System Calibration and Self Calibration
Part I: Rotationally Symmetrical Lens Distortion and Image Deformation

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ABSTRACT: Photographs of three testfields taken with two cameras of a different make, both equipped with a front-projected réseau, have been used to investigate the effectiveness of system calibration as compared to several self-calibration procedures. Image coordinates were corrected to a varying extent for image deformation and for rotationally symmetrical lens distortion and then used repeatedly in a bundle adjustment which permitted the determination of several sets of self-calibration parameters.

INTRODUCTION

MOST PHOTOGRAMMETRIC CONSIDERATIONS are based on the assumption of a mathematically ideal camera: the light-sensitive recording surface is a perfectly flat plane; the optical axis of the lens is perpendicular to this plane and intersects it in the principal point; and an object point, the projection centre, and the resulting image point are located on a straight line.

An ideal camera cannot, of course, be realized in practice. The best that can be hoped for is a perfectly made camera with no manufacturing defects whatsoever. Such a camera would not include a lens free of distortion—that is, object point, projection center, and image point are no longer located on a straight line—but this distortion would be small and completely rotationally symmetric about the optical axis.

In a real aerial survey camera there are many departures, not only from the mathematically ideal camera, but also from a perfectly manufactured camera. It is not possible to center all lens elements perfectly; hence, the lens will most likely not have a single optical axis on which the centers of all lens surfaces are located. In addition, the materials used to manufacture the camera are not completely invariant with changes in ambient conditions. Other departures include (a) the inability to generate a perfectly flat recording surface at the instant of exposure; (b) manufacturing imperfections for camera filters and camera port windows; and (c) dimensional changes of the light-sensitive recording material between the instants of exposure and measurement as a result of processing, storage, and copying.

Some of these departures can be mathematically modeled and included into the general projective equations used in numerical photogrammetry, for example, departures due to rotationally symmetrical lens distortion. Other departures cannot be modeled nearly as well, but their effects can be included into the general projective equations with a larger number of parameters. We shall attempt to demonstrate to what extent mathematically modeled departures and various sets of selected parameters will lead to comparable results.

Some of the results reported in this paper were presented at the International Society for Photogrammetry and Remote Sensing (ISPRS) Commission I symposium in Canberra in 1982, others at the ISPRS congress in Rio de Janeiro in 1984. This paper will present root mean square image residuals for a larger number of adjustments of six blocks of aerial photographs of identical configuration, from over three different testfields with two simultaneously operated cameras of different type. Both cameras were equipped with a front-projected réseau. The paper will show the effect of the correction of rotationally-symmetrical lens distortion and image deformation. A similar evaluation with regard to decentring distortion will be reported in Part 2 of this investigation, and an overall evaluation of self-calibration models in Part 3.

GEOMETRIC-OPTICAL PERFORMANCE PARAMETERS

The general projective equations of phtogrammetry can include a larger number of parameters defining the geometric-optical performance of a photogrpahic data acquisition system.

In a perfect camera, the principal point is the image formed in the recording plane of the camera by a lens from an incident beam of parallel light which, in the object space, is perpendicular to the recording (emulsion) plane. Because an imperfectly centered lens does not possess a single optical axis, several possibilities exist for the definition of a point serving as the principal point. One point useful in laboratory calibrations is the point found when the foregoing definition for the principal point is applied to a real

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camera: this point has become known as principal point of autocollimation. In general, this point is neither identical with the center(s) derived from fiducial marks, nor the center point of a réseau, nor the point of best symmetry derived by an adjustment of lens distortion values aiming at best rotational symmetry.

For aerial photographs taken over practically flat terrain, as is the case for all three testfields, the differences between various points defining the image center are negligible.

Lens distortion can be defined in various ways. The formulation used most commonly for the rotationally symmetrical lens distortion is

$$\Delta x = x(s_0 + s_2 r^2 + s_4 r^4 + \ldots)$$
$$\Delta y = y(s_0 + s_2 r^2 + s_4 r^4 + \ldots)$$

in which $x = \ell_x - x_o$ and $y = \ell_y - y_o$, ℓ_x and ℓ_y are measurements, and x_o and y_o are the coordinates of the principal point chosen as origin of the image coordinate system. The origin for the rotationally symmetrical lens distortion is typically the point of best symmetry. The term s_0 is a function of the difference between equivalent focal length and calibrated focal length. It reduces the lens distortion determined with the equivalent focal length such that it becomes as small as possible, using one of several suitable criteria.

One departure from the ideal mathematical projection, which is difficult to model because it is of a systematic but changing nature, is image deformation. All photographs used were taken with two cameras equipped with a front-projected réseau, which permits a significant improvement in the correction of image deformation consisting of film deformation and the effect of major departures of the recording emulsion from the intended perfectly flat recording plane. All points of the 1-cm by 1-cm spaced réseau were measured. This made various types of image deformation corrections possible. The correction procedures chosen for this investigation were as follows: no correction (identified as N in Table 1 to 4); a bilinear transformation based on the four réseau points bracketing an image point (identified as B in the tables); and least-squares interpolation procedures based on the same four réseau points (identified as L in the tables), the nearest 24 réseau points (identified as I in the tables), and the nearest 52 points (identified as T in the tables). The correlation functions used in the least-squares interpolation were computed for each photograph from the residuals Δx, Δy remaining after a linear conformal transformation of the réseau measurements to an ideal grid: the sums of $\Delta x_i \Delta x_j$, $\Delta y_i \Delta y_j$, and $\Delta x_i \Delta y_j$ were first computed as a function of the distance between points i and j and then approximated by a Gaussian function for the sums of $\Delta x_i \Delta x_j$ and $\Delta y_i \Delta y_j$ and by a polynomial for the sum of $\Delta x_i \Delta y_j$. Experiments with sparse réseau data indicated that the use of the cross correlation polynomial could lead to nonsensical image coordinate corrections if only a few réseau points randomly selected were used for the correction of the image deformation. Therefore, only the two Gaussian correlation functions were used. While the bilinear transformation fits to the bracketing réseau points exactly, the least-squares interpolation procedures do not because of the presence of measuring inaccuracies at these points.

SYSTEM CALIBRATION

A camera calibration can be carried out with several objectives in mind: (1) an evaluation of the performance of a lens, (2) an evaluation of the stability of a lens, (3) the determination of geometric-optical performance parameters of the lens, (4) the determination of geometric-optical performance parameters of a lens-camera system, and (5) the determination of geometric-optical performance parameters of an aerial photographic data acquisition system. The first three objectives, and possibly also the fourth, are well served by a laboratory camera calibration procedure. Photographic laboratory procedures are usually set up in a controlled environment resembling the actual photogrammetric imaging process as closely as possible: Targets projected by collimators and appearing as located at infinity in known directions in front of a vertically downward looking camera are photographed on an emulsion in the camera's film plane. The illumination corresponds in its spectral distribution to typical mid-day light, and the emulsion in its spectral sensitivity to that of typical aerial photographic emulsions. At the National Research Council in Ottawa, the photographic emulsion coated on glass is made nearly perfectly flat prior to the exposure.

A laboratory calibration excludes many factors contributing to the overall performance of an aerial photographic data acquisition system by intent, and therefore deviates significantly from the concept of measurement system calibration developed by Eisenhart and championed for photogrammetric use by Merchant, who began relevant investigations in 1965 (Merchant, 1971, 1972, 1977):

> "Eisenhart considers that calibration is a refined form of measurement designed for the purpose of assigning numbers to specific properties of the procedure with approximate expressions of their systematic errors and precision. This task is accomplished by analysis of results of repeated applications of the measurement procedure (or subprocedure) performed over a random sampling of the range of circumstances allowed within the measurement specification. Predictions of error that are intended to characterize the process are obtained only after the measurement procedure has attained a stability known as 'State of Statistical Control.'

TABLE 1. RESULTS FOR THE RC8 RÉSEAU CAMERA FOR IMAGE COORDINATES NOT CORRECTED FOR LENS DISTORTION. ALL VALUES ARE RMS POINT RESIDUALS FOR CONTROL POINT IMAGES IN μm.

Self calibration	Image deformation correction				
	N	B	L	I	T
		Rheidt			
—	8.3	8.2	8.2	7.9	7.9
S	6.5	6.3	6.3	6.1	6.1
O	5.7	5.6	5.6	5.3	5.3
F(11)	6.2	6.0	6.0	5.8	5.8
E	7.4	7.3	7.3	7.1	7.1
F(14)	5.9	5.6	5.6	5.4	5.4
B	5.7	5.5	5.5	5.2	5.2
F(20)	5.8	5.5	5.5	5.3	5.3
		Jämijärvi			
—	5.0	5.1	5.0	4.7	4.7
S	4.3	4.5	4.4	4.0	4.0
O	3.8	4.0	3.9	3.5	3.5
F(11)	4.1	4.2	4.1	3.7	3.7
E	4.5	4.9	4.8	4.5	4.5
F(14)	3.9	4.1	3.9	3.5	3.5
B	3.6	4.0	3.9	3.4	3.4
F(20)	3.9	4.0	3.9	3.5	3.5
		Sudbury			
—	10.0	9.0	8.9	8.8	8.8
S	7.2	7.1	7.0	6.9	6.9
O	6.4	6.3	6.2	5.9	5.9
F(11)	7.8	6.7	6.6	6.4	6.4
E	8.0	8.4	8.3	8.2	8.2
F(14)	7.4	6.3	6.2	6.0	6.0
B	6.0	6.1	6.0	5.8	5.8
F(20)	7.4	6.2	6.1	5.9	5.9

TABLE 2. RESULTS FOR THE RMK RÉSEAU CAMERA FOR IMAGE COORDINATES NOT CORRECTED FOR LENS DISTORTION. ALL VALUES ARE RMS POINT RESIDUALS FOR CONTROL POINT IMAGES IN μm.

Self calibration	Image deformation correction				
	N	B	L	I	T
		Rheidt			
—	8.1	7.4	7.3	7.4	7.3
S	7.0	6.7	6.7	6.7	6.6
O	6.7	6.4	6.4	6.4	6.3
F(11)	7.4	7.1	7.1	7.1	7.0
E	6.7	6.6	6.6	6.6	6.5
F(14)	6.8	6.5	6.5	6.5	6.4
B	6.2	5.9	5.9	5.9	5.8
F(20)	6.8	6.4	6.4	6.4	6.3
		Jämijärvi			
—	6.8	5.9	5.9	5.7	5.7
S	5.3	5.4	5.3	5.1	5.1
O	5.0	5.0	5.0	4.7	4.7
F(11)	6.2	5.6	5.6	5.3	5.3
E	5.2	5.4	5.3	5.1	5.1
F(14)	5.6	4.9	4.8	4.6	4.7
B	4.1	4.2	4.2	4.0	4.0
F(20)	5.5	4.8	4.8	4.6	4.6
		Sudbury			
—	10.3	7.4	7.4	7.3	7.3
S	7.8	7.1	7.1	6.9	6.9
O	7.6	6.8	6.8	6.7	6.7
F(11)	8.8	7.3	7.2	7.1	7.1
E	7.7	7.1	7.0	6.9	6.9
F(14)	8.4	6.7	6.7	6.6	6.6
B	7.0	6.2	6.2	6.1	6.1
F(20)	8.4	6.6	6.5	6.4	6.4

"The guidelines necessary for achieving a realistic calibration of the photographic system are then the same as those for achieving the 'State of Statistical Control.' "

Again, according to Eisenhart:

" . . . the desired state of control can be achieved only after the following has been accomplished:

1. The establishment of measurement procedure specifications which define (along with allowable ranges or variations):
   a. apparatus
   b. operations
   c. sequence
   d. conditions
2. The establishment of consistency of the measurement (calibration) procedure:
   a. obtained measurements conducted within the established specification and sampled randomly within the stated range of conditions.
   b. analysis of the measurements to determine consistency or stability according to an arbitrary standard."

This concept of calibration provides a basis for systematically conducting calibrations of photographic systems. Mechant notes:

"Many procedures intended for calibration have been devised and are currently in use. With few exceptions, however, these schemes fall short of Eisenhart's concept of calibration. Such calibration procedures may well yield superior results in terms of the fit of the mathematical model to the observations. If the procedures are not conducted within the expected ranges of operational circumstances, the results do not represent realistic characteristic properties of the measurement procedure as intended. In many applications, a serious compromise in measurement accuracies will result using the results from such calibrations." (Merchant, 1971)

It is noted that the concept of measurement system calibration—or, for short, system calibration—requires an arbitrary standard of comparison. Such a standard for photogrammetric purposes is a testfield with known well-defined points. Photographs over three such testfields were available for the investigation.

TABLE 3. RESULTS FOR THE RC8 RÉSEAU CAMERA FOR IMAGE COORDINATES CORRECTED FOR ROTATIONALLY SYMMETRICAL LENS DISTORTION. ALL VALUES ARE RMS POINT RESIDUALS FOR CONTROL POINT IMAGES IN μm.

Self calibration	Image deformation correction N	B	L	I	T
		Rheidt			
—	6.2	6.1	6.1	5.7	5.7
S	5.8	5.6	5.6	5.3	5.3
O	5.8	5.7	5.7	5.4	5.4
F(11)	5.8	5.6	5.6	5.3	5.3
E	5.9	5.7	5.7	5.4	5.4
F(14)	5.8	5.6	5.6	5.3	5.3
B	5.7	5.5	5.5	5.3	5.3
F(20)	5.8	5.5	5.5	5.2	5.2
		Jämijärvi			
—	4.3	4.2	4.1	3.7	3.7
S	3.9	4.1	4.0	3.6	3.6
O	3.8	4.0	4.0	3.5	3.5
F(11)	4.0	4.1	4.0	3.5	3.5
E	3.8	4.1	4.0	3.6	3.6
F(14)	3.9	4.0	3.9	3.5	3.5
B	3.7	4.0	3.9	3.5	3.5
F(20)	3.9	4.0	3.9	3.5	3.5
		Sudbury			
—	8.3	6.5	6.4	6.2	6.2
S	6.4	6.3	6.2	6.0	6.0
O	6.3	6.2	6.1	5.9	5.9
F(11)	7.3	6.2	6.1	5.9	5.9
E	6.2	6.4	6.3	6.1	6.1
F(14)	7.3	6.2	6.1	5.9	5.8
B	6.1	6.2	6.1	5.9	5.8
F(20)	7.3	6.1	6.0	5.8	5.8

TABLE 4. RESULTS FOR THE RMK RÉSEAU CAMERA FOR IMAGE COORDINATES CORRECTED FOR ROTATIONALLY SYMMETRICAL LENS DISTORTION. ALL VALUES ARE RMS POINT RESIDUALS FOR CONTROL POINT IMAGES IN μm.

Self calibration	Image deformation correction N	B	L	I	T
		Rheidt			
—	7.7	6.7	6.6	6.7	6.6
S	6.4	6.0	6.0	6.0	5.8
O	6.5	6.0	6.0	6.0	5.9
F(11)	6.7	6.4	6.4	6.4	6.3
E	6.4	6.0	5.9	6.0	5.9
F(14)	6.7	6.4	6.3	6.3	6.2
B	6.2	5.9	5.9	5.9	5.8
F(20)	6.7	6.3	6.3	6.3	6.2
		Jämijärvi			
—	6.6	5.0	5.0	4.8	4.8
S	4.3	4.3	4.2	4.0	4.0
O	4.4	4.3	4.2	4.0	4.0
F(11)	5.4	4.7	4.7	4.5	4.5
E	4.6	4.4	4.3	4.0	4.0
F(14)	5.4	4.7	4.6	4.4	4.4
B	4.2	4.2	4.1	4.0	4.0
F(20)	5.4	4.7	4.6	4.4	4.4
		Sudbury			
—	10.1	6.8	6.8	6.7	6.7
S	7.1	6.2	6.1	6.0	6.0
O	7.1	6.2	6.1	6.0	6.0
F(11)	8.2	6.4	6.3	6.2	6.2
E	7.4	6.4	6.3	6.2	6.2
F(14)	8.1	6.3	6.3	6.2	6.2
B	6.9	6.0	6.0	5.9	5.9
F(20)	8.1	6.3	6.2	6.1	6.1

In the case of vertical photography over a practically flat plane testfield—the three testfields all fall into this category—an almost strict linear dependency exists between the following paired elements of interior and exterior orientation:

- Principal point coordinate (x_0) and exposure station coordinate (X_0) when $\kappa \sim 0°$ or 180°
- Principal point coordinate (y_0) and exposure station coordinate (Y_0) when $\kappa \sim 0°$ or 180°
- Calibrated focal length (or camera constant, f_c) and the terrain clearance $(H - h)$

It is clear that computational problems will arise as calibration is attempted when such geometry is given. For vertical photography, which is the conventional aerial case and the case at hand, there are basically two alternative approaches which have been suggested to supress these unfavorably high correlations existing between interior and exterior elements of orientation: the determination of the exposure station coordinates during the flight mission and the "Method of Mixed Ranges" proposed by Merchant. Neither approach was available to us. Hence, the principal point coordinates and calibrated focal lengths determined for the used cameras with camera calibration procedures were held, and only the parameters defining lens distortion were determined. There was no need to consider either shear factor or differential scale change for the image coordinates corrected by means of the réseau.

System calibration within the context of this paper includes the rotationally symmetrical component of the distortion of the lens, the effects of the camera port window, dimensional changes of lens/camera components caused by changes in the ambient conditions, uncorrected film deformation and deviation of the emulsion from the intended recording surface, and photogrammetric refraction.

System calibrations were carried out for each of the six blocks of photography using image coordinates corrected for image deformation by means of least-squares interpolations based on the 52 réseau points nearest to an image point and not more distant from that point than 8 cm.

SELF CALIBRATION

Self calibration is the name commonly used for an approach to a photogrammetric triangulation procedure with a configuration permitting the recovery of parameters defining the geometric-optical performance of the employed data acquisition system. In this approach all image points contribute to the determination of the parameters while only the given control points do so in laboratory and system calibration.

Self calibration had been used by the Photogrammetric and Geodetic Service Division of DBA Systems, Inc. (now Geodetic Services, Inc.) since 1965 in a bundle adjustment program used to calibrate parabolic radio reflectors by means of terrestical photographs (Brown, 1974). The first paper dealing with self calibration of aerial photographs presented at a meeting of the International Society of Photogrammetry was that by Bauer and Müller (1972) given at the congress in Ottawa. Since that time, self calibration enjoyed an increasing attention in photogrammetry. After the 1976 congress, a working group, "Compensation of Systematic Errors of Image and Model Coordinates," was established to study (1) component calibration, (2) testfield calibration, (3) self calibration, and (4) other possible methods. The results achieved by the working group are presented in Kilpelä (1980) and show that the members of the group concentrated on self calibration. Some results for system (testfield) calibrations and component calibrations are reported as well, but the report does not include a comparison of system and self calibration results.

The interest in self calibration using aerial photographs was rather limited after 1980. Yet a systematic comparison of system calibration and self calibration, and of different procedures for self calibration, is still outstanding.

From Appendix A of Kilpelä (1980), the following self-calibration parameter sets were selected:

Parameter set a proposed by Brown, to be referred to later as B:

$$
\begin{aligned}
dx = {} & a_1x + a_2y + a_3xy + a_4y^2 + a_5x^2y + a_6xy^2 + a_7x^2y^2 \\
& + \frac{x}{c}\left(a_{13}(x^2 - y^2) + a_{14}x^2y^2 + a_{15}(x^4 - y^4)\right) \\
& + x\left(a_{16}(x^2 + y^2) + a_{17}(x^2 + y^2)^2 + a_{18}(x^2 + y^2)^3\right) \\
dy = {} & a_8xy + a_9x^2 + a_{10}x^2y + a_{11}xy^2 + a_{12}x^2y^2 \\
& + \frac{y}{c}\left(a_{13}(x^2 - y^2) + a_{14}x^2y^2 + a_{15}(x^4 - y^4)\right) \\
& + y\left(a_{16}(x^2 + y^2) + a_{17}(x^2 + y^2)^2 + a_{18}(x^2 + y^2)^3\right)
\end{aligned}
$$

Parameter set c proposed by El-Hakim and Faig, to be referred to later as F. It was modified in regard to a_1 and a_2, and the harmonic series was extended by fourth order terms. The used parameter set is

$$dx = q\frac{x}{r}$$

$$dy = a_1\,y + a_2\,x + q\frac{y}{r} \text{ where}$$

$$
\begin{aligned}
q = {} & a_3r\cos\lambda + a_4r\sin\lambda + a_5r^2 + a_6r\cos2\lambda + a_7r\sin2\lambda \\
& + a_8r^5\cos\lambda + a_9r^3\sin\lambda + a_{10}r^3\cos3\lambda + a_{11}r^3\sin3\lambda \\
& + a_{12}r^4 + a_{13}r^4\cos\lambda + a_{14}r^4\sin\lambda + a_{15}r^4\cos2\lambda + a_{16}r^4\sin2\lambda \\
& + a_{17}r^4\cos3\lambda + a_{18}r^4\sin3\lambda + a_{19}r^4\cos4\lambda + a_{20}r^4\sin4\lambda \\
r^2 = {} & x^2 + y^2 \text{ and } \lambda = \arctan\left(\frac{y}{x}\right)
\end{aligned}
$$

The parameter set was used with 11, 14, and 20 coefficients, respectively.

Parameter set f proposed by Grün, to be referred to later as E:

$$
\begin{aligned}
dx &= a_1x + a_2xy + a_3xy^2 + a_4x^2y + a_5y^2 + a_6x^2y^2 \\
dy &= b_1\,x + b_2\,xy + b_3xy^2 + b_4x^2y + b_5y^2 + b_6x^2y^2
\end{aligned}
$$

Parameter set m proposed by Schut, to be referred to later as S:

$$
\begin{aligned}
dx &= c_3xy + c_5y^2\ (+\ c_7x^2y + c_9xy^2 + c_{11}x^2y^2 + c_{13}x^3) \\
dy &= c_1y + c_2x + c_4x^2 + c_6xy\ (+\ c_8x^2y + c_{10}xy^2 + c_{12}x^2y^2 + c_4y^3)
\end{aligned}
$$

This parameter set was used with only six parameters.

Parameter set n proposed by Salmenperä and Kilpelä, to be referred to later as O:

$$
\begin{aligned}
dx &= b_1x + b_2y + b_3xr^2(1-r_0/r) + b_4xr^4(1-r_0/r) + b_5xr^6(1-r_0/r) + b_6\cdot 2xy + b_7(r^2 + 2x^2) \\
dy &= -b_1y + b_2x + b_3yr^2(1-r_0/r) + b_4yr^4(1-r_0/r) + b_5xr^6(1-r_0/r) + b_6(r^2 + 2y^2) + b_7\cdot 2xy,
\end{aligned}
$$

where r_0 is a given constant (first radial distance, where radial distortion is wanted to be zero).

These parameter sets were incorporated into Schut's bundle adjustment program (Schut, 1978). This program was written to allow, in a simple way, the introduction of any choice and number of parameters. This has been achieved by dimensioning all affected arrays to variable length, and by restricting the program to the use of one of many subroutines defining additional parameters. This subroutine is defined by its name and the number of parameters in the input to the program. For this investigation, the program has been used repeatedly with one of the parameter sets defined above at a time.

RESULTS

Photographs of the three testfields—Rheidt, Jämijärvi, and Sudbury—were used. Each testfield was simultaneously flown with two wide-angle réseau cameras, a Wild RC8R and a Zeiss RMK-AR. The available six sets of photographs each consists of 13 photographs, namely, a 3 by 3 block with 60 percent overlap in both directions, and four photographs each covering the area of the block in one of the four different orientations. Glass diapositives of all photographs were measured on Zeiss PSK1 stereocomparators used in the monocomparator mode. All réseau and targeted image points were measured twice. The second measurements were carried out after completion of the first, using a reversed point order. The reading differences between first and second measurement were nearly normally distributed and resulted in standard deviations of the averages of the two measurements given in Table 5.

All testfield points were used as control points because the geometry of the block does not enable a good determination of points located in only one photograph of the 3 by 3 block. It should be noted that the photographs had been taken to carry out system calibration, and that the use of all ground points as control points had been intended.

In order to avoid distortion of the final results by gross errors of any source, all points exceeding three times the root-mean-square (RMS) image residual lengths after a bundle adjustment with self calibration were eliminated from the input prior to the correction of the measured image coordinates for lens distortion and/or image deformation. Hence, all the different adjustments of a block use an identical number of control point images, namely:

	RC8	RMK
Rheidt	771	767
Jämijärvi	1567	1587
Sudbury	1562	1555

The three coordinates of all the points of the three testfields were determined by geodetic methods: those of the Rheidt testfield "with RMS errors of less than ±10 mm" (Kupfer, 1972), those of the Jämijärvi testfield with errors of less than ±5 mm (Kilpelä and Savolainen, 1972), and those of the Sudbury testfield with approximately ±10 mm (documentation in preparation). In order to verify the quoted accuracies, adjustments using fully corrected image coordinates were carried out using the program GEBAT (El-Hakim, 1982). This program treats control points as weighted observations using an *a priori* standard error for each. One point each in each block corner was given an *a priori* standard deviations of 0.1 mm for each coordinate, while all other points were given the standard deviations listed in Table 6. The improvements for the residuals of the image coordinates resulting from the relaxation of the weighting of the control point coordinates is similar for all blocks, thus confirming a similar level of random error for all three testfields.

The results are given in Tables 1 to 4. They were obtained for Tables 1 and 2 without lens distortion correction, and for Tables 3 and 4 after correction of the rotationally symmetrical lens distortion determined for each block with corrected image coordinates. The first row in each table gives results obtained without self calibration, the first column results obtained without image deformation correction. Thus, the left-most value of the first lines in Tables 1 and 2 gives the result for uncorrected image coordinates, and the

TABLE 5. STANDARD DEVIATIONS FOR THE AVERAGES OF TWO MEASUREMENTS, IN μm.

		Targeted points			Réseau points		
Testfield	Camera	No. of points	σx	σy	No. of points	σx	σy
Rheidt	RC8	771	1.6	1.7	3856	1.7	1.6
	RMK	767	1.6	1.8	3404	1.7	1.6
Jämijärvi	RC8	1616	1.2	1.4	6871	1.5	1.7
	RMK	1634	1.6	1.7	6848	1.7	1.8
Sudbury	RC8	1617	1.8	1.9	6877	1.6	1.8
	RMK	1596	1.7	1.8	6851	1.8	1.7

TABLE 6. RMS RESIDUALS OF IMAGE COORDINATES ($\sigma x/\sigma y$), IN μM, OBTAINED WITH THE BUNDLE ADJUSTMENT PROGRAM GEBAT USING INCREASINGLY RELAXED WEIGHTS FOR THE CONTROL POINT COORDINATES.

		A priori standard deviations, in mm			
Testfield	Camera	$\sigma_{X,Y,Z} = 0$	$\sigma_{X,Y,Z} = 10$	$\sigma_{X,Y,Z} = 20$	$\sigma_{X,Y,Z} = 30$
Rheidt	RC8	3.9/4.0	3.0/3.2	2.9/3.2	2.9/3.1
	RMK	4.6/4.6	3.9/3.8	3.9/3.7	3.9/3.7
Jämijärvi	RC8	2.4/2.7	2.0/2.1	2.0/2.1	2.0/2.1
	RMK	3.2/3.4	2.7/2.9	2.5/2.7	2.4/2.7
Sudbury	RC8	4.4/4.2	3.8/3.6	3.5/3.3	3.4/3.3
	RMK	4.7/4.4	4.1/3.8	3.8/3.5	3.7/3.5

right-most value of the first lines in Tables 3 and 4 the results for the system calibrations.

The self calibration results are listed in the tables in the order of an increasing number of self-calibration parameters: 0, 6, 7, 11, 12, 14, 18, and 20. The given values are the root-mean-square residuals for the control point images, given in μm in the image scale.

INTERPRETATION OF RESULTS

The interpretation will be carried out with regard to several aspects, namely overall error level, film deformation correction, lens distortion correction, and self-calibration parameter sets.

OVERALL ERROR LEVEL

The results obtained for the different test areas are obviously different. However, there is no apparent reason why this should be so. The positional coordinates for all three testfields were determined by triangulation and the elevations by levelling, and can be epxected to have standard deviations not exceeding 1 cm. With photoscales of 1:5,000 and 1:10,000 for the Rheidt and Jämijärvi photography, and 1:8,000 and 1:15,000 for the Sudbury photography, the given control cannot be expected to be responsible for the different overall error levels.

All photographs were taken with two-engine aircraft commonly used for aerial photography using the same cameras. It is unlikely but, of course, possible that differences in aircraft account for the differences.

The targetting in the different testfields differs. The targets in the Jämijärvi testfields have built-in contrast and were most easy to measure. However, past experience with different target types as well as the obtained reading differences from repeated measurements of all points indicate that the differences in targetting are not likely to have caused the differences in the overall error levels.

The differences may be related to the location of the testfields: the Rheidt and the Sudbury testfields are located close to industrial plants, and one wonders whether the differences may have been caused by different levels of air pollution.

Comparison of the results for the two different cameras shows a persistent difference. This difference has also been observed in other projects flown simultaneously with these two cameras and is believed to be the result of differences in the image contrast of photographs produced with these two cameras: the RC8 imagery is of higher contrast, which results persistently in better pointing accuracy. It should be noted that only one film was used for each testfield; it was cut in half prior to insertion into the two magazines. Also, the photographic treatment of both halves of the film after exposure was identical.

IMAGE DEFORMATION CORRECTION

Differences between columns N and columns B, L, I, and T in Tables 1 to 4 are an indication of the complexity of the image deformation pattern. The photographs show very little scale affinity and shear effect. Overall scale changes and second-order deformation are compensated by changes in the exterior orientation parameters. Higher order deformations may not be completely compensated and are causing the differences between column N and the other columns in the case of the RMK photography over all three testfields.

Differences between columns B, L, I, and T indicate differences in the effectiveness of image deformation correction procedures. Columns B and L were derived from image coordinates corrected for image deformation by means of the four réseau points surrounding an image point; B using a bilinear transformation forcing an exact fit, and L using a least-squares interpolation procedure allowing for a certain noise level. The two columns are almost identical throughout; occasional small differences favor procedure L.

Columns I and T are identical with few exceptions in Tables 2 and 4. These two columns are based on image coordinates corrected by a least-squares interpolation using up to 24 and 52 points, respectively, nearest to the image point. The results indicate that the increase in the number of points on which the least-squares interpolation was based is, in general, not justified. Differences between the two columns are observed primarily for the Rheidt RMK photography. While all réseau points were measured for the other testfields, here this was done only for the smaller-scale photographs. In the larger-scale 3 by 3 block only a 5-cm by 5-cm grid was selected from the réseau for measurement in addition to the four réseau points surrounding a target. Because the 24-point pattern is located within a radius of approximately 52 mm around the image point, and the 52 points within approximately 77 mm, only a few additional points are gained when replacing least-squares interpolation procedure L by I and I by T, respectively.

The small but consistent improvements between columns L (or B) and I (or T) indicate that the use of more than four réseau points is desirable, presumably to eliminate detrimental effects caused by measuring inaccuracies for a réseau point located near an image point to be corrected. Because the réseau crosses are, in general, more difficult to measure than targeted points, their measured coordinates are, in general, less accurate than those for targeted pointed. This fact also causes occasionally values in column B (or L) to exceed those reported in column N. On the other hand, the values in column I (or T) exceed those in column N only in one case by a small amount, namely in Table 1 for Sudbury, self-calibration E.

Correction of the Rotationally Symmetrical Lens Distortion

Differences between corresponding values in the odd-numbered and even-numbered tables (Tables 1 and 3 and Tables 2 and 4, respectively,) reflect the effect of the correction of the rotationally symmetrical lens distortion.

Both lenses were calibrated three times at the National Research Council laboratories in Ottawa: prior to the photographic missions in Europe (Rheidt, Jämijärvi), after these missions but prior to the Sudbury photography, and after the Sudbury photography. The results of these calibrations were reported in Ziemann (1978).

The RC8 camera was equipped with a Universal Aviogon lens which was practically free of decentring distortion according to these repeated calibrations. The rotationally symmetrical distortion curve of this lens shows only a single point of inflection. This type of rotationally symmetrical distortion is handled well by most of the parameter sets with the notable exception of set E. The lack of decentring distortion of this lens is illustrated by the Jämijärvi results for this camera (Table 3), where essentially the same results is obtained without self calibration and with all the different self-calibration approaches. This is not true, however, for the other two testfields, where a somewhat larger spread between the results is observed.

The RMK camera was equipped with a Pleogon lens having a rotationally symmetrical distortion curve with two points of inflection. The more complex rotationally symmetrical lens distortion is less well handled by the self-calibration parameter sets with fewer elements. This can be seen from a comparison of Tables 2 and 4 for the sets identified as S, O, F(11), and E with 6, 7, 11, and 12 terms, respectively.

The Pleogon lens employed to obtain all the photography used in this investigation showed a significant amount of decentring distortion in all three laboratory calibrations. All the different self-calibration parameter sets yield for all three testfields better results than were obtained by system calibration using corrections for image deformation and rotationally symmetrical lens distortion only.

Self-Calibration Parameter Sets

Tables 1 to 4 report results for five different self-calibration sets. The effectiveness of these sets is not directly related to the number of included terms; if this were so, the reported values would decrease from top to bottom for each column.

It has already been pointed out that some of these sets are less effective than others in compensating either image deformation or rotationally symmetrical lens distortion. The effectiveness of compensation is related to the complexity of either effect degrading the image geometry; both, image deformation and lens distortion, cause more complex image error patterns in the RMK camera. The effectiveness is best demonstrated by comparison of column N in Tables 1 and 2 and column T in Tables 3 and 4.

It is further worth noting that an increase in parameters does not necessarily result in a better result. Tables 1 to 4 show this for parameter set F which was used with its first 11, 14, and 20 terms, respectively. While the first addition of three parameters improved the result for both cameras, in particular for the RMK, the further addition of six parameters causes only a marginal improvement, if any. However, it should be noted that the function q of parameter set F can only accomodate radial components of systematic errors. Thus, the inclusion of additional terms of the function q does absolutely nothing to reduce effects of tangential components of systematic image errors, which may explain why the use of higher order terms of the function remained virtually without effect.

TABLE 7. SUMMARY OF RESULTS. PRESENTED ARE THE BEST SYSTEM CALIBRATION RESULTS FROM TABLES 3 AND 4, AND THE BEST SELF-CALIBRATION RESULT FOR UNREFINED IMAGE COORDINATES (N) FROM TABLES 1 AND 2.

Testfield	RC8 System	RC8 Self Calibration	RMK System	RMK Self Calibration
Rheidt	5.7	5.7	6.2	6.2
Jämijärvi	3.7	3.6	4.2	4.1
Sudbury	6.1	6.0	6.9	7.0

CONCLUSIONS

Table 7 summarizes the results presented in Tables 1 to 4: the best system calibration result (from the first lines of Tables 3 and 4) is compared with the respective best self-calibration result for unrefined image coordinates (from the first columns of Tables 1 and 2). The system calibration and self-calibration results are practically identical for the three sets of RC8 photographs. The self-calibration results are, for two of the sets of RMK photographs, better than those for system calibration, and slightly worse for the third. The Jämijärvi results agree with expectations; the results for the other two testfields are larger than expected.

The differences between system and self calibration are expected to be caused, at least in part, by the omission to correct for decentring distortion. This aspect will be discussed in the second report on this investigation. It is suspected that the image deformation is somewhat more complex for the RMK Sudbury photography and, therefore, more effectively corrected by means of the réseau than self calibration.

The employed self-calibration models vary in their effectiveness more than anticipated. It will, therefore, be of interest to conduct an analysis of the effectiveness of all the different employed parameters. It will be interesting to see whether the most successful model can still be improved upon. The intended analysis will also attempt to show relationships between system calibration and self-calibration parameters.

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