A Comparison of Methods to Measure the Modulation Transfer Function of Aerial Survey Lens Systems from the Image Structures

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ABSTRACT: For the image quality analysis of photogrammetrical systems using the Modulation Transfer Function (MTF), the edge gradient analysis (EGA), using a Hanning function, and the grating pattern method are compared. Artificial edge and grating patterns were photographed from an airplane and were analyzed to determine the quality of the photogrammetric system. The pictures were scanned with a microdensitometer. For comparison with the artificial patterns, a natural roof edge was examined. Good agreement of all MTF measurements was found. Furthermore, the resolution found from MTF curves agreed well with the resolution obtained from three-bar targets. Generally, the MTF curves obtained from patterns in the flight direction were lower than the MTF's perpendicular to the flight direction due to airplane movement. The influence of linear image motion and its compensation was examined and is discussed.

INTRODUCTION

In photogrammetry the image quality of photographs is of great importance. With digital image processing the image quality can be improved under certain conditions. Flight conditions usually lead to reduced image quality (Holdermann, 1976). The optical transfer function (OTF) and especially the modulation transfer function (MTF) are useful techniques for measuring the image quality of photographs. The MTF is the result of the contributions of the links in the image transfer forming chain, namely, the camera, the film, image motion (in systems without forward motion compensation, FMC), vibrations, and the atmosphere (Tiziani, 1977).

Artificial or natural edges are used for the edge gradient analysis (EGA), and varying spatial frequency patterns are used for the grating pattern technique. The aim of this paper is to compare both methods and to test their reliability. Results are shown and discussed. Furthermore, the influence of linear image motion on image quality is examined.

METHODS FOR DETERMINING THE MTF FROM AN EDGE IMAGE

During the past 20 years different calculation methods for edge gradient analysis, EGA, have been developed (Jones, 1967; Jones and Yeadon, 1969; Lei and Tiziani, 1986; Sievers, 1976). One method, based on the differentiation of the edge, has often been used for MTF determination. The OTF is determined from the Fourier transform of the spread function, obtained from differentiating the edge image. It can be called an indirect method. Recently, a direct method using a Hanning window was developed (Lei and Tiziani, 1986). The method is based on the Fourier spectrum amplitude ratio between the real and an ideal edge image. It can be called an edge spectrum ratio method. There are several other methods for EGA (Scott *et al.*, 1963). In the following only the differentiation method and the edge spectrum ratio method will be discussed.

EDGE DIFFERENTIATION METHOD

In linear space invariant systems the image obtained can be described by the convolution integral

$$B'(u',v') = \iint_{-\infty}^{\infty} B(u,v)\cdot G(u'-u,\, v'-v)\,du\,dv = B(u',v')^{*}G(u',v') \tag{1}$$

with the intensity distributions $B(u,v)$ and $B'(u',v')$ in the object and the image, respectively, and $G(u',v')$ the spread function.

If $B(u,v)$ is an edge function it can be written one-dimensionaly as

$$K(u) = \begin{cases} 0 & -\infty < u < 0 \\ 1 & 0 \le u < \infty \end{cases} \tag{2}$$

which results in the edge image function

$$K'(u') = \int_{-\infty}^{u'} \mathrm{L}(u)du \tag{3}$$

By differentiating the edge image function, the line spread function can be obtained, i.e.,

$$\frac{dK'}{du'} = L(u'). \tag{4}$$

The Fourier transformation of Equation 4 leads to the OTF

$$D(R) = FT[L(u')] \tag{5}$$

The absolute value of $D(R)$ is the MTF. Figure 1 shows the calculation process schematically. In order to suppress photographic film noise, Jones and Yeadon (1969) introduced a smoothing function for the differentiation method.

High frequency noise is eliminated by convolution of the spread function with a smoothing function or by multiplication of the OTF with a rectangular function. Low frequency noise can be suppressed by multiplying the spread function with a chosen Gauss function.

EDGE SPECTRUM RATIO METHOD

In the new and relatively simple EGA method, the MTF is obtained by dividing the spatial frequency spectrum of a scanned edge by the spatial frequency spectrum of an ideal edge. From convolution theory and Equation 1, it follows that

$$b'(R,S) = b(R,S)\cdot g(R,S) \tag{6}$$

where $b'(R,S)$, $b(R,S)$, and $g(R,S)$ are the Fourier transforms of $B'(u',v')$, $B(u,v)$, and $G(u',v')$, respectively.

From the definition of the MTF, it follows that

$$T(R,S) = |g(R,S)| = \left|\frac{b'(R,S)}{b(R,S)}\right| \tag{7}$$

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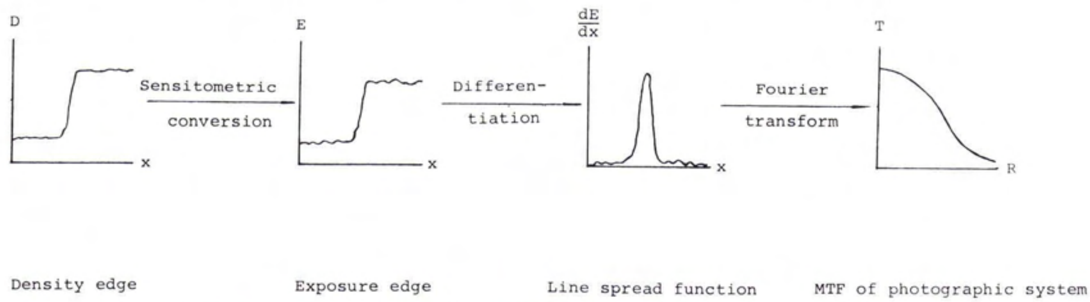


FIG. 1. Principle of the differentiation method to obtain the MTF from an edge image.

The division of the direct numerical Fourier transforms of the edge in the image and object space leads to a problem because infinitely expanded edges must be limited by a multiplication with a rectangular function (window), leading to zero amplitudes at even harmonic frequencies in the spectrum of the ideal edge. To avoid a division by zero, the rectangular function is replaced by a more suitable window function such as the Hanning function (Brigham, 1974). Figure 2 shows the Hanning function and its spectrum.

The Hanning function can be written as

$$H(u) = \begin{cases} 1/2 - 1/2\cos(2\pi u/L) & 0 \le u \le L \\ 0 & u < 0,\ u > L \end{cases} \tag{8}$$

with the length L of the limited edge.

The Fourier transform of the Hanning function is

$$h(R) = \frac{1}{2}\,Q(R) - \frac{1}{4}\,[Q(R + \frac{1}{L}\ ) + Q(R - \frac{1}{L}\ )] \tag{9}$$

$$\text{with } Q(R) = \frac{\sin(\pi L R)}{\pi R}.$$

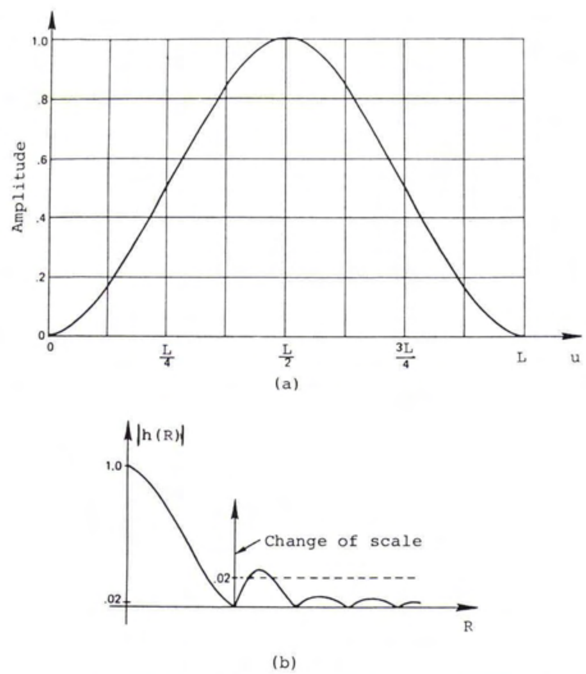


FIG. 2. (a) Hanning function. (b) Spectrum of the Hanning function.

Figure 3 shows the spectra of an edge function (a - multiplied with the Hanning function, b - without multiplication).

A smoothed curve is obtained from the spectrum of an edge image multiplied by the Hanning function, which is in good agreement with the spectrum's envelope. Zero amplitudes are no longer present.

Figure 4 shows schematically the procedure for the MTF determination. Before applying the Fourier transformation, both scanned and ideal edges must be multiplied by the Hanning function. By the introduction of the Hanning function, noise on top of an edge image can also be suppressed. The method of Blackman (1968) can be used additionally (convolution of the MTF with a chosen Gauss function) to suppress photographic grain noise.

It can be shown that a multiplication with a Hanning function does not affect the MTF. The method was also analyzed with a computer simulated edge function and good agreement with the expected MTF was found. The ratio method requires little numerical effort in comparison to other edge detection methods. For the EGA of images, we used the ratio method.

DETERMINATION OF THE MTF FROM A GRATING PATTERN

For comparison with EGA, a variable spatial frequency grating pattern was used for the determination of the MTF. The density profile of the grating pattern is shown in Figure 5a. The pattern was photographed from an airplane with an aerial survey camera. The contrast of the photographed pattern decreases due to diffraction and different disturbances with increasing spatial frequency. Figure 5b shows a density profile for a grating image

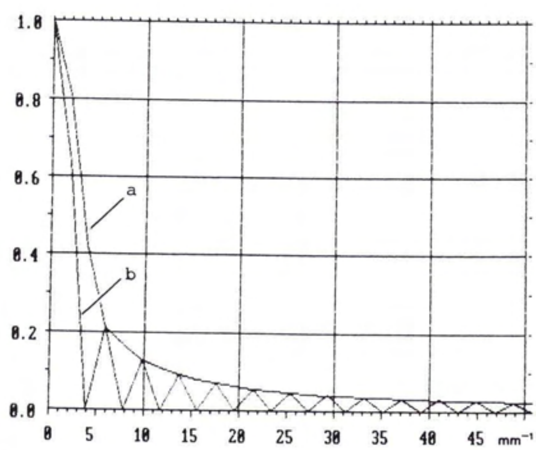


FIG. 3. Spectra of the edge images: a - multiplied with the Hanning function and b - without Hanning function.

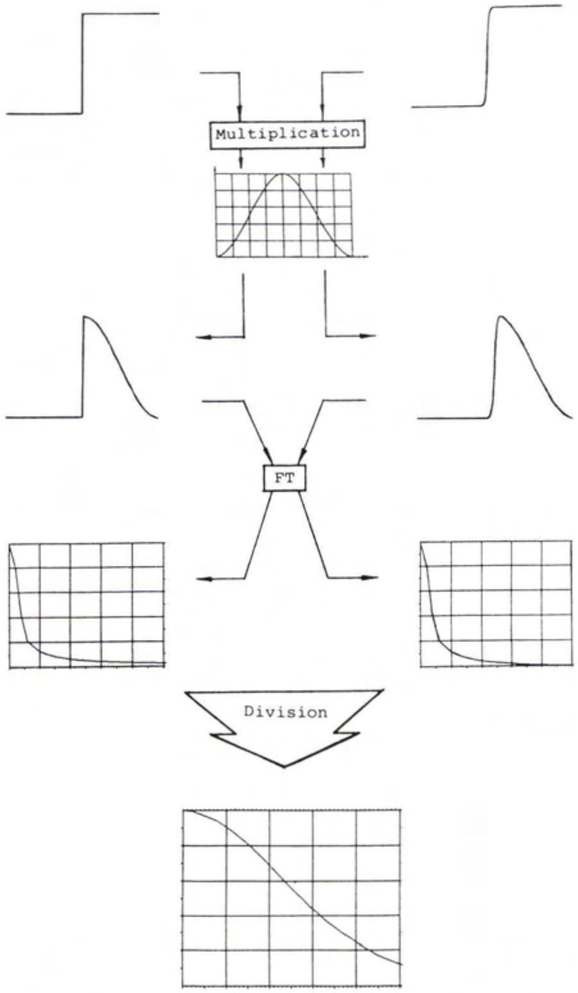


FIG. 4. Diagram to calculate the MTF using the Hanning function.

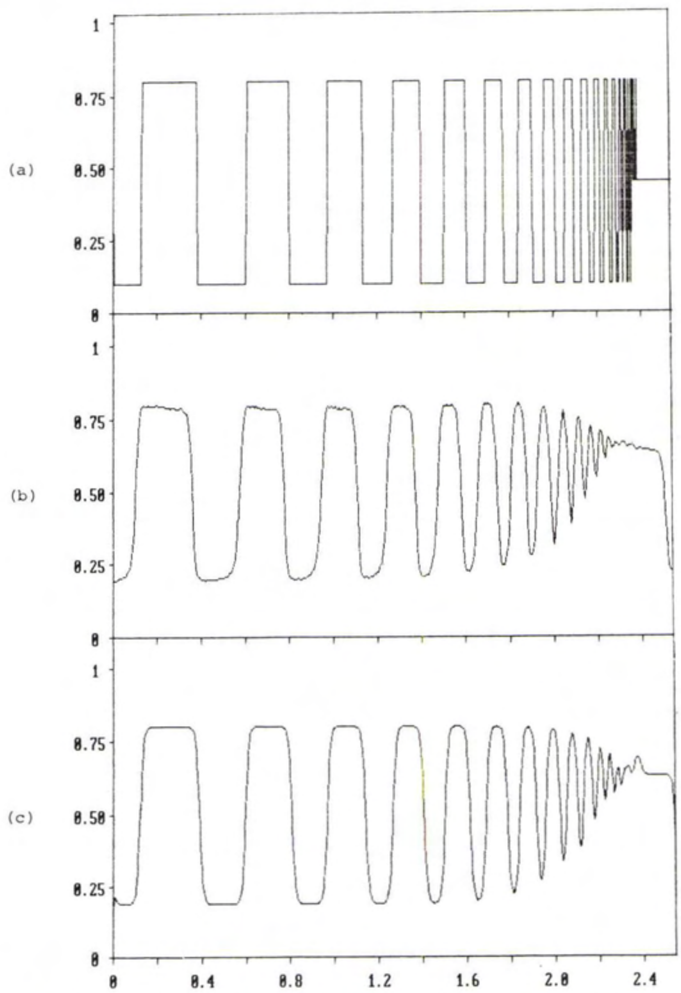


FIG. 5. (a) The density profile of an original grating pattern. (b) The density profile measured on a Kodak Panatomic-X film. (c) The corresponding computer-simulated density profile.

obtained with a microdensitometer. Using the convolution in Equation 1, the imaging process can be simulated with a computer. The intensity distribution of the grating pattern is convoluted with a computer simulated model of the spread function and the result, shown in Figure 5c, compares well with that of the scanned image of the grating.

Three different models of spread functions were suggested by Kölbl (1985). A Gauss function, which was found to be a very rough approach, and two different combinations of two Gauss functions have been examined. We used the following spread function:

$$I(u) = 0.85\exp\left[-4\left(\frac{u-u_0}{\sigma}\right)^2\right] + 0.15\exp\left[-1\left(\frac{u-u_0}{\sigma}\right)^2\right] \quad (10)$$

with the coordinate u_0 of the spread function maximum and a parameter σ controlling the width of the spread function.

For the convolution of a spread function with the intensity distribution of a grating pattern, the parameter σ was steadily varied until the envelope of the scanned image was in best agreement with the simulated pattern. For the comparison of the simulated with the scanned pattern, the exposure curve of the film needs to be taken into account. After determination of σ, the MTF is calculated as the absolute value of the Fourier transformed spread function. According the Fourier theory, it is a combination of two Gauss functions.

COMPARISON BETWEEN THE MTF'S OBTAINED FROM AN EDGE IMAGE AND A GRATING PATTERN

To analyze the image quality of airborne photographs, artificial patterns were produced and photographed by the Wild

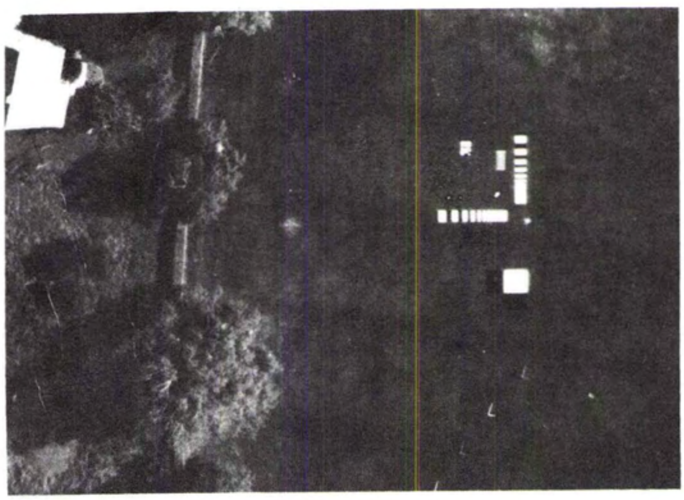
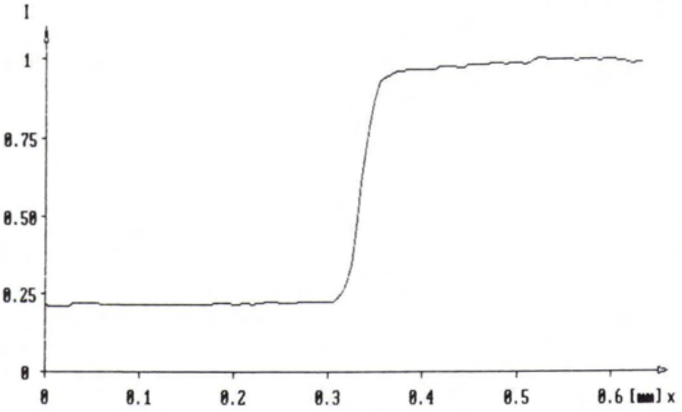


FIG. 6. Different test patterns.



(a)

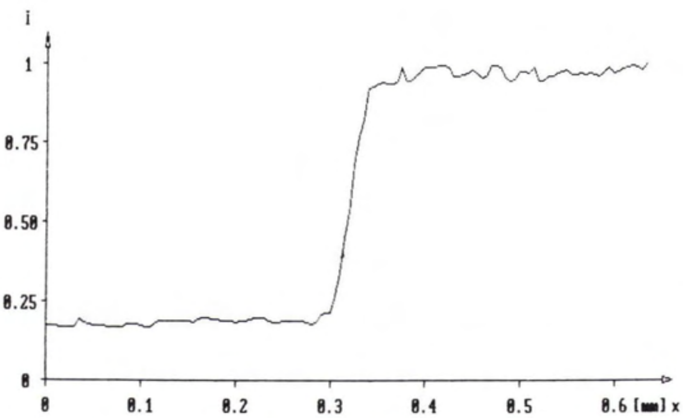


FIG. 7. (a) The density profile of an artificial edge image. (b) The density profile of a natural edge image.

Corporation. A pair of edge and grating patterns was oriented in the flight direction, and another pair was oriented orthogonally to it. Figure 6 shows a recorded image. The length of the grating pattern was 8 m and the width was 1.2 m. The length of the edge pattern was 4.5 m and the width was 2.5 m. Kodak Panatomic-X film was used. The photographs were taken with an RC10A camera manufactured by Wild. Objective focal lengths were 150 mm (effective aperture ratio 1:4) and 300 mm (1:5.6). The test pattern images were scanned with a computer controlled microdensitometer and the measurements were evaluated by a computer.

Figure 5b shows the results obtained from a grating pattern oriented in the flight direction. Very good agreement was obtained between the theoretical and the measured curves. The spread function parameter σ was 27μm. The calculated width of the spread function of a grating pattern perpendicular to the flight direction was 23 μm.

Figure 7 shows the density profiles obtained for an artificial edge and a natural edge. A higher signal-to-noise ratio of the density profile is obtained when scanning an artificial edge in comparison to a natural edge. Figure 8 shows the MTFs obtained from grating patterns and from artificial and natural edges both in the flight direction (a) and perpendicular to the flight direction (b). It should be noted that there is little difference in the MTF of a natural and an artificial edge. We found good agreement of the results obtained with different test patterns.

For an additional test of the MTF, we examined three-bar patterns with different spatial frequencies. They were put close to the grating and edge patterns in order to determine resolution limits. With a 300-mm focal length lens, the minimum detectable period in the flight direction was 100 mm, and the corresponding spatial frequency on the film was 31.5 mm^{-1}. The minimum detectable period perpendicular to the flight direction was 70.7 mm with a corresponding film spatial frequency of 44.6 mm^{-1}. By introduction of a threshold in Figure 8, the resolution mentioned above is obtained as an intersection between the MTF and the threshold (Tiziani, 1978). The spatial frequencies at the intersection of the MTF, for well-defined artificial edges, and the threshold curves compare well with the resolution limits obtained from the three-bar pattern in the flight direction and perpendicular to it, respectively. This comparison proves the reliability of the different methods for the determination of image quality.

INFLUENCE OF LINEAR IMAGE MOTION ON IMAGE QUALITY

There are different types of image motions such as linear, parabolic, sinusoidal, and random (Tiziani, 1977). This paper deals only with linear motion. The quality of airborne photo-

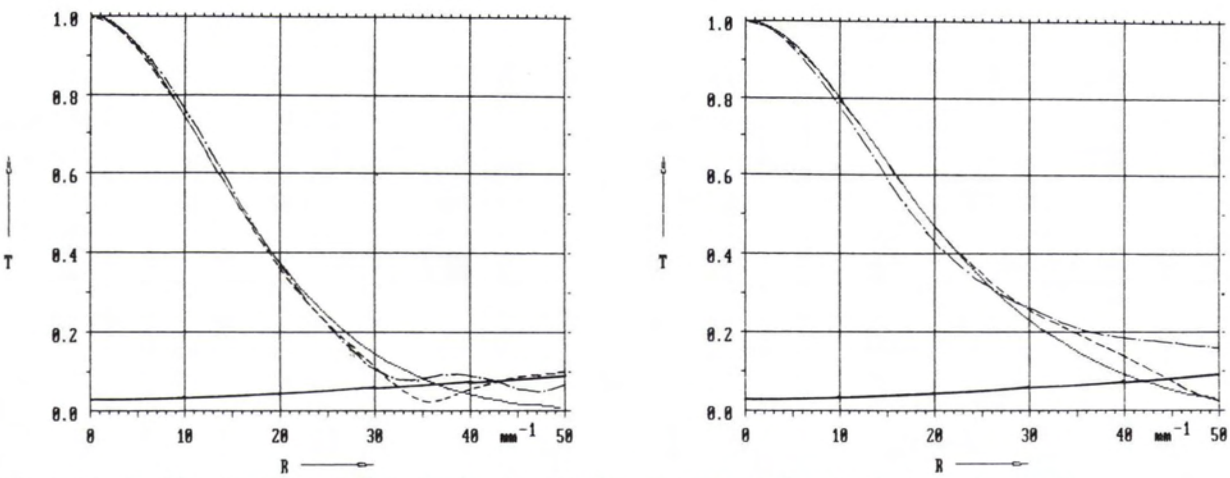


FIG. 8. (a) MTF of the test patterns in flight direction. (b) MTF of the test patterns perpendicular to the flight direction.
———— MTF curve of a grating pattern.
------------ MTF curve of an artificial edge image.
— • —•— • MTF curve of a natural edge image.
━━━━ Threshold value SW.

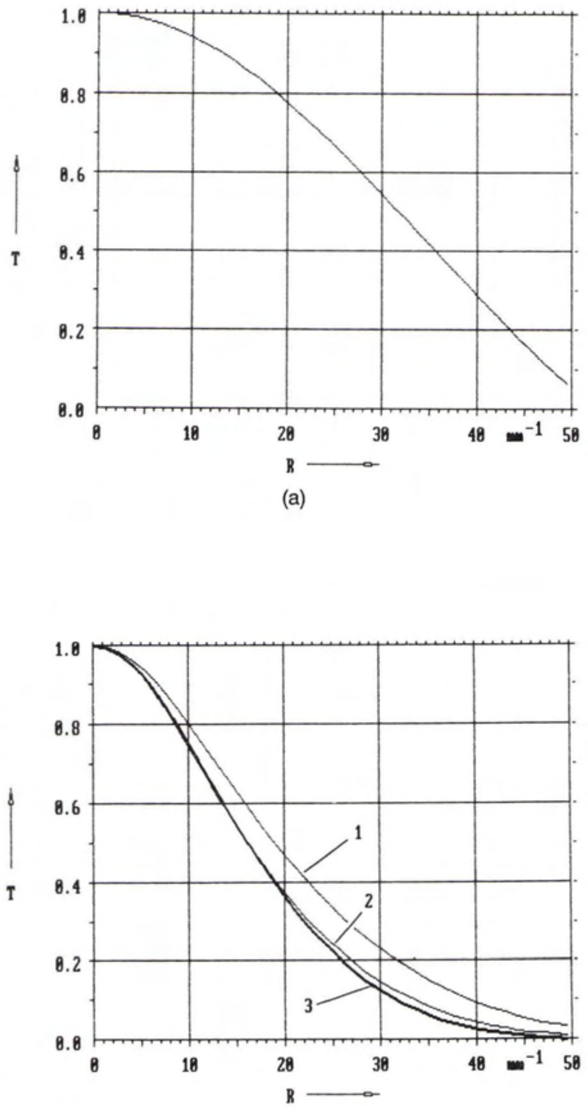


FIG. 9. (a) MTF of an image motion with $a = 19$ μm. (b) MTF_F, MTF_P, and $MTF_B \cdot MTF_P$.
1 - MTF_P, 2 - MTF_F, 3 - $MTF_B \cdot MTF_P$.

graphs can be improved by applying forward motion compensation (FMC). The photographs thus far referred to in this paper were obtained without using FMC, and this explains why in Figure 8 the MTFs of test patterns oriented on the flight direction are worse than those perpendicular to it. From the difference of the MTFs in the two directions, the image motion can be calculated. It will be compared with the theoretical image motion.

The image displacement during exposure time T is

$$a = vTf/H \tag{11}$$

where v is the speed of the airplane, f is the focal length of the objective, and H is the altitude of the airplane. The spread function corresponding to the movement is a rectangular function of width a. The corresponding MTF is its Fourier transform (Meier, 1960)

$$MTF_B = a \frac{\sin \pi R a}{\pi R a} \tag{12}$$

where R is the spatial frequency and a is the image motion. This is a sinc-function. If the MTF is measured perpendicular to the flight direction and multiplied by this sinc-function, the result measured in the flight direction should be obtained providing the signal-to-noise ratio is not to bad. According to linear response theory, the MTF_F in flight direction can be written as

$$MTF_F = MTF_B \cdot MTF_P \tag{13}$$

where MTF_P is perpendicular to the flight direction and MTF_B is in the direction of linear image motion. The image motion was therefore extracted from the measured MTF. The flight parameters were the following:

$$v/H = 0.04342 \text{ 1/sec}$$

$$T = 1/700 \text{ sec}$$

$$f = 303.64 \text{ mm}$$

From Equation 11 the image motion is obtained as

$$a = 19 \ \mu\text{m}.$$

Figure 9a shows the MTF of image motion with $a = 19$ μm. In Figure 9b MTF_F, MTF_P, and $MTF_B \cdot MTF_P$ are shown together. Good agreement was obtained between MTF_F and $MTF_B \cdot MTF_P$.

Two airborne photographs of the same scene, taken with and without FMC, were compared to investigate the linear image motion. Corresponding roof edges were chosen and scanned with the microdensitometer. From the parameters of the photograph ($v/H = 0.05491$ 1/sec, $T = 1/160$ sec, and $f = 303.75$ mm) the image motion was calculated according to Equation 11 to be 104 μm as shown in Figure 10a. The MTF without FMC can be written as

$$MTF_O = MTF_B \cdot MTF_C \tag{14}$$

with MTF_B as the MTF of the linear image motion and MTF_C that of the airborne photograph with FMC. Figure 10b shows the combined curves MTF_O, MTF_C, and $MTF_B \cdot MTF_C$. A major improvement of the image quality with FMC can be seen by comparing MTF_O and MTF_C. MTF_O compares well with $MTF_B \cdot MTF_C$. One should notice that the first minimum and maximum of the sinc-function can be seen regarding the MTF_O.

CONCLUSION

A comparison of the MTFs measured with two different methods is presented. The theory involved is first explained. For EGA a new and relatively simple method was developed, characterized by applying a Hanning function. In the same photograph a grating pattern was examined. The MTFs of both methods were found to be in good agreement. The two techniques can be used in a complementary manner and increase the reliability of image quality determination. EGA however has, from a practical point of view, the advantage that it does not rely on artificial edges, but works almost as well on natural roof edges. The measured MTFs have been compared with the resolution from three-bar targets where again good agreement was found. Furthermore, the influence of linear image motion was examined. We found an improvement of image quality with FMC. For photographs taken with high resolution films, FMC is found to be necessary. Due to longer exposure times, additional disturbances such as vibrations need to be considered.

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REFERENCES

Blackman, E. S., 1968. Effects of Noise on the Determination of Photographic System Modulation Transfer Functions: *Phot. Sci. Eng.*, Vol. 12, No. 5, pp. 244–250.

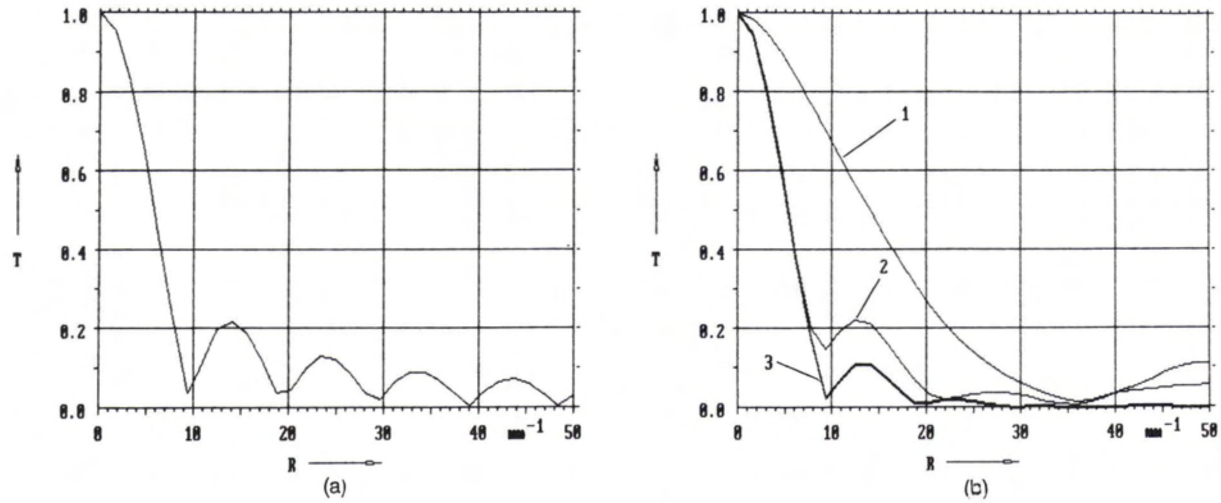

FIG. 10. (a) MTF of an image motion with $a = 104$ µm. (b) MTF_O, MTF_C, and $MTF_B \cdot MTF_C$ 1 - MTF_C, 2 - MTF_O, 3 - $MTF_B \cdot MTF_C$.

Brigham, E. O., 1974. *The Fast Fourier Transform*. Prentice-Hall, Inc. Englewood Cliffs, New Jersey.

Holdermann, F., 1976. Methoden zur Bildverbesserung: *Bildmessung und Luftbildwesen*, 44, Heft 2, pp. 53–61.

Jones, Robert A., 1967. An Automated Technique for Deriving MTF's from Edge Traces: *Phot. Sci. Eng.*, Vol. 11, No. 2, pp. 102–106.

Jones, Robert A., and Edward C. Yeadon, 1969. Determination of the Spread Function from Noisy Edge Scans: *Phot. Sci. Eng.*, Vol. 13, No. 4, pp. 200–204.

Kölbl, O., 1985. Arbeitstagung: Kammerkalibrierung in der photogrammetrischen Praxis. *Deutsche geodätische Kommission*, Reihe B, Heft 275, S. 48–64. Verlag der Bayerischen Akademie der Wissenschaften, herausgegeben v. G. Kupfer u. W. Wester-Ebbinghaus.

Lei, Fang, and H. J. Tiziani, 1986. Eine neue Methode zur Bestimmung der MTF aus dem Kantenbild: *Optik* 74, No. 1, pp. 17–21.

Meier, H.-K., 1960. Diskussion der Bewegungsunschärfe bei Luftbildern mit Hilfe einer Kontrastübertragungsfunktion: *Bildmessung und Luftbildwesen*, Nr. 2, pp. 65–77.

Scott, Frank, Roderic M. Scott, and Roland V. Shack, 1963. The Use of Edge Gradients in Determining Modulation Transfer Functions: *Phot. Sci. Eng.*, Vol. 7, No. 6, pp. 345–349.

Sievers, J., 1976. *Die Kantenbildanalyse als Mittel zur Bestimmung von "Bildparametern."* Beitrag für die Arbeitsgruppe "semantische Information" der Kommission E der OEEPE.

Tiziani, H., 1977. Beurteilung der Bildqualität von Luftbildkammern: *Vermessung, Photogrammetrie, Kulturtechnik*. 4-77, pp. 137–146.

———, 1978. The Use of Optical Transfer Function for Assessing the Quality of Optical Systems: *Photogrammetria*, 34, pp. 45–68.

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