Analytical Independent Model Triangulation Strip Adjustment Using Shore-Line Constraints

Mohamed Shawki Elghazali
Department of Civil Engineering, Cairo University, Cairo, Giza, Egypt

ABSTRACT: The objective of this research is to test the effect of adding points on shore lines as absolute constraints to the basic linearized mathematical model of independent model triangulation involving seven parameters representing three rotations (Ω, ϕ, and χ), three translations (X_o, Y_o, and Z_o), and a scale factor (λ). A strip of eight aerial photographs having an approximate scale of 1:7500 for part of Kuwait City showing shore line for an area on the Arabian gulf extending over seven stereoscopic models is used. The strip contains five full ground control points, five planimetric check points, and ten height check points. Eight shore-line points were used, and the strip adjustment was performed twice with and without the additional constraints to compare results, while keeping the check points outside the adjustment procedure to guarantee proper accuracy assessment. Photographic measurements were performed on the Kern DSR 11 in the comparator mode followed by image refinements from main sources of errors. Stereoscopic models were formed analytically before the simultaneous strip adjustment was carried out. Results showed an improvement in the RMS errors in height and planimetry of approximately 19.5 percent and 8.5 percent, respectively, when adding shore-line points as absolute constraints. The only side effect of adding these constraints for shore lines extending over a number of stereoscopic models would be the increase of the bandwidth of the coefficient matrix of normal equations.

INTRODUCTION

Aerial triangulation procedures have been widely used as a method of control-point densification for the past few decades. Since the introduction of this technique during the thirties, photogrammetrists have been continuously trying to improve the resulting accuracies through the use of ingenious designs of analog plotters, notably in Europe, followed by more rigorous analytical treatments that were particularly developed in North America. A historical account tracing the development of analytical aerial triangulation is given by Tewinkel (1984).

One way of classifying aerial triangulation is based on the smallest independent unit in the block adjustment. Figure 1 shows such a classification, which has been generally agreed upon among practitioners. The block or strip adjustment starts with an instrumental part followed by an analytical treatment. It is commonly accepted that accuracy increases by minimizing the instrumental part and maximizing the computational treatment. The methodology followed in the paper is indicated in Figure 1, where photographic coordinate measurements are performed analogically whereas the rest of the work is done analytically. The aim of this paper is to assess the contribution of incorporating absolute constraints, utilizing the fact that shore-line points should have the same height, on the final resulting accuracies at some check points within the strip. In Canada, where conditions are favorable for this type of constraint, earlier work has been encouraging regarding this point as well as the

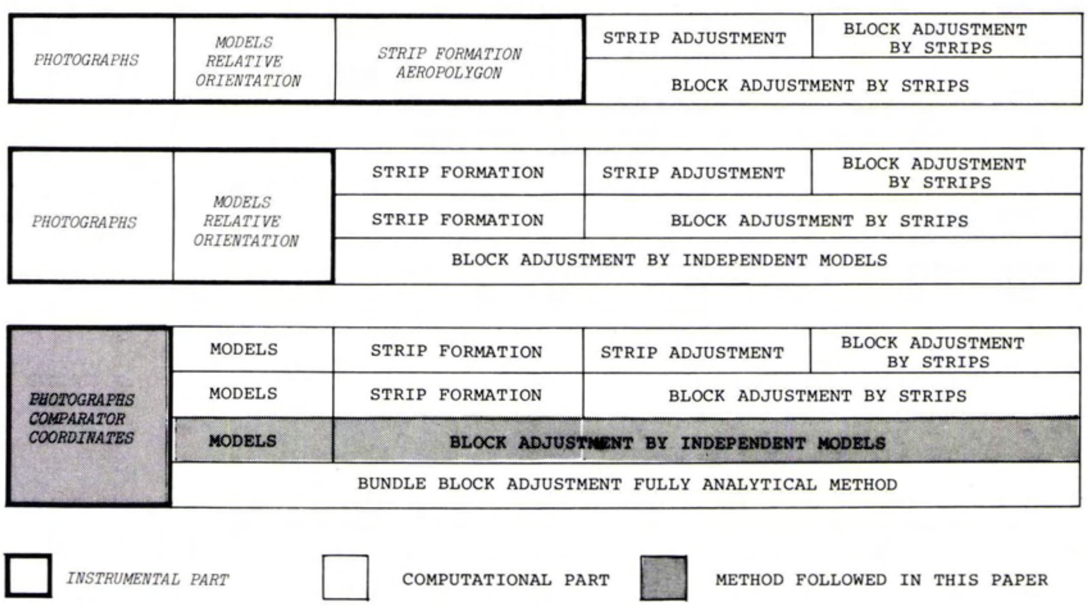

FIG. 1. Classification of aerial triangulation adjustment procedures based on the smallest independent unit in the block, showing the instrumental and computational parts.

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incorporation of auxiliary data in the block adjustment (Blais, 1977, 1984).

DATA ACQUISITION

A brief description is given regarding aerial photography used, distribution, type, acquisition of ground control points, as well as measurements of photographic coordinates.

Aerial Photography

A small strip of eight aerial photographs showing part of the shore line of the Arabian Gulf was used. Details of photography are as follows (Figure 2):

- Approximate scale of photography 1:7500.
- Area covered is located on the northern part of Kuwait City overlooking the Arabian Gulf.
- Number of photographs forming strip: 8.
- Frame numbers: 7182-7189.
- Date of photography: April 1980.
- Foreward overlap ~ 60 percent.
- Altitude: 1125 metres.
- Camera model: Zeiss RMK A 15/23.
- Maximum aperture *F*/5.6.

A full laboratory camera calibration report using the multicollimator method was provided, giving the calibrated focal length (CFL), the nominal focal length, the radial lens distortion curves, as well as the coordinates of the fiducial marks of the camera.

Type and Distribution of Points in the Strip

Refering to Figure 2, the type and number of points within the strip are as follows:

- Full ground control points (X,Y, and Z) : 5 points
- Tie points : 10 points
- Projection centers : 6 points
- Shore line points : 8 points
- Planimetric check points (X,Y) : 5 points
- Height check points (Z) : 10 points.

Horizontal control points used in the strip were densified from the control point system completed by the end of 1979 by Kuwait Institute for Scientific Research (KISR) with the assistance of the Kuwait Ministry of Defence and the Surveying Department of Kuwait Municipality. It is related to the European datum through Ain El-Alod station in Saudi Arabia and consists of some 45 stations spaced at about 18 km, along three adjacent traverse loops, with azimuth orientation given for each fifth station on average. The densification traverse satisfied second-order accuracy requirements such that the ratio of the closing error to the unadjusted horizontal distance did not exceed 1 in 10,000. Also, the angular closing error of the unadjusted horizontal angles was less than $10\sqrt{N}$ seconds, where N is the number of traverse sides. Vertical control, based on mean sea level in Shuwaikh harbor for observations over a three months period, was established by a second order leveling procedure with a maximum error of less than 8.4 $\sqrt{K}$ mm, where K is the length of the circuit covered in km (Bannister and Raymond, 1984).

For the connection of models, points in common between adjacent models have to be observed each in its own coordinate system. For that reason tie points within the physical three-dimensional models are not enough to secure a proper connection. In addition, we have to include projection centers to guarantee a strong solution for the common phi (Φ) parameter (Figure 3). The types of tie points used in aerial triangulation are either signalized points, artificially marked points, or natural untargeted points. Signalized points are very expensive and, therefore, are restricted to high precision aerial triangulation and cadastral applications. Artificially marked points produced by point transfer devices are very inexpensive and, therefore, most widely used for standard aerial triangulation projects (Ackermann and Bettin, 1980). Natural untargeted points proved to be accurate enough, but their application in large projects requires considerable organizational efforts. Because we were dealing with a small strip, ten natural untargeted points were used as tie points between adjacent models. In addition, eight shore-line points extending over four stereoscopic models were utilized as added absolute constraints to the basic mathematical model (Figure 2).

Coordinate Measurements

Photographic coordinates of all points were measured on the Kern DSR 11 analytical plotter installed in the Surveying Department of Kuwait Municipality. It was used in the comparator mode, i.e., moving in the x and y directions and eliminating x- and y-parallaxes, with 10× magnification and an illuminated round dot measuring mark of 0.02 mm diameter.

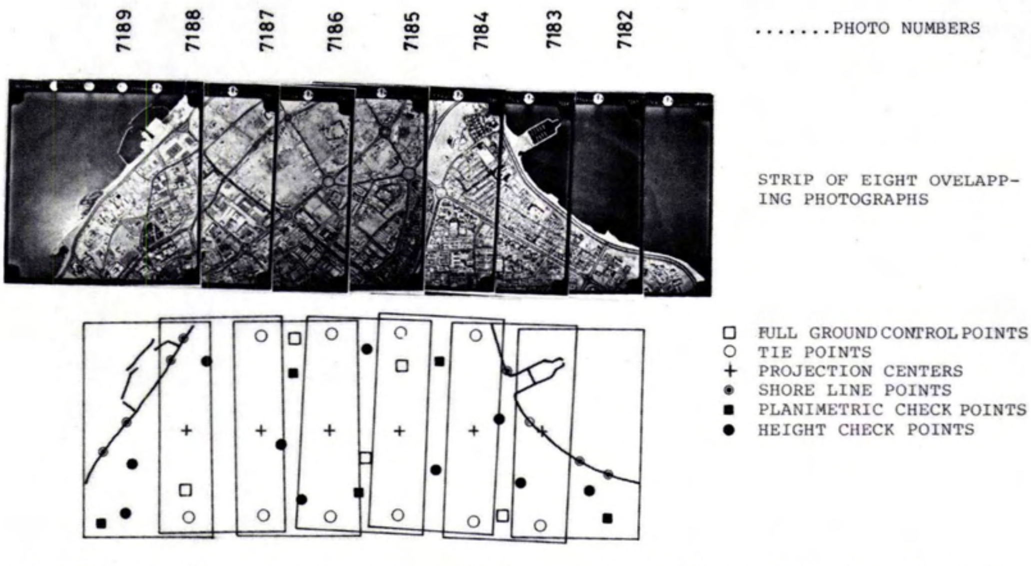

Fig. 2. Strip used in the study for an area located in the northern part of Kuwait City overlooking the Arabian Gulf, together with a schematic diagram showing the distribution and type of different points.

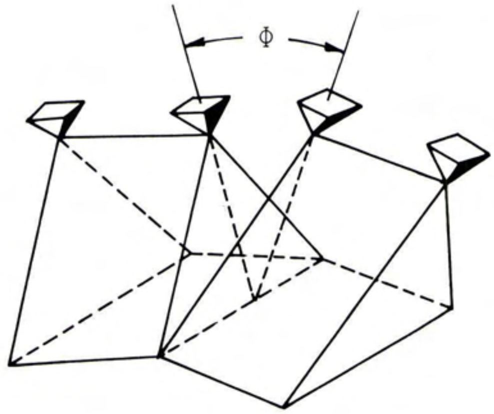

FIG. 3. Use of coordinates of projection centers secures proper determination of Φ.

During measurement, the current results were displayed along with the differences between common points, and commands are available for editing, remeasuring, adding additional points, etc.

DATA PROCESSING

REFINEMENTS OF PHOTO COORDINATES

After the mensuration process, software was developed to refine photo coordinates for the following error sources:

- Lack of coincidence between principal point and fiducial center.
- Shrinkage or expansion of photographic film.
- Objective lens distortion correction.
- Atmospheric refraction correction.
- Earth curvature correction.

Correction of the first three factors was based on data from the camera calibration report. Radial effect on image displacement due to atmospheric refraction was assumed to conform to the Air Research and Development Command (ARDC) of the U.S. Air Force Model Atmosphere (Merchant, 1973; Moffitt & Mikhail, 1980).

MATHEMATICAL MODEL

The basic three-dimensional similarity transformation involving seven independent parameters — three rotations (Ω, Φ, χ), three translations (X_o, Y_o, Z_o), and a scale factor (λ) — is used in its linearized form using Taylor's expansion to transform independent analytical models into a unified strip coordinate system (Equation 1) (Elghazali, 1985).

$$\underbrace{\begin{bmatrix} X_i \\ Y_i \\ Z_i \end{bmatrix}}_{\substack{\text{Ground}\\ \text{Coordinates}\\ [\mathbf{C}_i]}} = \underbrace{\begin{bmatrix} x_{ij} & 0 & z_{ij} & -y_{ij} & 1 & 0 & 0 \\ y_{ij} & -z_{ij} & 0 & x_{ij} & 0 & 1 & 0 \\ z_{ij} & y_{ij} & -x_{ij} & 0 & 0 & 0 & 1 \end{bmatrix}}_{\substack{\text{Coefficient Matrix of}\\ \text{Model Coordinates } [\mathbf{A}_{ij}]}} \underbrace{\begin{bmatrix} \Delta\lambda_j \\ \Delta\Omega_j \\ \Delta\phi_j \\ \Delta\chi_j \\ \Delta X_{oj} \\ \Delta Y_{oj} \\ \Delta Z_{oj} \end{bmatrix}}_{\substack{\text{Corrections to Unknown}\\ \text{Parameters}\\ [\Delta\mathbf{P}_j]}} + \text{constant} \quad (1)$$

Equation 1 is applied to point (i) in model (j). However, a distinction should be made between control points (known ground coordinates), tie points (unknown ground coordinates), and shore-line points (having similar but unknown height). The constant term appearing in Equation 1 is added symbolically to account for neglecting higher order terms due to Taylor's expansion and will thus be ignored in subsequent equations.

Accordingly, for each control point we have

$$[\mathbf{A}_{ij}]\ [\mathbf{P}_j] = [\mathbf{C}_i], \quad (2)$$

for each tie point in each model we have

$$[\mathbf{A}_{ij}]\ [\mathbf{P}_j] - [\mathbf{C}_i] = \text{zero, and} \quad (3)$$

for each shore line point we have a constraint in the form

$$[\mathbf{B}_{ij}]\ [\mathbf{P}_j] - [\mathbf{Z}_w] = \text{zero} \quad (4)$$

where

$[\mathbf{B}_{ij}]$ is a submatrix of dimensions (1×7) similar to the last row of the coefficient matrix of model coordinates $[\mathbf{A}_{ij}]$; and
$[\mathbf{Z}_w]$ is a submatrix of dimensions (1×1) representing the unknown height of water level at the shore-line points.

In spite of the apparent similarity between Equations 2 and 3, it should be pointed out that the mathematical treatment is different. For control points, the term $[C_i]$ is known and therefore appears on the right hand side of Equation 2, whereas for tie points the same term is unknown and therefore appears on the left hand side of Equation 3.

ADJUSTMENT WITHOUT CONSTRAINTS

The independent model triangulation strip adjustment was performed using the well-known program, package Aerial Triangulation with Models (PAT-M), performing the spatial similarity transformation with simultaneous determination of all seven transformation parameters for each model (Ackermann *et al.*, 1972). It offers a theoretical rigorous solution with a stochastic model providing the possibility to assign different weights for model coordinates, projection centers, as well as ground control points. Based on equations 2 and 3, the observation equations were formed followed by reduction to the normal equations and subsequent solution for the unknowns. Figures 4 and 5 show the patterns of the observation equations as well as the coefficient matrix of the normal equations, respectively. Without adding constraints, the system resulted in 117 observations and 97 unknowns (49 transformation parameters and 48 spatial coordinates).

ADJUSTMENT WITH CONSTRAINTS

Based on Equations 2 and 3 and adding the effect of shore-line point constraints as formulated in Equation 4, the PAT-M7 program was run again. This time the number of observations was increased by eight because we have eight shore-line points, while the number of unknowns was increased by only one from the added unknown height $[Z_w]$. The total system thus includes 125 observations and 98 unknowns (49 transformation parameters, 48 spatial coordinates, and one shore line point height Z_w). Figures 6 and 7 show the resulting patterns of the observation equations and the coefficient matrix of normal equations where the effect of added constraints is obvious. As a result of these constraints, the bandwidth of the coefficient matrix of normal equations is increased.

RESULTS AND ANALYSIS

Results of the independent model triangulation strip adjustment were assessed based on discrepancies in the spatial coordinates (DX, DY, DZ) at ten height and five planimetric ground check points (Figure 8). These check points did not play any role in the simultaneous adjustment procedure so as to ensure unbiased and meaningful comparisons between the two sys-

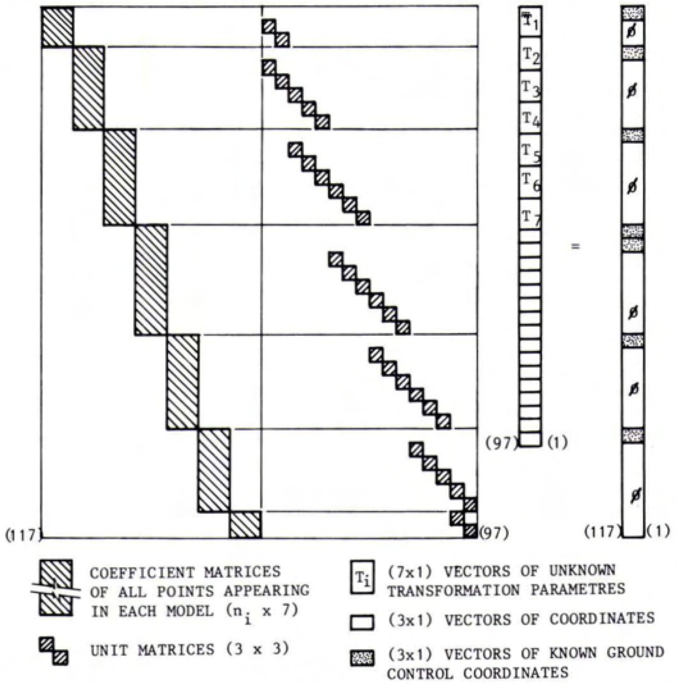

FIG. 4. Pattern of observation equations without adding shore-line point constraints.

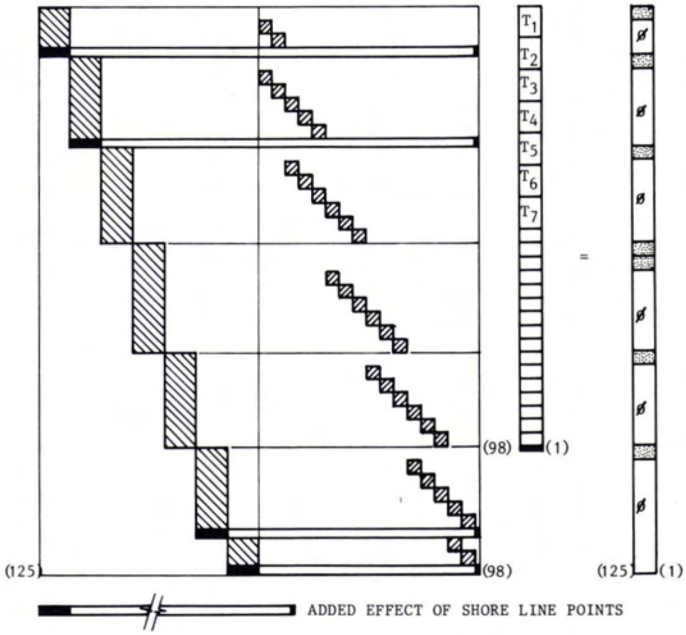

FIG. 6. Pattern of observation equations showing added effect of shore-line point constraints on models number 1, 2, 6, and 7.

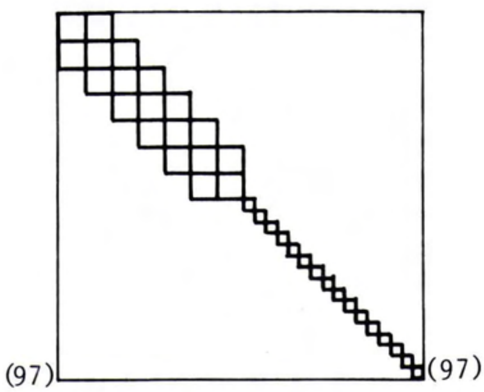

FIG. 5. Pattern of coefficient matrix of normal equations (without constraints).

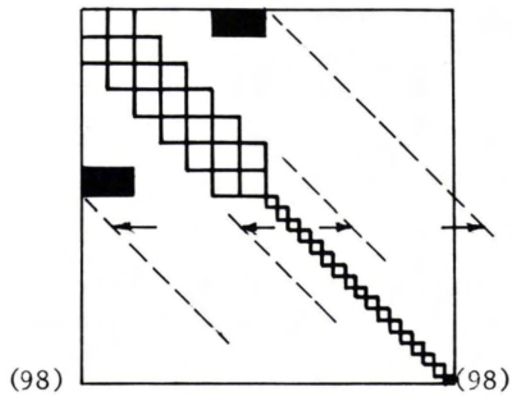

FIG. 7. Pattern of coefficient matrix of normal equations (note the effect of increasing the band width due to the added constraints).

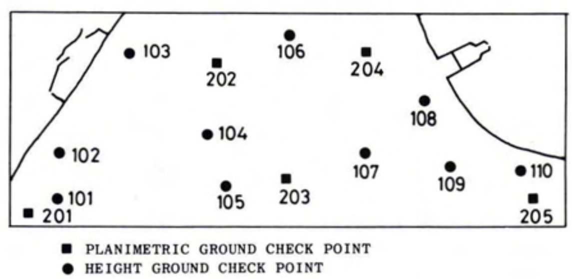

FIG. 8. Distribution of planimetric and height ground check points.

tems with and without added constraints. Results are summarized in Tables 1, 2, and 3, giving height discrepancies (*DZ*), planimetric discrepancies (*DX, DY*), and root mean square errors (RMSE), respectively.

From Tables 1 and 2, it is clear that adding constraints resulted in reducing the discrepancies at all ground check points except for the single case of height check point #105 where the values of *DZ* increased by an insignificant value of 0.6 μm at photo scale. This point lies approximately in the middle of the strip and is, thus, the least affected by the constraints where their maximum effect occur at both ends of the strip. It should be pointed out that adding the constraints did not change the general behavior of the strip in spite of reducing the numerical values of discrepancies, which is evident from the fact that the *X, Y, Z* discrepancies maintained the same algebric signs. Generally, values of Table 3 show that the RMSE in *X, Y, Z* were reduced by 7.7 percent, 9.3 percent, and 19.3 percent, respectively. It is natural that heights are the most affected because of the nature of the shore-line point constraints. This is generally desirable because most of aerial triangulation adjustment methods result in higher accuracies in planimetry as compared to altimetry. Therefore, adding height constraints, when pos-

TABLE 1. HEIGHT DISCREPANCIES (DZ) AT HEIGHT CHECK POINTS IN μM AT PHOTOS SCALE 1:7500

Point Number	Height discrepancies (DZ) in μm at photo scale without constraints	with constraints
101	+ 48.3	+ 39.2
102	+ 49.5	+ 38.7
103	− 42.2	− 32.8
104	− 38.7	− 36.3
105	+ 37.9	+ 38.5
106	− 46.7	− 40.7
107	+ 39.7	+ 31.6
108	− 48.8	− 35.7
109	+ 43.4	+ 30.1
110	+ 52.3	+ 37.9

TABLE 2. PLANIMETRIC DISCREPANCIES (DX, DY) AT PLANIMETRIC CHECK POINTS IN μM AT PHOTO SCALE 1:7500

	Without constraints		With constraints	
Point Number	DX	DY	DX	DY
201	+ 32.5	+ 28.4	+ 30.2	+ 26.7
202	+ 34.6	− 37.2	+ 31.5	− 34.1
203	+ 28.7	− 35.5	+ 27.1	− 33.2
204	− 31.8	− 30.9	− 29.5	− 27.5
205	− 37.9	+ 41.2	− 34.6	+ 35.8

TABLE 3. ROOT MEAN SQUARE ERRORS (RMSE) FOR PLANIMETRIC AND HEIGHT CHECK POINTS

	Root Mean Square Errors (RMSE) At Photo Scale		
Check Points	X (μm)	Y (μm)	Z (μm)
Without Constraints	33.241	34.937	45.005
With Constraints	30.679	31.673	36.307

sible, will enhance the homogeneity of the resulting accuracy of the spatial coordinates.

CONCLUSIONS

Use of shore-line points as height constraints proved to be an effective and economical way of improving the height accuracy. Wherever points of known absolute or relative height exist, as in the case of shore lines or lakes extending over several models, it is certainly recommended that they be incorporated in the basic mathematical model. This proved to be an attractive alternative to enhance the height accuracy in spite of very limited additional computational effort. The only drawback is the increase of the bandwidth of the coefficient normal equations and, consequently, the resulting additional computational time. This can be easily rectified by dividing shore lines or lakes extending over several models into segments extending over a limited number of models and assigning for each segment an independent unknown height, thus breaking the correlation pattern. This will, of course, increase the number of unknowns, but will maintain the increase in bandwidth within controlled limits.

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