Precision of Stereoscopic Height Measurements

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ABSTRACT: Stereoscopic height measurements have been made on targets recorded on wideangle and super wide-angle aerial photography. The precisions of height measurements were independent of target size and shape, being of the order of 0.03°_{100} of the projection distance for wide angle black-and-white aerial photography, but decreased by a factor of almost two as overlap increased from 60 percent to 80 percent. Under high magnification the precisions of height measurements may be as high as 0.02°_{100} of the projection distance. Height precisions were converted to equivalent *x*-parallaxes in a stereoplotter, and were then compared with results that were obtained with monocular pointing observations.

INTRODUCTION

IN INTENSIVE STUDIES on pointing observations, Trinder (1971, 1973, 1984) analyzed the characteristics of visual monocular pointing observations to photographic targets subject to noise and blur. Trinder (1972, 1975) also studied the precision of stereoscopic observations, that is, x-parallax clearance, and described some of the visual factors affecting these observations. This paper is the final stage in this study on pointing observations in which stereoscopic observations of height, on artificial ground targets, are investigated. Parameters considered are the percentage overlap and base to height ratio of the photography and the dimensions and shape of the ground targets. Black-and-white and color aerial photography of the same area and the same targets was available. The height precisions obtained are compared with those obtained previously with monocular observations, and the effects of optical magnification on precisions are discussed. An examination is then made of the processes associated with stereoscopic height measurements compared with those of monocular observations.

INVESTIGATIONS

Aerial photographs used for height measurement observations, described fully by Trinder (1984), were obtained in 12 black-and-white photography runs using Kodak 2405 aerial negative film, and one color photography run using Kodak 2445 color negative film, with wide-angle and super wide-angle cameras and with up to 90 percent overlap. Extensive investigations of the image quality of this photography have been reported by Trinder (1984) and will not be repeated in this paper. Twenty-two circular and cross-type ground targets were placed prior to photography. A Wild A8 with $6 \times$ optical magnification was used for height measurements on photographs with overlaps of 60, 70, and 80 percent (90 percent overlap cannot be accommodated in the plotter). The base/projection distance in the stereoplotter was identical to that of the photography for all wide-angle photography. The super wideangle photographs, however, were observed with a reduced ratio of base to projection distance, because of the limited range of focal length of the Wild A8. All height observations were made at a linear magnification of 2.3 in the model scale, and converted to a precision at a projection distance of 150 mm. In addition to height measurements, x and ycoordinates were recorded to confirm that observing in the stereoscopic mode did not affect the precision of *x* and *y* measurements; this indeed proved to be the case.

A group standard deviation of the height observations was derived from individual standard deviations of seven sets of 15 observations on a selection of targets of different sizes and shapes, following appropriate statistical tests to determine consistency within and between the individual sets of observations. Further, standard deviations derived from observations to four targets of different size and shape, but of the same photographic quality and percentage overlap, proved to be statistically similar, based on a Fisher test at the 95 percent confidence level. Therefore, a group standard deviation was computed from the standard deviations of the four targets. The height measurement precisions, derived on black-and-white and color photography with overlaps varying from 60 to 80 percent, are presented in Table 1, each entry being based on 420 observations. These standard deviations have been expressed as a ratio of the stereoplotter projection distance in model units or flying height in ground

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	Overlap		Height Measurement Precisions			Overall Precision		Equivalent Precision of X-Parallax Clearance
Photography	%	B/H		(µm)		(µm)	%00 Z	(µm)
Wide angle B/W	80%	0.31	6.5	7.3	8.0	7.3	0.047	2.3
(focal length	70%	0.46	5.0	5.7	6.2	5.6	0.037	2.6
setting 150 mm)	60%	0.60	4.6	4.5	4.1	4.1	0.030	2.5
Wide angle color	80%	0.31	5.9			5.9	0.039	1.8
0	70%	0.46	3.9			3.9	0.026	1.8
	60%	0.60	3.1			3.1	0.021	1.9
Super-wide angle B/W								
(focal length	70%	0.73	5.7			5.7	0.038	2.9
setting 105 mm)	60%	0.87	4.9			4.9	0.032	3.0

 TABLE
 1.
 PRECISIONS OF HEIGHT MEASUREMENT (μM) FOR A PROJECTION DISTANCE, Z, OF 150 MM. EACH ENTRY IS DERIVED

 FROM 105 OBSERVATIONS ON FOUR TARGETS.

units, and in terms of equivalent *x*-parallaxes on the photography.

DISCUSSION OF RESULTS

Standard deviations of height measurements for sharp targets, presented by Trinder (1972), were independent of target size, remaining constant for targets as large as 20 mrad (830 µm at 6× optical magnification). This study confirmed that result, and further indicated no significant difference in height measurement precision for circular and cross-type targets. Height measurement precisions plotted against percentage overlap are given in Figure 1, which demonstrates the deterioration in precision as the overlap increases. There is approximately a 50 percent deterioration of precision of height measurement when the overlap is increased from 60 to 80 percent, i.e., reduced ratio of base to projection distance, for both wide-angle and super wide-angle photography.

Standard deviations of height measurement on the color photography were consistently about 30 percent smaller than those on the black-and-white photographs. The two sets of photography are approximately of the same resolution. This same phenomenon was revealed by Trinder (1984) for monocular observations. Conversations with the one observer used for these observations revealed that she was more at ease viewing the color photography because the targets were more clearly defined and easier to observe. It has been possible to reveal these small differences in pointing precisions for color and black-and-white photography by the carefully controlled experimental procedures adopted, based on a large number of repetitive observations of high precision. It should be noted that three observers-KB, PY, and EW-were used for observations reported by Trinder (1984). All observers obtained the same pattern of results, but variations in the absolute magnitude of precisions did occur. Observer



FIG. 1. Relationship between percentage overlap and heighting precisions. Separate interpolated lines have been drawn for black-and-white and color photography.

characteristics of EW, used for this study, are consistent with the other two observers.

Height measurement precisions expressed as a ratio of projection distance in model units or flying height in ground units, as shown in Table 1, are on the order of 0.03% of the projection distance in model units for black-and-white photography with 60 percent overlap and slightly better for color photography. This high precision indicates the true ability of observers on stereoscopic plotters. It also indicates that the visual observations themselves are not the limiting factor in stereophotogrammetric methods such as aerial triangulation and spot height measurement. Schwarz (1982) has shown that stereoscopic observations of topographic features result in height measurement precisions varying from 0.07 to 0.15‰ of the projection distance. An improvement of the observations to ground targets would be expected; therefore, the precisions of 0.03% of the projection distance are considered consistent with Schwarz's results.

STEREOSCOPIC HEIGHTING AS A PROCESS OF X-PARALLAX CLEARANCE

Equivalent *x*-parallax differences σ_{x-par} derived from the height measurement precisions σ_z in Table 1 can be expressed by the formula

$$\sigma_{x-\text{par}} = (b/z) \cdot (f/z) \cdot \sigma_z \tag{1}$$

where *f* is the focal length setting in the plotter,

- *z* is the projection distance, and
- b/z is the ratio of the base to projection distance in the stereo plotter.

Equivalent x-parallaxes, given in Table 1, are almost constant and therefore independent of b/z, being about 2.5 µm for wide angle black-and-white photography, 2.9 µm for super wide, and 1.8 µm for color photography. Small variations do exist in the figures for x-parallax, but the differences do not appear to be significant. Converted to angular subtense at the observer*, these figures are equivalent to about 60, 70, and 44 µrad, respectively, which are similar to the monocular precisions derived by Trinder (1984) for very small targets, shown in Figure 2. Trinder (1972) also showed that the precision of x-parallax clearance was independent of target size, and equal to the monocular pointing for very small targets. This result has been duplicated in this study for standard aerial photographs.

The effects of target blur on monocular observations were investigated thoroughly for aerial photography by Trinder (1984). It was revealed that the quality of the photography was such that monoc-

* Angular subtense is derived from the formula:angular subtense (mrad) = linear dimension (μ m) × optical magnification/250.

ular observations were unaffected by target blur for $6 \times$ optical magnification. Trinder (1972) showed that stereoscopic height measurement precisions tend to deteriorate more rapidly in the presence of blur than do precisions of monocular observations. However, for this photography observed at $6 \times$ magnification, it can be assumed that the stereoscopic elevation observations were unaffected by blur in the photography.

Because stereoscopic observations can be considered as a task of *x*-parallax clearance, and the visual system performs as a function of the angular subtense of those parallaxes, an increase in the optical magnification should lead to an even higher height measurement precision than that obtained in this study for 6× optical magnification. Optical magnification increases the effects of blur on the observations, because it reduces the slope of the density profile of the target. The slope of the density profile, which has been shown by Trinder (1972) to be related to pointing precision for blurred targets, is the ratio of the density difference of the target above background over the dimension of the blurred edge of the target. According to Trinder (1972) observations to targets with density profiles less than about $0.3 \triangle D$ /mrad will be affected by the blur, the effect being approximately proportional to the increase in blur. Slopes of the density profiles of the targets observed at $6 \times$ optical magnification were about $0.6 \triangle D$ /mrad, but they would decrease for larger optical magnifications. Estimates of precisions made for optical magnifications of $12 \times$ and $24 \times$ on blackand-white photography are shown in Table 2. Slightly better precisions would be expected for color photography.

Further increases in magnification would cause a deterioration in height measurement precision. A precision between 0.015 and 0.02‰, however, seems possible using standard color or black-and-white



Fig. 2. Relationship between pointing precisions for monocular and precision of x-parallax clearance for stereoscopic observations, expressed as angular subtense, in relation to the annulus width for circular targets and distance between the edge of the measuring mark and ends of the cross for +-type targets. Equivalent linear dimensions for observations at $6 \times$ optical magnification are also shown.

Optical Magnification	X-Parallax Clea	Proportion of Projection	
	(µrad)	(µm)	Distance (Z) % Z
6×	65	2.7	0.03
$12 \times$	75	1.4	0.015
$24 \times$	130	1.4	0.015

TABLE 2. ESTIMATES OF HEIGHTING PRECISIONS AS A FUNCTION OF OPTICAL MAGNIFICATION

aerial photography at high optical magnification. Looking to cases where the quality of photography is higher than that used for this study, it is theoretically possible that height measurement precisions of 0.01‰ of projection distance or better could be obtained provided sufficiently precise measuring equipment is available. Similar projections were made for monocular observation precisions by Trinder (1984).

In studies on block adjustment by the bundle method with additional parameters on blocks with dense control, Kilpela (1981) revealed height accuracies of 4 µm to 6 µm, equivalent to x-parallaxes of 2.4 µm to 3.6 µm or 0.026‰ to 0.04‰ of the flying height. Jacobsen (1982) obtained similar results, but also achieved a height accuracy of 2 µm, equivalent to 1.6 µm in x-parallax or 0.018‰ of the flying height, for a four-fold block in the Jämijärvi test range. These results overall indicate that there is still some difference between block adjustment accuracies and precisions of height measurement, but improvements in height measurement precisions will be necessary in the future. As image quality is the limiting factor, apart from the measuring equipment itself, efforts should be made now to improve precisions by choosing color photography or high resolution black-and-white photography and the best available aerial camera lenses. For the future, special efforts should be made to improve the image quality of aerial photographic materials and equipment.

WIDE-ANGLE VERSUS SUPER WIDE-ANGLE PHOTOGRAPHY

Studies have been made in the past on the accuracy of height measurements on super wide-angle photography compared with that of wide-angle photography. Because the base to height ratio of super wide-angle photography is greater than that of wide-angle photography, precisions of height measurement should be better for the super wideangle photography. However, the poorer image quality of super wide-angle photography has often been blamed for the lack of improvement in precision (Harley, 1979). Trinder (1984) showed that the image quality of the super wide-angle photography used for this study is only marginally lower than that of the wide-angle photography, and that monocular pointing precisions on both sets of photographs were substantially the same.

It is noted in Table 1 that height measurement precisions on the super wide-angle photography are statistically the same as those for wide-angle photography. Further, the equivalent x-parallaxes of the height measurement precisions for super wide-angle photography are marginally higher than those for wide-angle photography. This may be due to the slightly lower image quality of the super wide-angle photography. The scale of the measurements on super wide-angle photography, also converted to a precision at a projection distance of 150 mm because they were observed with a focal length setting in the Wild A8 of 105 mm, was 1.43 times larger than that shown for the wide-angle photography. For height measurements at the same scale as this photography, precisions would have been a factor 1.43 higher on the super wide-angle photography. It can therefore be deduced that super wide-angle photography taken at the same scale as wide-angle photography, and therefore at a lower flying height, will produce height measurement precisions, assuming that focal length ranges in the stereoplotter will accommodate the focal length of super wideangle photography, approximately 1.7 times better than on the wide-angle photography. This difference, however, is due only to the lower flying height of the super wide-angle photography. Conversely, for wide-angle and super wide-angle photography taken from the same flying height, height measurement precisions would be the same.

GENERAL COMMENTS ON STEREOSCOPY

Stereoscopy has always been a topic of interest in photogrammetry because of its importance to the success of photogrammetric measurement and, indeed, interpretation of features. The fundamental visual and cortical functions which lead to stereopsis are not fully understood, but progress is being made (Frisby, 1979). The fact that the pattern of stereoscopic precisions vary so significantly from those of monocular observations is difficult to explain.

Stereoscopy involves the fusion of two similar images using the boundaries of the objects in the images rather than the area, as shown in a study of the autocorrelation in stereoscopy by Trinder (1975). Monocular and binocular (as opposed to stereoscopic) observations are a function of a vernier acuity task in which the width of the annulus on each side of the measuring mark must be equated. It is known that stereoscopy assists observers in the interpretation of features on imagery. Konecny *et al.* (1982), for example, showed the superiority of stereoscopic observations over monocular observations in interpretation of features. Their investigations proved that features could be interpreted on stereoscopic images with only about half the resolution required to interpret the same features on single images. Frisby (1979), in reviewing the characteristics of human vision, speaks of stereoscopy as a "camouflage-breaking system." The ability to observe the depths of features adds a major clue to the interpretation of features on images. This factor has been known by photointerpreters, and is further confirmed by the results of Konecny *et al.*

Frisby claims that the ability to decode camouflage may well have been the initial advantage of primitive human vision, but clearly the ability to detect depth through retinal disparity has proved to be an important factor which stereophotogrammetry has used. Ultimately, stereoscopic depth perception is a function of the way in which the visual system uses the disparity in the two images, i.e., *x*parallax, the borders being the important properties of the objects which are used by the visual system to detect the disparity. This paper reveals the high precision with which these observations can be made.

CONCLUSIONS

Tests of stereoscopic observations on aerial photographs reveal that standard deviations of height measurement equivalent to 0.03‰ of the projection distance in the stereoplotter in model units or of the flying height in ground units were achieved with black-and-white aerial photography. Results for color photography were about 30 percent better than those for black-and-white photography. Under high magnification, it is probable that height measurement precisions could be about 0.02‰ of the projection distance to flying height or better. For very high image qualities, height measurement precisions could approach 0.01‰ of the projection distance in stereoplotter model units or 0.01‰ of the flying height in ground units.

Standard deviations of height measurements will deteriorate approximately linearly as the overlap of the photography increases. For an overlap of 80 percent, standard deviations of heighting will be about 50 percent worse than those for an overlap of 60 percent.

The precisions of height measurement converted

to an equivalent precision of *x*-parallax clearance are approximately equal to the monocular pointing precisions for very small targets but are independent of target size and shape and base to projection distance ratio in the stereoplotter, i.e., base to height ratio, for wide angle photography. The visual functions of stereoscopic observations based on retinal disparity are clearly different from monocular observations of vernier acuity.

Because the image qualities of wide-angle and super wide-angle photography are substantially the same, height measurement precisions, expressed in ground units, will be a function only of the flying height of the photography.

Because image quality is the major factor affecting heighting precisions, the highest image quality photographic materials and cameras should be selected if the most precise measurements are to be achieved.

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