Evaluation of the Effects of Photo Measurement Errors on Predictions of Stand Volume from Aerial Photography

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ABSTRACT: A simple and flexible technique was developed for assessing the impact of photo measurement errors on aerial-photo-based volume predictions. The method utilizes random numbers from distributions as substitutes for actual measurement errors, and thus is able to handle errors of many types and sizes without making restrictive assumptions. The procedure allows the user to examine the properties of the fitted models that are of interest. The proposed technique is demonstrated on Southern pine and Douglas-fir data sets.

INTRODUCTION

THE CONCEPT of estimating quantitative stand characteristics from aerial photography is not new, but it is rarely put into practice. A persistent problem with its development and application has been the need to select between ground collected and photo-measured data for constructing prediction equations. Many researchers have used the assumption of fixed, perfectly measured predictor variables (Johnston, 1963) to justify the use of ground collected data in equation construction, even though this has necessitated correcting or adjusting photo-measured data to ground before substituting into the prediction function. At this juncture, however, the effect of typical measurement errors on the prediction equations has not been theoretically or empirically investigated. If the effect were found to be minimal, photo-measured data could be used to construct the prediction function, and thus a complicating step in their application could be eliminated in many circumstances.

The objective of this paper is to develop a technique for use in assessing the impact of photo measurement errors on predictions of stand volume made from aerial photography. The technique will be demonstrated on two previously published data sets, one for Southern pine and one for Douglas-fir.

PREVIOUS WORK

Measurement errors are an accepted fact in forest photogrammetry. Several researchers have attempted to characterize photo measurement errors. Most have learned that the size and structure of the errors is dependent on several factors, such as the scale of the photography, skill of the interpreter, and the types of forest stands being examines (Spurr and Brown, 1946). Typical errors in height measurements and crown closure estimates are of particular importance because most aerial photo volume tables utilize these two variables as predictors (Paine, 1981). Paine (1981) estimated errors in height to be 10 to 20 percent of average stand height and to be relatively independent of photoscale. Spurr and Brown (1946) reported three-foot errors in the average height of 20 Southern pine plantations. Crown closure, which is rapidly and easily estimated, is subject to interpreted bias (Avery, 1958). The consensus of researchers is that two of three crown closure estimates will be within 10 percent of the true value (Spurr, 1960). None of the studies, however, address the problem of the effect of these errors on the variable of interest, stand volume.

Econometricians have supplied much of the information about the effect of variable regressors because of the inexact nature of the data in their applications. The theory is difficult and requires the use of expansions, limits, and assumptions about error sizes which are often not well defined.

Excepting special experimental circumstances, ordinary least-squares parameter estimates derived from error contaminated predictor variables are biased. In the case of simple linear regression, if it can be assumed that the various errors are distributed independently of each other and the true values, the estimate of the slope approaches

$$\text{plim }(\hat{\boldsymbol{\beta}}) = -\ \boldsymbol{\beta}/(1 + (\sigma_u^2/\sigma_v^2))$$

where
$\hat{\boldsymbol{\beta}}$ = parameter estimate vector,
$\boldsymbol{\beta}$ = parameter vector,
σ_u^2 = variance of the measurement error in the regressor variable,
σ_v^2 = variance of the error term, and
plim = probability limit

in which case the true slope will be underestimated because variances are always positive (Johnston,

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1963). For the multiple regression situation (Hodges and Moore, 1972), the bias of least-squares parameter estimate can be approximated with the expression

$$E(\hat{\boldsymbol{\beta}}) - \boldsymbol{\beta} = -(n\text{-}m\text{-}1)(\mathbf{X}'\mathbf{X})^{-1}\mathbf{S}\boldsymbol{\beta}$$

where $\hat{\boldsymbol{\beta}}$, $\boldsymbol{\beta}$ are as previously defined;
n = sample size;
m = number of parameters;
$\mathbf{X}$ = data matrix; and
$\mathbf{S}$ = variance-covariance matrix of the measurement error in the regressors.

This result requires that restrictive assumptions be made concerning the distribution and sizes of the measurement errors. Stewart (1980) stated that large errors in the predictor variables are likely to bias the regression coefficients downward by moving the data away from collinearity. Bias in the estimated regression coefficients is not always a problem, however. According to Hodges and Moore (1972), if the predictor variables are drawn from a stationary distribution, and if the values used to make the forecast are from the same distribution, no prediction bias is incurred. In the case of photo-measured data and aerial photo volume tables, if the photo-based estimates of stand characteristics are similar and consistent through time or among interpreters, unbiased predictions of volume per acre could be made from regression functions based on error contaminated data.

Little research into the precision of the parameter estimates or predicted values for regressions constructed from error contaminated data has been conducted. Hodges and Moore (1972) indicated that estimated σ^2 should "be a reasonable estimate of"

$$\sum_{i=1}^{n} \boldsymbol{\beta}_i^{\,2}\, \sigma_i^2 + \sigma_v^{\,2}$$

where $\boldsymbol{\beta}_i$ = parameter i,
σ_i^2 = variance of the measurement errors in variable i, and
$\sigma_v^{\,2}$ = error variance.

Undoubtedly, the precision of regression equation is hampered by measurement errors (σ_i) in the regressors.

Various indices or bounds have been derived to determine the sensitivity of the coefficients to changes in the data. For the most part, however, they have restrictive assumptions and undesirable properties or are limited in their information content (Stewart, 1980). Single number indicators, such as the condition number or pertubation index, are limited by their assumptions concerning the size and structure of the measurement errors, and they contain no information on how much the coefficients may change (Stewart, 1980). They are useful for pointing out potentially troublesome data sets which contain significant mutlicollinearity or error contamination. An upper bound for the effect of small changes in the regressor variables has been constructed by Stewart (1980). To derive this bound, Stewart assumed that the standard deviation of the measurement errors in the variable was less than the smallest singular value of cross product matrix divided by the number of observations in the data set. A test by Smith (1981) indicated that even optimistic photo measurement errors did not satisfy this requirement. A further disadvantage of Stewart's bound was that joint effects of measurement errors in several variables could not be assessed.

Researchers have described the theoretical ramifications of the presence of measurement errors in predictor variables. Much of this theory requires restrictive assumptions concerning the characteristics of the errors which can occur. Further, the practical implications of the problem have not been fully addressed. For example, Draper and Smith (1981) state that problems may arise in least-squares analysis if the predictor variable measurement error is not small in comparison to the population variation. However, they failed to place bounds on the allowable sizes of these errors, or to state how small the errors must be in order to have an insignificant effect on the analysis. Those involved in the development of photo volume tables have knowledge of the sizes and structures of the photo measurement errors present in their data, but no method for evaluating their effect on the prediction models.

Rounding errors stimulated Beaton *et al.* (1976) to develop a technique for assessing the effect of changes in the predictor variables on ordinary least-squares parameter estimates. Rounding errors of a logical size and distribution were created with random number generators and added to each observation in the data set. The fitted regression line for the simulated, unrounded data was then compared to the corresponding line for the rounded data. The process was repeated 1000 times and the outcomes averaged so that trends in the parameter estimates could be discovered. This technique is flexible because pertubations of any size or structure could be introduced into the data set. The only assumptions required by this technique are that the assumed structure of the pertubations is logical and of interest, enough repititions are performed to correctly indicate the effects of these perturbations, and the random number generation process is correct. A modification of this technique may be applicable to the photo measurement problem because of its simplicity and flexibility.

EVALUATING THE EFFECTS OF PHOTO MEASUREMENT ERRORS

In order to assess the effects by measurement errors on stand volume predictions from aerial photographs, specific types of data are needed. Each observation should have predictor variables which contain measurement error (photo-estimated), and the same variables without (or with insignificant amounts of) observational error. The effect of those errors could then be evaluated by fitting regression models to both types of data, and noting the changes in the fitted equations. However, the inferences of such a study would be strictly limited to the type of error present in that single data set.

A procedure similar to that of Beaton *et al.* (1976) was developed which will permit an investigator to examine many different sizes and types of photo measurement error for their effect on photo-volume predictions. The method is flexible and simple, and it requires few assumptions. The empirical nature of the results of this method would allow researchers to determine the effect of a particular type of error, or the level of photo measurement error that significantly effects the volume prediction. The proposed error evaluation technique is composed of three major steps: (1) selection of data base, (2) assumption of error structures and simulations, and (3) model fitting and comparisons. The method will first be described and then demonstrated on two previously published data sets.

METHOD DESCRIPTION

The first step in the proposed error evaluation technique is to prepare a base data set which is relatively free of measurement errors. These base data will typically consist of the ground measurements of the dependent and independent variables for each observation. Ground collected information is used as the base because it represents the alternative to photo measurement and prediction, and because it can be assumed to contain smaller amounts of observational error. Further, ground information is usually available because it is needed in the photo volume table construction process. When accurately measured ground data are not available, such as occurs when crown closure is utilized, the average of several independent photo estimates is acceptable as the base value. These "error-free" data will be the basis for all simulations and comparisons.

The second step in the error evaluation process is to create simulated photo measurement errors by contaminating the predictor variable values in the base data set with random variates of known structure. The dependent variable (volume) is left unchanged, but the base values for the independent variables are increased or decreased by random amounts which represent photo measurement errors. The random pertubations are dependent upon the photo measurement error structure assumed by the investigator. The investigator selects an appropriate statistical distribution for the simulated photo measurement errors, i.e., normal, F, uniform, etc. By varying the parameters of the assumed distribution, the simulated photointerpretation errors may be biased or unbiased, precise or imprecise. The characteristics of the photo measurement errors could even vary across the range of a variable. Using this technique of error simulation, many forms, types, and sizes of photo measurement error can be rapidly created and examined.

The results of a simulation procedure such as that described above are dependent on the properties of the basic data, and on the characteristics of the random numbers that are used as proxies for actual photo measurement errors. It is thus desirable to minimize the simulation error. First, the quality of the "pseudo-random" number generator used to create the photo measurement errors must be determined through literature searches or statistical tests. Second, the simulation process must be repeated until the effects of the random number generation are minimized. The number of repetitions needed will likely vary for different data sets and types of measurement error. A typical sample size determination method is to stabilize the variance of the characteristic of interest. It is recommended that the necessary number of repetitions be determined in each situation in a preliminary study prior to the actual simulations.

The final step in the proposed error evaluation process is to fit regression models to the base data and to each of the error contaminated data sets. Parameter estimates, predicted values, fit statistics, etc., are then averaged over the repetitions within an error structure, and compared to the corresponding result for the base data set. If desired, t-tests may be used to test for significant differences between the error structures and the base data and between the error structures themselves. Trends or patterns in the effects of photo measurement error on various regression characteristics may be searched for and recorded.

Two important factors must be considered prior to performing the final step in the proposed error evaluation technique. Although the method is not limited in application to any particular type of model, the form of the model fit to the data sets may affect the results of the error analysis. Certain models or variable transformations may increase or decrease the effect of photo measurement errors. Investigators are strongly urged to give serious consideration to model specification before attempting to analyze the effect of errors. Consideration should also be given to the selection of comparison criteria. Photo measurement errors are likely to have different effects upon the various regression characteristics. The choice of which properties to compare should depend upon the intended use of the fitted equations, i.e, prediction or modeling.

The proposed error analysis technique is simple to perform, but it does require that researchers be creative and knowledgeable about their particular situations. The applicability of the results of this procedure will depend to a large extent upon factors under the control of the investigator.

DEMONSTRATION OF THE TECHNIQUE

To facilitate understanding, the photo measurement error evaluation technique proposed in this study is demonstrated on two previously published data sets. The models and error structures assumed in this demonstration are intended only as examples. The user of the proposed technique is expected to utilize models and error structures which are appropriate for his particular situation. Further, a relatively low number of repetitions within each error structure were performed in this demonstration. For these two reasons, general inferences concerning the effect of photo measurement errors on volume predictions from aerial photography should not be made from this example. This demonstration is intended only as a vehicle for providing understanding of the proposed error evaluation technique.

Base Data

Two actual data sets were selected for use in this study. One contained observations on Southern pines (*Pinus taeda* L., *P. echinata* Mill. and *P. palustris* Mill.) in Mississippi, while the other was observed on Douglas-fir (*Pseudotsuga menziesii* (Mirb.) Franco) in the Pacific Northwest. Both data sets were collected for the purpose of constructing an aerial photo volume prediction equation.

Southern pine. One data set to be used in this study came from a seven-county area in central Mississippi (Smith and Mead, 1981). Ground measurements were made, and the aerial photography was flown by the Forest Insect and Disease Management Team, State and Private Forestry, U.S. Forest Service.

Fifty, one-fifth acre circular plots were located on 1:8000 color infrared aerial photographs. Twelve plots were located in plantations, with the remaining 38 being in natural pine stands (less than 25 percent hardwood). Each tree on each ground plot was measured for dbh and total height. Individual tree volumes (total, outside bark) were computed with an appropriate equation, and were summed over each plot for expansion to a per acre basis. These actual volumes ranged from 250 to 4900 cubic feet per acre.

The predictor variables of interest were crown closure and tree height. The range of crown closure (0 to 100 percent) was divided into ten intervals of 10 percent each. The value given to each interval was its midpoint. Five interpreters independently estimated crown closure for each plot, and their average value was categorized (10 percent classes) and taken to be the "actual" crown closure for that plot. In this particular study, height was defined as the ground measured (with a clinometer) average total height of the four tallest trees in the plot. Table 1 contains a summary of the basic characteristics of this data set.

Douglas-fir. A second data set containing 282 observations for Douglas-fir in western Washington and Oregon was obtained from Dr. David Paine of Oregon State University. The data came from a study conducted by Pope (1962).

The Douglas-fir data differed from the Southern pine data in the method of acquisition and definition as well as the species covered. Pope had several interpreters estimate crown closure as did Smith and Mead (1981). However, the crown closure estimates were measured on 1:12000, black-and-white panchromatic photography, and the average of the individual interpreter estimates were retained in a point form (not categorized). Pope defined height as the field measured total height of the dominants and codominants in the one-fifth acre plots. Volume was defined as the gross volume, Scribner Decimal C, for all trees 11.0 inches dbh and larger. A 100-observation test data set was created from the original 282 observations using stratified (by volume class) random sampling. Only 182 observations were thus available for equation construction. A summary of the properties of this data is presented in Table 1.

Procedures

Simulated Photo Measurement Errors. A logical size and structure for the photointerpretation errors likely to be encountered in the Southern pine and Douglas-fir data sets was inferred from personal experience and past research. It is not implied, however, that the error structures assumed for this demonstration apply to all situations. In fact, the advantage of this technique lies in its ability to handle different types and amounts of photo measurement error. Three levels of measurement error (none, optimistic, pessimistic) in crown closure and tree height were investigated for the two basic data sets (Table 2). For Southern pine and Douglas-fir, the nonzero values for error in the tree height estimate correspond to approximately 5 percent and 15 percent of the average height for all the plots in the data sets. Thus, error sizes were consistent for the two data sets in both variables. Combining the three levels of error for each variable resulted in nine combinations or levels of simulated photo measurement error. Independent normal structures were assumed for the measurement errors in crown closure and height, because the two variables had exhibited a low correlation coefficient in the original data. The normal assumption is not unduly restrictive, because measurement errors by an experienced interpreter are in theory symmetric, random and large errors are far less common than small errors. The standard deviations for the normal distributions were the error sizes assumed for each of the nine combinations (Table 2). For example, the measurement errors for

TABLE 1. SUMMARY STATISTICS FOR THE TWO ORIGINAL DATA SETS

Data Set	Number of Observations	Crown Closure %			Height (ft)			Volume[a]		
		Min	Max	Avg	Min	Max	Avg	Min	Max	Avg
Southern Pine	50	15	85	55	31	116	71	256	4937	2800
Douglas-fir	182	15	97	71	50	270	139	0	400	75

[a]Cubic feet per acre for Southern pine and MBF per acre for Douglas-fir.

TABLE 2. LEVELS OF SIMULATED PHOTO MEASUREMENT ERROR TO BE INVESTIGATED, AND THEIR DESIGNATIONS

Level of Crown Closure Error (%)	Southern Pine: Level of Height Error 0	3 (Feet)	9	Douglas-fir: Level of Height Error 0	7 (Feet)	20
0	I	II	III	I	II	III
5	IV	VI	VIII	IV	VI	VIII
15	V	VII	IX	V	VII	IX

Level VI were assumed to have normal distributions with standard deviations of 5 percent in crown closure and 3 feet in height for Southern pine. The mean of all the error distributions was assumed to be zero, meaning that the photo measurement errors are theoretically unbiased.

Eight data sets were constructed from each of the two original data sets by contaminating their observations with simulated photo measurement errors of the form assumed above. Level I had no introduced error and was thus identical to the original data, which represents the most error free condition. For each of the eight remaining error levels, the crown closure and height for each observation were perturbed by a random deviate drawn from the error distribution assumed for that level, i.e.,

$$\mathrm{VOL}_{ij} = \mathrm{VOL}_i$$
$$\mathrm{CC}_{ij} = \mathrm{CC}_i + N(0, \sigma^2_{CCj})$$
$$\mathrm{HT}_{ij} = \mathrm{HT}_i + N(0, \sigma^2_{HT_j})$$

where VOL_i = volume per acre for plot i,
HT_i = average tree height for plot i,
CC_i = average crown closure for plot i,
$\mathrm{VOL}_{ij}, \mathrm{CC}_{ij}, \mathrm{HT}_{ij}$ = i^{th} contaminated observation in error level j,
σ^2_{ccj} = variance of the simulated photo measurement errors in crown closure in the j^{th} error level, and
C^2_{HTj} = variance of the simulated photo measurement errors in height in the j^{th} error level.

The IMSL (Anon, 1982) routine GGNML was used to generate the random numbers. Because crown closure is constrained to fall between 0 and 100 percent, and was categorized in the Southern pine data, normality of the errors in crown closure was not maintained. Volume was left unchanged for each observation and error level. Five repetitions of the error contamination process were performed within each error level. This number was pre-specified for this demonstration, not derived in the recommended way with a preliminary study. Thus, the sample size may not be sufficient for determining the effect of the simulated photo measurement errors at all the desired levels.

Changes in the Fitted Models. A regression model was fit to each of the five independently perturbed data sets within each error level for the Southern pine and Douglas-fir observations using ordinary least-squares estimation methods. A simple linear model of the form

$$\text{Volume per acre} = b_0 + b_1\,(\mathrm{CC}) + b_2\,(\mathrm{HT})$$

where b_0, b_1, and b_2 are regression coefficients;
CC = crown closure (%); and
HT = average tree height (ft)

was used in all instances in this demonstration. Any model(s) of interest could be specified and utilized with this technique, however.

Several characteristics of the fitted regression models were investigated in this demonstration. The averages of the five sets of parameter estimates for each error level were noted, as were the average of the usual fit statistics, namely, R-squared, standard deviation about the regression, and the absolute values of the residuals. The average predicted stand volume for a range of values of crown closure and height was computed and compared across error levels.

RESULTS OF THE DEMONSTRATION

CHARACTERISTICS OF THE PHOTO MEASUREMENT ERRORS

The results of an investigation of the type proposed here are dependent on the properties of the random numbers used as proxies for photointerpretation errors. It is thus important to examine the pertubations and to note their effect on the basic data.

Means and standard deviations of the simulated photo measurement errors are presented in Table 3. As desired, the means are near zero and the variances approach the values assumed for each error level. Deviations from the planned characteristics of the pertubations could have indicated improper or inadequate random number generation. Examination of the minimum, maximum, average absolute value, or some other characteristic of the pertubations may also be desirable to determine if they represent the correct population of photo measurement errors. Although the pertubations in this study were adequate, a close examination of their properties may at times necessitate defining a new structure for the photo measurement errors, or reviewing the process used to generate the random numbers.

The effect of the simulated photo measurement errors on individual observations and on the relationship between crown closure, height, and volume cannot be determined from the overall characteristics listed in Table 3. The major effects of the simulated errors were an extension of the range of the observations, and a small increase in the spread of the data throughout its range. Space limitations prevent the presentation of all the data sets, but examples will be used to illustrate the results. The original correlation coefficients between crown closure and volume, and between height and volume for the 50 Southern pine observations, were 0.61 and 0.75, respectively. For the data sets in Error Level IX (maximum pertubation of both variables), the correlation between crown closure and volume ranged from 0.35 to 0.52, and averaged 0.44. Correlations between height and volume averaged 0.60 in Error Level IX and ranged from 0.50 to 0.70. Clearly, correlations between the dependent and independent variables were reduced by measurement errors of the type assumed in this study. Overall, however, the effects of even the largest amount of simulated photo measurement error seemed to weaken the relationships between crown closure, height, and volume only minimally, possibly due to the relative weakness of these relationships in the original data.

The effects of the pertubations on the original data were variable within an error level, as evidenced by the differences in the correlation coefficients for the five repetitions in Error Level IX for Southern pine. More repetitions will be needed for each error level to determine more precisely the amount of change caused by the pertubations. A smaller number of repetitions, such as the five used here, did indicate the direction and general magnitude of the effects of the pertubations.

Changes in the Fitted Regressions

The effects of the simulated photo measurement errors on the original observations have already been examined, and it was argued that they were not great. However, the effects on the fitted regression equations are of major interest. In particular, the effects of photo measurement errors on the parameter estimates, on the quality of fit of the regression models, and on the predicted values were selected as the comparison criteria for this demonstration. Standard t-tests could have been used to check for significant differences in the average regression characteristics across the various levels of error. However, they were not presented in this report to reduce its length, and

Table 3. Average[a] Characteristics of the Simulated Photo Measurement Errors

		Crown Closure Pertubations (%)		Height Pertubations (ft)	
Data Set	Error Level	Mean	Standard Deviation	Mean	Standard Deviation
	II	—	—	−0.31	2.94
	III	—	—	−0.68	9.50
Southern	IV	+0.20	5.79	—	—
Pine	V	−0.32	15.3	—	—
	VI	−0.32	5.78	−0.04	3.06
	VII	−0.64	14.4	−0.04	3.06
	VIII	+0.68	5.72	+0.11	8.87
	IX	−0.20	15.5	−0.65	10.0
	II	—	—	−0.08	7.00
	III	—	—	−0.50	20.0
	IV	+0.07	4.90	—	—
Douglas-	V	−1.00	14.1	—	—
fir	VI	−0.09	5.06	+0.06	6.85
	VII	−0.59	13.7	+0.52	6.90
	VIII	+0.23	4.84	−0.47	20.1
	IX	−0.79	13.8	+0.51	20.4

[a]Average of the five repetitions in each error level.

to limit the inferences taken from this demonstration.

Average parameter estimates across the various levels of error are presented in Table 4. Clearly, the three parameter estimates reacted differently to the various amounts of error introduced into the two predictor variables. However, the general tendency is for the coefficient to move toward zero with increasing amounts of simulated photo measurement error. This tendency concurs with the theoretical results derived for simple linear regression (Johnston, 1963), and with Stewart's (1980) hypothesis of probable downward bias in multiple regression situations.

Some interesting specific results can be found in Table 4. The more optimistic level of simulated photo measurement error had only a small effect on the parameter estimates. For the Southern pine and Douglas-fir data sets, the intercept consistently decreased with increasing error in both independent variables, as did the coefficient for crown closure (b_1) in the Douglas-fir data. In these instances, the minimum parameter estimate occurred at the level of maximum simulated error in both variables (Level IX). In contrast, the coefficient for crown closure (b_1) in the Southern pine data was smallest when no simulated measurement error was introduced into the height values (Level V). For both species, the coefficient for height (b_2) was smallest at Error Level III, where no error was introduced into the crown closure estimates.

The differences in response to the simulated photo measurement errors between the two species are likely due to the strengths of the relationships in the original, unperturbed data. Crown closure was only a marginal contributor to the volume prediction equation in the original Douglas-fir data, whereas crown closure in the Southern pine data was only slightly less important than height for explaining volume variation. The effects of simulated photo measurement errors on ordinary least-squares parameter estimates seem to be variable and dependent on the properties of the basic data as well as on the structure of the photo measurement errors. The technique demonstrated here offers a simple and flexible method for exploring those dependencies.

Three statistics were used to compare the quality of the fit for the regression models based on error contaminated data: R-squared, standard deviation about the regression, and the absolute value of the residuals. Even though the three chosen criteria are closely related, each is presented here because of its widespread application to the model selection problem.

Averages of these quality-of-fit criteria across the levels of simulated photo measurement error are presented in Table 5. As expected, the fit of the regression models decreases, or remains approximately constant, as more photo measurement error is introduced into the data. The results did differ slightly for the two species, however. The Douglas-fir data seemed to be slightly less sensitive to the pertubations, and nearly insensitive to errors in crown closure. The Southern pine data set was approximately equally affected by errors in crown closure and height. For both species, the effect of the lower level of error in either or both variables on the fit of the models was negligible.

The implications of the results presented above are that photo measurement errors do not necessarily have a significant, deleterious effect on the fit of regression models. The technique demonstrated in this report offers a way to determine this type of information under varying conditions.

The final characteristics of the regression models

TABLE 4. AVERAGE[a] PARAMETER ESTIMATES ACROSS ERROR LEVELS FOR THE SOUTHERN PINE AND DOUGLAS-FIR DATA SETS

		Southern Pine			Douglas-fir		
	Level of Crown Closure	Level of Height Error					
Parameter[b]	Error	0	3	9	0	7	20
	0	−2585	−2526	−1924	−131648	−127389	−94181
b_0	5	−2453	−2449	−1818	−130314	−124314	−93265
	15	−2185	−2120	−1297	−115132	−111151	−91596
	0	28.29	28.30	31.67	512.7	501.0	397.7
b_1	5	25.13	25.83	26.77	494.2	468.0	377.3
	15	15.64	17.99	18.42	327.5	303.2	300.8
	0	49.27	48.50	37.52	1228	1204	1021
b_2	5	49.97	49.54	39.85	1228	1198	1025
	15	54.21	51.46	39.68	1206	1185	1046

[a]Average of the five repetitions within each error level
[b]Using the model VOL $= b_0 + b_1$CC $+ b_2$HT

TABLE 5. AVERAGE[a] FIT STATISTICS ACROSS THE VARIOUS LEVELS OF ERROR FOR THE SOUTHERN PINE AND DOUGLAS-FIR DATA SETS

		Southern Pine			Douglas-fir		
	Level of Crown Closure	Level of Height Error					
Variable	Error	0	3	9	0	7	20
R^2	0	0.73	0.72	0.64	0.77	0.76	0.66
	5	0.70	0.70	0.63	0.77	0.75	0.65
	15	0.64	0.65	0.54	0.76	0.75	0.65
$S_{y.x}$[b]	0	652	664	750	31.7	32.8	38.7
	5	688	688	762	31.7	32.8	39.0
	15	758	740	853	32.3	33.4	39.0
Average Absolute	0	532	537	601	22.7	23.6	28.5
Value of the	5	564	558	603	22.7	23.3	28.5
Residuals	15	614	583	683	22.7	23.9	28.3

[a]Average of the five repetitions in each error level.
[b]ft^3/acre for Southern pine, MBF/acre for Douglas-fir.

to be examined were their resultant predicted values of volume per acre. Because aerial photo volume tables are predominantly used for prediction purposes, it could be argued that changes in the predicted values are the most important effect of photo measurement errors. Theory predicts unbiasedness for these predictions under many circumstances, but unbiasedness is a large sample property, and it does not imply that individual predictions will not be different under varying error structures.

Tables 6 and 7 present the average predicted volume per acre for each error level for a range of crown closure and height values. Individual predictions were changed little by the lower level of error in either variable, although some large differences are present at the upper levels of error. Toward the center of the data, predicted values were affected less than at the extremes because the overall mean of the observations was not greatly affected by the unbiased pertubations. It should be remembered, however, that these predicted values, regardless of the type or size of the error introduced into the data, are unbiased if the conditions of Hodges and Moore (1972) are met.

SUMMARY OF THE EXAMPLE

The technique developed in this report provided insight into the effect of certain types of photo measurement errors on two aerial photo volume equations. However, the results indicated that more

TABLE 6. AVERAGE[a] PREDICTED SOUTHERN PINE VOLUME (FT^3/ACRE) ACROSS THE LEVELS OF SIMULATED PHOTO-MEASUREMENT ERROR

		Height (ft)								
		40			70			100		
Crown Closure	Level of Crown Closure	Level of Height Error								
(%)	Error	0	3	9	0	3	9	0	3	9
25	0	90	120	370	1570	1580	1490	3050	3030	2620
	5	170	180	440	1670	1660	1640	3170	3150	2840
	15	370	390	750	2000	1930	2110	3630	3480	3370
55	0	940	970	1320	2420	2430	2440	3900	3880	3570
	5	930	950	1250	2430	2440	2440	3930	3930	3640
	15	840	930	1300	2470	2470	2660	4100	4020	3930
85	0	1790	1820	2270	3270	3280	3390	4750	4730	4520
	5	1680	1730	2050	3180	3210	3250	4680	4700	4440
	15	1310	1470	1860	2940	3010	3220	4560	4560	4480

[a]Average of the five repetitions in each error level.

TABLE 7. AVERAGE[a] PREDICTED DOUGLAS-FIR VOLUME (MBF/ACRE) ACROSS THE LEVELS OF SIMULATED PHOTO-MEASUREMENT ERROR

		Height (ft)								
		100			175			250		
		Level of Height Error								
Crown Closure (%)	Level of Crown Closure Error	0	3	9	0	3	9	0	3	9
25	0	4	6	18	96	96	94	188	186	171
	5	5	7	19	97	97	96	189	187	172
	15	14	15	21	104	104	99	195	193	177
55	0	19	21	30	111	111	106	204	201	183
	5	20	21	30	112	111	107	204	201	184
	15	24	24	30	114	113	108	204	202	186
85	0	35	36	42	127	126	118	219	216	195
	5	35	35	41	127	125	118	219	215	195
	15	33	33	39	124	122	117	214	211	195

[a]Average of the five repetitions in each error level.

than five repetitions were needed to precisely define those effects. Only the general magnitudes of the effects, and their direction, could be reliably ascertained from the small number of repetitions utilized in this example. Also, using more than three levels of error in the variables would likely simplify the task of identifying aberrant responses. It is sometimes difficult to determine which result does not fit the pattern of the others when there are only three to select from.

The practical significance of the effects of the photo measurement errors introduced into the two data sets during this demonstration is a matter of personal judgment. The results do indicate that the more optimistic levels of error examined in this study had very little effect on the estimated parameters, the quality-of-fit of the regression models and their predicted values. Because aerial photo volume tables are properly used to obtain a preliminary volume estimate, a rough, first-look volume forecast, or as a corollary variable to a ground cruise in a double sampling framework, the effects of smaller, unbiased, normally disturbed photo measurement errors may be ignored without practical consequence. However, other error structures and data sets require further investigation.

SUMMARY AND CONCLUSION

The inevitability of photo measurement error has led to the use of ground measured data in the construction of aerial photo volume tables. This has necessitated the adjustment of photo-measured information before substitution into the volume prediction function. The need for this procedure has not been questioned in the past.

A review of the existing theory found it generally inappropriate for the problem of assessing the effect of photo measurement errors, because of its restrictive assumptions, and because no method has been developed for evaluating the effects of observational error. It was found, however, that predictions made from equations constructed from error-contaminated regressor variables can be unbiased if the data used to make the forecast are drawn from the same distribution as that of the regressor variables used to construct the prediction function. This is an important result because the predominant purpose of aerial photo volume tables is prediction.

A simple and flexible technique for assessing the effect of photo measurement errors on predictions of stand volume from aerial photography was developed and demonstrated. The technique uses random numbers of known distribution as proxies for measurement errors, and thus has only a small number of easily satisfied assumptions. The technique can handle a wide variety of structures for the photo measurement errors, and lets the user examine whatever property of the fitted regression functions is of interest. The proposed method could be used to determine thresholds of significant measurement error, or the effects of a selected type of error. The proposed procedure could be used in other situations where imperfectly measured data must be used to construct or apply a prediction function.

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