Filtering of Digitally Correlated Gestalt Elevation Data

*A. L. Kok** and *J. A. R. Blais*
Department of Surveying Engineering, The University of Calgary, Calgary, Alberta T2N 1N4, Canada
R. M. Rangayyan
Department of Electrical Engineering, The University of Calgary, Calgary, Alberta T2N 1N4, Canada

ABSTRACT: Digitally correlated Gestalt data are generally less accurate than manually digitized data because of limitations of image correlation. The data must, therefore, be filtered to remove the degradations and to improve accuracy. Results from the estimation of the unit sample response of the degradation function suggested that the responses vary from one Gestalt patch to another. Consequently, filtering for restoration cannot be performed unless the unit sample response of the degradation function is known for every patch. One alternative, as adopted in this investigation, is filtering for noise removal only. Conventional (linear and multilinear) and weighted least-squares filters in the spatial domain, and the Wiener filter in the spectral domain, were applied to the Gestalt data. The results suggest that accuracy of the degraded Gestalt data can be improved through noise removal only.

INTRODUCTION

THE GESTALT PHOTOMAPPER II, hereafter called the GPM II, is a system that uses the digital correlator to derive elevations. A given stereomodel is divided into approximately one thousand 8-mm by 9-mm patches, and the parallaxes of the 2444 points (or a 47 by 52 matrix) within each patch are derived. Only the center portion of a 40 by 40, 32 by 32, or 24 by 24 grid matrix is written out. The successive processing of individual patches is carried out column-wise to cover the whole stereomodel.

Currently, the automatic image correlators do not recognize objects. In the presence of steep terrain, the two images can be very different and the image correlators could then fail. Difficulties have also been reported when correlating over areas with buildings and other tall structures (Allam, 1982; Dowman and Haggag, 1977). Most image correlators, however, can perform geometric corrections for distortions caused by terrain relief. The process of image correlation could also fail if it is performed over imageries with low contrasts, such as over water- and snow-covered surfaces.

This paper presents various linear filtering algorithms, in the spatial and spectral domains, which could be used to improve the accuracy of the Gestalt data. The filtering algorithms investigated here take advantage of the regularly gridded structure of the Gestalt data. The findings of this investigation are also valid for filtering regularly gridded data generated by other digital correlators or acquired manually.

DESCRIPTION OF TEST AREA AND DIGITIZED DATA

Manually digitized data, which have higher accuracy than the Gestalt data, were required for the evaluation of the performance of the various filters and also for the estimation of the unit sample response of the degradation function. These data were obtained on a Wild AC-1 stereoplotter at locations corresponding exactly to those of the Gestalt data. Also, the same two pairs of aerial diapositives were used to ensure that the terrain being manually digitized was identical to the digitally correlated terrain.

The Gestalt digital elevation matrix (DEM) used in this investigation corresponds to one stereomodel in the National Research Council of Canada's Sudbury Test Area (hereinafter called the Sudbury Model or the Sudbury Gestalt Model). The pair of overlapping diapositives are from frame numbers 70 and 72 of the aerial photography at a scale of 1:16 000.

The overall root-mean-square (RMS) errors of eight targetted control points used in the absolute orientation were 0.057 m, 0.061 m, and 0.078 m (at ground scale) in X, Y, and Z, respectively. These same eight control points used for the absolute orientation on the GPM II prior to the correlation process yielded the overall RMS errors 0.145 m, 0.176 m, and 0.312 m (at ground scale) in X, Y, and Z, respectively.

The precision of the data digitized on the Wild AC-1 was evaluated using two determinations of the same elevation points. With a total of 992 points, each with two elevation measurements, the precision σ_d was 0.41 m.

DEGRADATION MODELS AND FILTERING STRATEGIES

Two linear filtering models that could be used for the processing of Gestalt data are presented here. The first model involves both restoration and noise removal and the second involves noise removal only.

In the first model, the Gestalt data $g(x,y)$ are assumed to have been degraded by a linear shift-invariant degradation function with unit sample response $h(x,y)$, and additive noise $\eta(x,y)$: i.e.,

$$g(x,y) = f(x,y) * h(x,y) + \eta(x,y) \tag{1}$$

where $f(x,y)$ represents the undegraded elevation data. This model could be used efficiently only if the degradation function is linear and shift-invariant when going from one patch to another within the Gestalt DEM. If this is not true, then the application of this model is severely limited, unless there is some knowledge of the unit sample response $h(x,y)$ of the degradation function for each Gestalt patch. In the absence of the above information regarding $h(x,y)$ for all patches, a second model should be used, where the only errors present are due to additive noise, i.e.,

$$g(x,y) = f(x,y) + \eta(x,y). \tag{2}$$

Depending on the model chosen, restoration filters may be designed for removal of the convolutional degradation and/or the additive noise.

In the context of the Gestalt data, $h(x,y)$ is not known explicitly. Moreover, access to the GPM II has not been possible for this investigation. Hence, $h(x,y)$ has to be determined analytically using data digitized manually on the Wild AC-1 stereoplotter (see previous section) and the erroneous Gestalt data. The following discussions will explain how $h(x,y)$ could be determined.

Equation 1 can be rewritten in matrix notation as

$$\mathbf{g} = \mathbf{F}\mathbf{h} + \boldsymbol{\eta} \tag{3}$$

*Currently with Intergraph Corporation, One Madison Industrial Park, Huntsville, AL 35807.

PHOTOGRAMMETRIC ENGINEERING AND REMOTE SENSING,
Vol. 53, No. 5, May 1987, pp. 535–538.

0099-1112/87/5305–535$02.25/0

where **F** is the convolution matrix, **g** is the vector of the degraded Gestalt data, and **h** and η are the vectors of the unit sample response of the degradation function and noise, respectively. The unknown unit sample response **h** of the degradation function could be determined by least-squares estimation. For an over-determined solution, the least-squares estimate $\hat{h}$ is well known to be expressible as

$$\hat{h} = (F^T C_\eta F)^{-1} F^T C_\eta^{-1} g \tag{4}$$

where C_η is the covariance matrix of the zero-mean noise and all other quantities are as defined in Equation 3. It is reasonable to assume that C_η is an identity matrix because no other *a priori* information is available (e.g., Blais, 1985).

The results obtained from investigations performed as described above revealed that $\hat{h}$ is not shift invariant for finite array sizes of 20 by 20 and 10 by 10 when going from one patch to another (Kok, 1986). Thus, to perform filtering for restoration, $\hat{h}$ must be obtained for and applied to each patch individually. Although this is not impossible, the second model was chosen in this study to attempt noise removal only as a first measure.

FILTERING IN THE SPATIAL DOMAIN

Two filters for the estimation of signals in the presence of additive noise in the spatial domain are discussed here. Both of these filters, *viz.* the conventional and weighted least-squares filters, are linear and shift invariant.

Conventional Least-Squares Filter

This is an optimal filter that uses the least-squares, also called minimum mean-square, criterion. It is assumed that the signal and noise are stationary random processes with known first- and second-order moments. Also, the signal and noise are assumed independent. The optimum solution is dependent on the autocovariance functions of the signal and noise. This filter has been dealt with in various references (Kraus and Mikhail, 1972; Mikhail, 1976; Kratky, 1978; Hall, 1979; Papoulis, 1977; Blais, 1985). Thus, mathematical details of the derivation are not given here but the implementation of this optimal filter is described.

In order to reasonably satisfy the stationarity assumption, the trend component in the data is first removed to obtain centered values. These centered data are used in the estimation of the signal as

$$\hat{s} = c_{sr}(C_{ss} + C_{\eta\eta})^{-1}\,\hat{r} \tag{5}$$

where $\hat{r}$ denotes the centered discrepancies. The diagonal elements of C_{ss} are equal to the variance of the signal, and the diagonal elements of $C_{\eta\eta}$ are equal to the variance of the white noise, while c_{sr} denotes the cross-correlation matrix between the signal and the data.

The model for the autocovariance function of the signal used was

$$C(d) = C_{ss}(0)\exp(-kd)^2 , \tag{6}$$

where $C_{ss}(0)$ is the variance of the signal to be estimated, d is the lag, and k is a constant to be estimated. This model completely characterizes the random signal, assumed here to be a stationary Gaussian process.

When the signal at location i has been estimated, it is added back to the value of the trend at the corresponding location

$$\hat{z}_i = \hat{s}_i + \mathbf{A}_i\hat{\mathbf{X}} , \tag{7}$$

where z_i is the filtered elevation, $\hat{s}_i$ is the estimated signal, and $\mathbf{A}_i\hat{\mathbf{X}}$ is the value of the estimated trend at location i.

The conventional least-squares filter is used here as an adaptive, or locally variable, filter in the context of filtering the Gestalt data. Each patch of the Gestalt DEM is assumed to have known first- and second-order moments for the random signal and noise, and the filtering is performed on one patch at a time.

Conventional Least-squares Filter with Multilinear Formulation. With the gridded structure of the Gestalt data, and with the assumption that the autocovariance function models of the signal and noise are separable in the two orthogonal directions (X and Y), conventional least-squares filtering can be performed using the *multilinear* formulation. Either the direct product or the more computationally efficient array algebra can be used. Details of direct product or array algebra are given in Rauhala (1980), Blaha (1977), Snay (1978), and Lancaster (1969).

The assumption of separable autocovariance function models may or may not be valid in the context of the Gestalt data, but the assumption is required so as to apply the multilinear formulation. In this investigation, the validity of this assumption is judged by how well this filter performed in the experimentation.

The conventional least-squares filter with multilinear formulation has the form (see, e.g., Kok, 1986)

$$\hat{s} = (c_{sxX}C_{rrX}^{-1}) \otimes (C_{sxY}\,C_{rrY}^{-1})\,\hat{r} \tag{8}$$

where c_{sxX} and c_{sxY} are the vectors of autocovariances between the centered data and the unknown signal to be estimated in the X and Y directions, respectively, C_{rrX} and C_{rrY} are the matrices of autocovariances for the signal part of the centered data in the X and Y directions, respectively, $\hat{r}$ are the centered data, and $\otimes$ denotes the direct product.

The two covariance matrices C_{rrX} and C_{rrY} are

$$C_{rrX} = C_{ssX} + C_{\eta\eta X} \tag{9}$$

$$C_{rrY} = C_{ssY} + C_{\eta\eta Y} , \tag{10}$$

where C_{ssX} and C_{ssY} are the covariance matrices of the signal part of the centered data in the X and Y directions, respectively, and $C_{\eta\eta X}$ and $C_{\eta\eta Y}$ are the covariance matrices of the noise part of the centered data in the X and Y directions, respectively.

The direct product formulation in Equation 8 can also be written in the array algebra formulation as (see, e.g., Kok, 1986)

$$\hat{s} = c_{sxX}\,C_{ssX}^{-1}\,\mathbf{R}\,C_{ssY}^{-1}\,c_{sxY}^T \tag{11}$$

where **R** is a matrix formed from the centered data and all other quantities are as defined in Equation 8. Snay (1978) has shown that the array algebra formulation is even more computationally efficient and requires less storage than the direct product solution.

To filter data in an array of size M by N, Equation 8 or Equation 11 requires solving one M by M and another N by N linear system of equations, while Equation 5 requires the solution of a MN by MN linear system of equations. Therefore, the multilinear formulation is more efficient than the conventional least-squares filter.

Weighted Least-Squares Filtering

This filter consists of two mathematical models with appropriate weights (see Kok, 1986 for details):

$$v_1 = f_i - g_i \tag{12}$$

$$v_2 = f_{i-1} - 2f_i + f_{i+1} , \tag{13}$$

where v_1 and v_2 are the discrepancies, or residuals, of the first and second models, f's are the unknowns to be estimated, and g_i is an observation corresponding to location i in the grid matrix. The first model states that the unknown f_i at location i should be estimated from the observation g_i. The second model corresponds to the Laplacian condition in the finite difference method of interpolation discussed in Lancaster and Salkauskas (1975). Note that the observation in this model can also be set equal to any value different from zero.

Rewriting Equations 12 and 13 in the matrix notation as $\mathbf{A}_1\mathbf{X} - \ell_1 = \mathbf{V}_1$ and $\mathbf{A}_2\mathbf{X} - \ell_2 = \mathbf{V}_2$, respectively, the weighted least-squares filter is then given by

$$\hat{\mathbf{X}} = (\mathbf{A}_1^t\mathbf{P}_1\mathbf{A}_1 + \mathbf{A}_2^t\mathbf{P}_2\mathbf{A}_2)^{-1}\,(\mathbf{A}_1^t\mathbf{P}_1\ell_1 + \mathbf{A}_2^t\mathbf{P}_2\ell_2)$$

where $\mathbf{P}_1$ and $\mathbf{P}_2$ correspond to the observables in the first and second model, respectively. Note that $\mathbf{A}_1$ is an identity matrix and, because the observation in Equation 13 is equal to zero, ℓ_2 is a null vector.

The finite element method of interpolation and filtering discussed in Ebner and Reiss (1978) can also be related to the filter discussed here.

FILTERING IN THE SPECTRAL DOMAIN

With the regularly gridded structure of the Gestalt data, filtering in the spectral domain can be implemented efficiently using the Fast Fourier Transform (FFT) (see Brigham, 1974; Bracewell, 1978). The Wiener filter for removal of additive noise is presented here.

WIENER FILTER

The Wiener filter is an optimal filter that minimizes the mean-square error. This criterion is identical to that for the conventional least-squares filter, discussed earlier.

In Equation 5, the covariance matrix $(\mathbf{C}_{ss} + \mathbf{C}_{\eta\eta})$ is positive-definite and symmetric. Another important property of this matrix is its Toeplitz structure, a direct result of the fact that autocovariances are dependent only on the distance between two data points, i.e., wide-sense stationarity. As explained in Andrews and Hunt (1977) and Hall (1979), a matrix with the Toeplitz structure can be aproximated by a circulant matrix, which can then be diagonalized by the Fourier transform.

Filtering centered data with the Wiener filter is performed as follows (Hall, 1979; Barrett and Swindell, 1981):

$$\hat{S}(u,v) = \frac{P_s(u,v)}{P_s(u,v) + P_\eta(u,v)} R(u,v) \,, \tag{15}$$

where $\hat{S}(u,v)$ is the estimate of the signal in the spectral domain, $R(u,v)$ is the Fourier transform of the centered data, and $P_s(u,v)$ and $P_\eta(u,v)$ are the power spectral density (PSD) functions of the signal and noise model, respectively.

The PSD of the signal is computed from its autocovariance model by Fourier transformation. The PSD of the noise, which is assumed white, is a constant and can be determined from the noise variance, which is equal to the mean-square value for a zero-mean process.

RESULTS AND DISCUSSIONS

Results obtained with the three filters, *viz.* the conventional least-squares filter with the multilinear formulation (LSM), the weighted least-squares filter (WLS), and the Wiener (WN) filter, were compared. The conventional least-squares filter (with the linear formulation) was excluded from the comparison because it is computationally very involved and yields results equivalent to the Wiener filter (Kok, 1986).

Six Gestalt patches were processed using the methods described. Figure 1 shows the graphical results for one of those patches (C18R11), including the results using a conventional moving average (MA) filter for comparison purposes. All the data within a patch, which forms a grid matrix of size 32 by 32, were used in the filtering. However, only an array of size 20 by 20, corresponding to rows 7 to 26 and columns 7 to 26 of each filtered patch, was used for comparisons. There are two reasons for using only the interior 20 by 20 grid of data. The first is that the filtered data are usually not reliable along the edges, and this is especially true with data filtered in the spectral domain. The second reason for considering only the interior 20 by 20 array is that the data on both sides of a patch boundary are usually less accurate than data closer to the patch center. Therefore, data near the patch perimeter should be considered separate from the data in the interior of the patch. Although results from filtering the data near the patch perimeter have not been analyzed explicity, these filters are applicable to any part of a Gestalt patch.

For the LSM and WN filters, the second-order trend was removed from the data. The 32 by 32 array of data filtered with the WN filter were extended to an array of size 64 by 64 and windowed before a Fourier transformation was performed. For the WLS filter, the weights used were 5.0 and 1.0 for Equations 12 and 13, respectively.

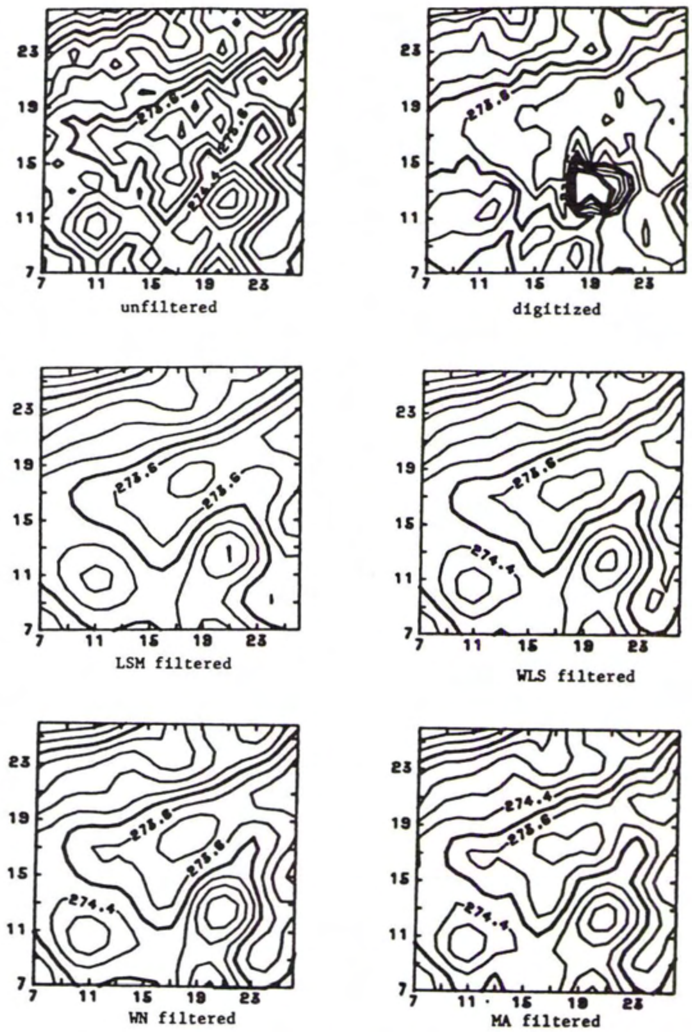

FIG. 1. Contour plots corresponding to C18R11

TABLE 1. SNRS AND RMS ERRORS (IN METRES) FOR THE SIX PATCHES

PATCHES	SNR	UNFILTERED σ_{dg} (m)	LSM σ_{df} (m)	WLS σ_{df} (m)	WN σ_{df} (m)
C6R13	2.96	0.71	0.65	0.63	0.65
C7R11	2.56	0.68	0.64	0.59	0.61
C9R9	2.52	0.60	0.54	0.51	0.51
C10R18	6.68	0.82	0.78	0.77	0.80
C18R11	2.27	0.71	0.63	0.64	0.65
C18R17	2.54	0.67	0.64	0.57	0.62

Table 1 shows the RMS errors σ_{dg} and σ_{df}, computed from the differences between the digitized and unfiltered and between the digitized and filtered data, respectively. The improvements range from 0.02 m to 0.10 m. These rather small numbers are misleading if they are not considered in conjunction with the accuracy of the digitized data, σ_d, which is 0.41 m. This is because σ_{df} is comprised of two components, which can be deduced by error propagation of the equation expressing the difference between the digitized and filtered data (assuming random errors only): i.e.,

$$\sigma_{df}^2 = \sigma_d^2 + \sigma_f^2 \quad (16)$$

where σ_d^2 is the variance of the digitized data, and σ_f^2 is the variance of the filtered data. As an example, consider one patch from Table 1, say C18R11. The improvements in accuracy of 0.05 m, 0.06 m, 0.07 m, and 0.08 m translate to about 17, 20, 23, and 27 percent of the maximum possible improvement of 0.30 m.

From Table 1, it can then be concluded that all three filters yielded approximately the same improvement in accuracy for each of the six patches. The rather similar improvements in accuracy of the LSM and WN filtered data also suggest that the assumption of separable autocovariance functions and variances in the LSM filter is valid. Note that the WN filter does not assume the autocovariance functions and variances to be separable.

The signal-to-noise ratios (SNRs) shown in Table 1 were computed from the empirically determined autocovariance functions of the signal and noise:

$$\mathrm{SNR} = \sigma_s/\sigma_\eta \, . \quad (17)$$

To obtain the SNRs in terms of variances, one simply squares the result obtained from Equation 17. The SNRs are a function of the photography, scanning spot diameter, and various other factors. Without access to the GPM II, the proper values of SNRs for the patches being investigated cannot be obtained. Rather, the validity of the SNRs shown in Table 1 is ascertained by comparison to the values determined in investigations carried out by Forstner (1982) and Helava (1976). All, except one, of the computed SNRs are at the lower end of the range of values of 1.0 to 5.0 cited in Forstner (1982), and all are in the range 1.1 to 10.0 cited in Helava (1976). This is because the values cited in those papers correspond to the digitized gray levels whereas the SNRs here correspond to the digitally correlated data, which have additional noise introduced due to correlation.

Removal of the degradation due to $h(x,y)$ was not performed in the filtering of the Gestalt data presented here. If the model of $h(x,y)$ for each patch is known, then restoration could be performed. In the context of Wiener filtering, variable and adaptive procedures such as those used in analysis of speech signals (Oppenheim, 1978) and modelling of electroencephalogram (EEG) signals (Bodenstein *et al.*, 1977) on a short-time basis could be extended to two-dimensional Gestalt data for "patch processing."

This paper has presented the implementation and results of several linear filtering algorithms in the spatial and spectral domains. These filtering algorithms are efficient when used with data which have a regularly gridded structure, such as the Gestalt and other digitally correlated elevation data.

ACKNOWLEDGMENTS

The contributions of Prof. M.A. Chapman in this investigation are greatly appreciated. This work was funded by the Natural Sciences and Engineering Research Council of Canada.

REFERENCES

Allam, M.M., 1982. Filtering and Editing of Automatically Generated Digital Elevation Models, *Proceedings of the Symposium on Mathematical Models, Accuracy and Quality Control*, ISPRS Commission III, Helsinki, pp. 12–23.

Andrews, H.C., and B.R. Hunt, 1977. *Digital Image Restoration*, Prentice Hall, Englewood Cliffs, New Jersey.

Barrett, H.H., and W. Swindell, 1981. *Radiological Imaging*, Vol. 1, Academic Press, New York.

Blaha, G., 1977. Least-Squares Prediction and Filtering in Any Dimensions Using the Principles of Array Algebra, *Bulletin Geodesique*, Vol. 51, pp. 265–286.

Blais, J.A.R., 1985. *Estimation and Spectral Analysis*, The University of Calgary Press, Calgary, Alberta. (In Press)

Bodenstein, G., and H.M. Praetorius, 1977. Feature Extraction from the Electroencephalogram by Adaptive Segmentation. *Proc. of the IEEE*, Vol. 65, No. 5.

Bracewell, R.N., 1978. *The Fourier Transform and Its Applications*, McGraw-Hill, New York.

Brigham, E.O., 1974. *Fast Fourier Transform*, Prentice Hall, Englewood Cliffs, New Jersey.

Dowman, I.J., and A. Haggag, 1977. Digital Image Correlation Along Epipolar Lines, *Proceedings of the International Symposium on Image Processing, Interactions with Photogrammetry and Remote Sensing*, Graz, pp. 47–49.

Ebner, H., and P. Reiss, 1978. Height Interpolation by the Method of Finite Elements, Paper presented to the Symposium of Commission III, ISPRS, Moscow, USSR, pp. 79–94.

Forstner, W., 1982. On the Geometric Precision of Digital Correlation, *Proceedings of the Symposium on Mathematical Models, Accuracy and Quality Control*, ISPRS Commission III, Helsinki, pp. 176–189.

Hall, E.L., 1979. *Computer Image Processing and Recognition*, Academic Press, New York.

Helava, U.V., 1976. *Digital Correlation in Photogrammetric Instruments*, Bendix Research Laboratories, Southfield, Michigan.

Kok, A.L., 1986. *Preprocessing and Filtering of Digitally Correlated Gestalt Data*, M.Sc. Dissertation, Department of Surveying Engineering, The University of Calgary, Calgary, Alberta.

Kratky, V., 1978. Reflexive Prediction and Digital Terrain Modelling, *Photogrammetric Engineering and Remote Sensing*, Vol. 44, No. 5, pp. 569–574.

Kraus, K., and E.M. Mikhail, 1972. Linear Least-squares Interpolation, Paper presented at the 12th Congress of ISP, Ottawa, Ontario.

Lancaster, P., 1969. *Theory of Matrices*, Academic Press, New York.

Lancaster, P., and K. Salkauskas, 1975. *An Introduction to Curve and Surface Fitting*, Department of Mathematics and Statistics, The University of Calgary, Calgary, Alberta.

Mikhail, E.M., 1976. *Observations and Least-squares*, IEP-A Dun-Donnelley Publisher, New York.

Oppenheim, A.V., 1978. Digital Processing of Speech. *Applications of Digital Signal Processing*, A.V. Oppenheim ed., Prentice Hall, Englewood Cliffs, New Jersey.

Papoulis, A., 1977. *Signal Analysis*, McGraw Hill, New York.

Rauhala, U.A., 1980. Introduction to Array Algebra, *Photogrammetric Engineering and Remote Sensing*, Vol. 46, No. 2, pp. 177–192.

Snay, R.A., 1978. *Applicability of Array Algebra*, NOAA Technical Memorandum NOS NGS-11, National Geodetic Survey, Rockville, Maryland.

(Received 16 May 1986; revised and accepted 6 January 1987)