye of observer.

The position of the stereometric camera station is determined based on the depth of field and horizontal stereoscopic coverage. This point is marked on the line which is perpendicular to the datum plane by using an optical square, a theodolite, or a level and a tape. The stereometric camera is set up at this point. After leveling the camera, the auxiliary apparatus is put on stereometric camera base. A third rod,  $J_3$ , is held at point A (Figure 6). The image of this rod must be superimposed onto the reflections of the miniature rods of the apparatus in the prism (Figure 6). In order to superimpose these images, the stereometric camera is rotated about its vertical axis. Thus, the base axis is made perpendicular to the line SA, and parallelism is obtained as a result of this procedure. This apparatus is easily used with a little practice.

A practical accuracy test was performed with the Zeiss SMK 120 Stereometric Camera. A schematic representation of the test area is shown in Figure 7. The datum plane contains three target points (*A*, *B*, and *C*) arranged on the same line. There are four meters between the targets, and the SMK 120 Stereometric Camera was set up ten meters away from the target plane. Orientation was accomplished with the auxiliary apparatus just described. The pair of photographs were measured in a Zeiss PSK2 Ster-

eocomparator. The comparator coordinates and transformed image coordinates are summarized in Table 1.

The object space coordinates of the target points, calculated with the parallax equations, are

$$X_A = 4685.399 \text{ mm}$$
  
 $X_B = 696.22 \text{ mm}$   
 $X_C = -3277.939 \text{ mm}$   
 $Y_A = 10030.914 \text{ mm}$   
 $Y_B = 10043.388 \text{ mm}$   
 $Y_C = 10065.642 \text{ mm}$ 

The rotation of the object plane from the datum plane can be calculated according to the geometry shown in Figure 7.

Consequently, the achievable accuracy is

$$\tan \phi = \frac{Y_C - Y_A}{X_C - X_A} = \frac{34.728}{7963.338} = 0.00436$$
$$\phi = 0^{\text{s}} \cdot 2776$$

The expected  $\phi$  rotation error can be calculated according to Equation 6 or Equation 13. For example, if  $d\phi$  is calculated with Equation 6, the expected  $\phi$  rotation error is

$$d\phi = \frac{0.5 \times 60.56 \times 63.66}{80 \times 50} \\ = 0^{\text{g}} \cdot 40$$



FIG. 6. Geometry of the parallel orientation using a double pentagonal prism.

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FIG. 7. A schematic representation of the test area.

TABLE	1.	COMPARATOR AND	TRANSFORMED	MAGE	
		COORDINATES			

	Comp coord	Comparator coordinates		Image coordinates	
Point	x	z	x	z	
βA	507.244	499.548	-28.292	-0.424	
B hote	531.340	500.054	- 4.199	0.197	
J C	555.272	499.462	19.725	-0.282	
L O A	501.364	498.716	-21.046	-0.505	
B lot	525.446	499.421	3.038	0.092	
in C	549.356	498.996	26.946	-0.441	

for the SMK 120 Stereometric Camera. This value is greater than the achievable accuracy, showing that the orientation of the stereometric camera was done precisely.

#### CONCLUSION

In this paper, new equations expressing the displacement of an object point due to a  $\phi$  rotation of the object plane relative to the datum plane were presented. These are applicable to various kinds of deformation studies. The camera focal planes can be made parallel to the datum plane with the help of an auxiliary apparatus. Thus, errors due to scale change and displacements, as well as orthogonal and affinity errors, would be minimized on the graphical outputs of analog plotting by using pairs of photographs which are obtained in proper position in the normal case of stereophotogrammetry.

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