Modeling and Evaluating the Effects of Stream Mode Digitizing Errors on Map Variables

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ABSTRACT: A statistical model based on time series analysis was developed to examine the effect of stream mode digitizing errors on GIS-based estimates of polygon area and line length. The model was used to evaluate the effect of changing the digitizing accuracy standard on the map measurements of interest. Using an ARMA (1,0) stationary time series to mimic digitizing errors under nine different accuracy standards, GIS-based polygon area estimates were found to be essentially unbiased, and their standard deviations ranged from approximately 0.1 percent of polygon area under the most stringent standard to slightly more than 1.5 percent under the loosest standard. However, line-length estimates appeared to be biased by digitizing, and the bias increased dramatically as the standard was loosened. The imprecision of line-length estimates also increased more than five fold as the digitizing accuracy standard was relaxed.

INTRODUCTION

GEOGRAPHIC INFORMATION SYSTEMS (GIS), CAD/CAM packages, AM/FM programs, and other computerized tools designed to store and manipulate spatially referenced information have developed into valuable tools for the display and analysis of spatial information used by private companies and public agencies. Computer technology has advanced so rapidly that research of the automated map making process and research of the reliability of mapping systems have lagged behind. Many users of computer generated maps assume that the high quality appearance of the product implies high accuracy (Vitek and Richards, 1978). However, this is not always a valid assumption.

Map accuracy has always been important whether map making is automated or manual, but its assessment has often been a confusing issue to the producer and user alike. Quite often a map user does not understand, or is completely unaware of, the accuracy level of the map being used. The accuracy of a map is normally expressed by reference to an accuracy standard, which may be stated on the map. Even when the accuracy standard is stated on the map, the user usually cannot use that standard to determine the accuracy and precision of common map measurements he/she understands and uses, such as polygon area or line length.

The most common form of map accuracy standards is that used in the National Map Accuracy Standard (NMAS), which was approved in 1947 and currently is used by USGS to produce paper topographic quad sheets (Marsden, 1960). The NMAS states that, for a given map, a specified percentage (90 percent) of well defined points must be within a specified map distance of their true ground location. For example, small scale maps (> 1:20,000) must have 90 percent of well defined points within 1/50 of an inch of true ground location (Thompson, 1979). With this type of standard, it is difficult to gauge the precision of an area measurement or the accuracy of the measured length of a particular linear feature.

As a result of progress in numerical and analytical cartography, the National Committee for Digital Cartographic Standards (NCDCDS) was established to develop digital standards. This committee was concerned that statements of map accuracy should be in terms familiar to the map user and in terms of quantities obtained from the map by the user (e.g., bounds on areas) (Chrisman, 1983; Merchant, 1982).

There are many components of GIS and other computer mapping systems which introduce error into the mapping process and thereby affect the accuracy of the map product. Sources of error can be broken down into two primary components that could be called source map error and operational error. Source map error includes the accumulated error in the map used as input into the GIS. Operational error is error produced by both data input and data manipulation. Digitizing is one component of data input that introduces error into the map. Although technological advances are increasing the use of automatic digitizing methods, manual digitizing has been the primary method of data capture used in the industry. Manual digitizing has been recognized as a significant source of error, but its magnitude and impact upon digital map accuracy has not been well studied (Chrisman, 1982). Digitizing is usually the most expensive part of a GIS, yet the error introduced into maps by digitizing is often overlooked or assumed to be negligible.

Digitizing error can be divided into line-following error and line-sampling error. Line-following error is the operator inability to trace the map line perfectly with the cursor. Line-following error is caused in part by problems in locating the intended boundaries because drawn boundaries have finite line width. The actual boundary should be the center of the line; however, the map area bounded by a line is rather ambiguous. Psychological and physiological factors affect line-following error. Psychological errors are misperceptions of the map line. Physiological errors stem from the inability of human muscles to keep the cursor on the map line (Jenks, 1981). The second part of digitizing error, line-sampling error, deals with the selection of points to be used to represent the map line. The number and locations of points sampled are sources of line-sampling error.

Only a few studies have been done to quantify the effect that human line-following error has on the accuracy of a digitized map. Psychologists have studied the ability of a person to trace a line accurately, although not in a cartographic setting (Conklin, 1957; Poulton, 1962). Jenks (1981) suggests that digitizing errors are highly correlated to the direction of movement of the cursor. Several studies have examined digitizing error empirically by comparing physically digitized features to their source material (Otawa, 1987; Baugh and Boreham, 1976). Otawa (1987) cited the need for the development of a model to predict digitizing errors. Traylor (1979) created an analytical model of digitizing error by studying and modeling the line-following pattern of a digitizing operator, but did not directly address the effect digitizing error had on the accuracy of the map product.

There is a trade-off between the stringency of the accuracy

[1] Presently with ITT Rayonier, Inc., Fernandina Beach, Florida. An abbreviated version of this article was presented at GIS/LIS '88, San Antonio, Texas.

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standards and digitizing costs. Increasing accuracy requirements must increase digitizing costs. Users need a way to compare the benefits of improved accuracy against the associated cost rise. The study reported here was undertaken to address this concern. Its objectives were (1) to develop a statistical methodology to examine the line-following error of digitizing, and (2) to use the methodology to evaluate the effect of digitizing error upon polygon area and line length at varying map accuracy standards. It is unlikely that the statistical model we develop can meet the three objectives of generality, realism, and precision (*cf.* Levins, 1966) equally well; however, we feet it provides a defensible foundation upon which to base analyses of digitizing error.

DEVELOPMENT OF THE STATISTICAL MODEL

Hypothesized Model

Manual digitizing is performed either in stream mode or point mode. By stream mode, we mean coordinates are recorded at a regular time or distance interval as the cursor is moved continuously along the map line. By point mode, we mean that the operator selects where to record the coordinates of a point. In point mode, the digitizer operator must use judgment in selecting the numbers and locations of points needed to characterize the line adequately, and, because of this, point mode is considered by some to produce a poor representation of the true feature (Douglas and Peucker, 1973). Moreover, stream model is considered to be faster and easier for the operator. Others promote point mode, arguing that stream mode produces more error in the digitizing process (Burrough, 1986; Jenks, 1981). In this paper, we will model the error of stream mode digitizing only, because of the intractability of including the subjective component of point-mode digitizing into the modeling framework.

Inasmuch as stream mode produces a series of points that are close together in space and time, it was hypothesized that the stream-mode data are serially correlated, or autocorrelated. This means that the magnitude and direction of the error of a digitized point are influenced by the magnitude and direction of preceding points. As the cursor is moved continuously along the map line, the operator never follows the line perfectly, but instead continually crosses from one side of the line to the other side. As the operator strays from the line and recognizes it, a correction is attempted to return to the line. Inasmuch as the psychological and physiological factors, which give rise to line-following error, interact in a complicated and unpredictable fashion, each of the errors in the series of digitized points can be regarded as a random or stochastic event.

It was hypothesized that stream-mode digitizing error can be modeled as a statistical time series. A time series is defined as a set of observations generated in sequence. The observations are usually dependent, such that the value of a particular observation is affected by previous observations in the series, as well as by a purely random component called white noise (Box and Jenkins, 1976). When the observations follow a normal distribution, the time series can be completely characterized by its mean, variance, and autocorrelation.

A time-series model can be fitted to stream-mode data because points are recorded at regular time or regular distance intervals. A stationary time series is one which is in equilibrium about a constant mean. We suggest that an assumption of stationarity of a series of stream-mode digitizing errors is justified because errors of a trained operator should occur, on average, on the left side of the line as frequently as they occur on the right side of the line and be similar in magnitude. Therefore, the mean of the series will be constant, namely zero, and the series will display uniform variance. Given the current evidence, this suggestion is arguable. Nonstationarity implies, among other things, that an operator tended to favor a particular side of the line; if true, we presume that this would be corrected by training. Errors may not be exactly normal; however, we present evidence that normality holds approximately.

Data Collection and Examination

Identifying and measuring the errors typically produced by manual digitizing is the first step in the process of modeling digitizing error. Five digitizing operators digitized a series of features to obtain a sample of digitizing error. For this study, trained, experienced digitizers were used who were not believed to have a digitizing bias, thereby limiting operator differences and allowing the use of stationary time series analysis.

Two sheets with elementary, map-like linear features were prepared and affixed to the digitizing table. The complexity of these calibration figures was varied intentionally in order to produce a range of digitizing situations. The sheets were registered to an arbitrary coordinate system. The sheets were not removed during the study, and the original registration was used for all repetitions. Acetate material was used, and the environment of the room was controlled to minimize variations due to temperature and humidity. Thus, there were no registration errors confounding the differences between repeated digitizations. Each figure was digitized very carefully once in a high-density point mode, by an individual who moved at a slow pace and entered each coordinate pair manually. This was considered to be the standard line to which all subsequent digitized lines were compared. The five operators then digitized the lines as if in a production situation in order to create sample digitized lines.

Two aspects of digitizing error were measured. First, the orthogonal distance from a point on the sample line to the standard line was calculated. Second, the error was assigned a sign of positive or negative depending on which side of the standard line the point from the sample line was. Positive errors were defined as errors to the left of the line segment when viewed in the direction of digitizing. Those errors to the right of the line segment were considered negative errors. Figure 1 provides a visual description of the perpendicular error distance. Many of the algorithms used to measure the error were based upon work done by Traylor (1979) and are described in detail in Keefer (1988).

Time series modeling is unnecessary if the data comprise purely white noise. Therefore, the pattern of digitizing error was tested for nonrandomness using a standard runs test (Zar, 1984). The null hypothesis was that the error is random. The runs test required the number of points to the left of the standard line

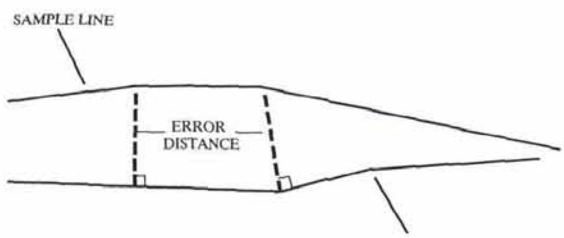

Fig. 1. Graphical depiction of perpendicular error distance definition.

(NL), the number of points to the right of the standard (NR), and the number of runs (R) (Zar, 1984). The expected number of runs was calculated as

$$ER = [2(NL \cdot NR)/(NL + NR)] + 1 \quad (1)$$

and the variance was determined as

$$VR = (ER - 1) \cdot \frac{2 \cdot NL \cdot NR - NL - NR)}{(NL + NR) \cdot (NL + NR - 1)} \quad (2)$$

The standardized z score was

$$z = (R - ER)/VR \quad (3)$$

If the errors were uncorrelated, large z scores would occur infrequently in repeated sampling. In large samples, values of ± 2 would occur approximately 5 percent of the time. Consistent z scores outside of that range would be cause for rejecton of the null hypothesis, thereby indicating that digitizing error is nonrandom. Of the 80 runs tests that were conducted ($\alpha = 0.05$), exactly 5 percent led to rejection of the null hypothesis that stream mode digitizing errors were random in nature. Both the plots of digitizing errors and runs strongly supported the hypothesis that stream mode digitizing was nonrandom. The next step was to ascertain whether the nonrandomness was due to serial correlation.

An initial means to determine if a process is serially correlated is simply to observe a graph of the observed variable over time. For this study of digitizing error, the observed variable was the distance from a point on the digitized line to the perpendicular intersection with the position of the standard line. Figure 2 shows digitizing error for an actual digitized line plotted over the digitizing interval. The non-random pattern exhibited typifies an autocorrelated process for digitizing error. Figure 3 shows the distribution of digitizing errors for all five operators. Clearly,

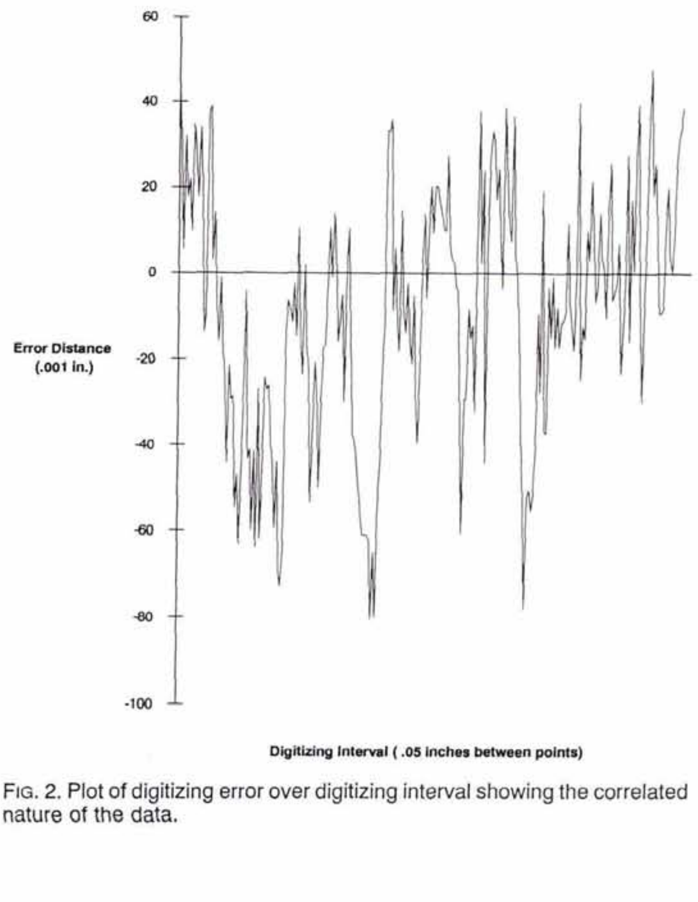

FIG. 2. Plot of digitizing error over digitizing interval showing the correlated nature of the data.

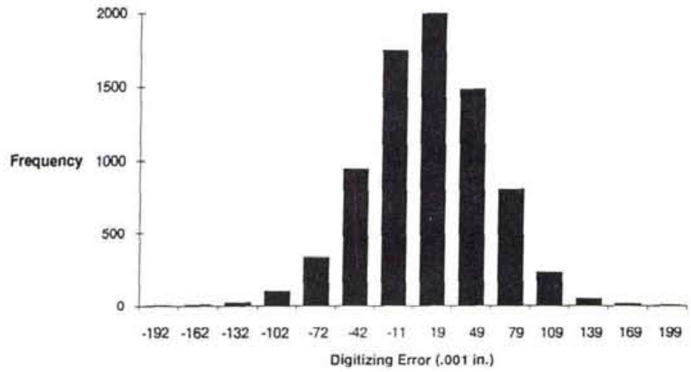

FIG. 3. Distribution of digitizing errors from five operators and 16 features. Interval labels are uneven due to rounding errors.

digitizing errors are at least approximately normally distributed. Because we did not intend to make inferences about model parameters, the possible lack of exact normality was a secondary concern. In addition, evidence indicated that the assumption of stationarity in the time series was fulfilled. Refer to Keefer (1988) for more details.

Model Fitting

The stochastic time domain approach using autoregressive integrated moving average (ARIMA) models was used to model digitizing error. The general ARIMA (p, d, q) model reduces to simply an ARMA (p, q) model because the parameter d is zero due to stationarity of the digitizing error time series. The lag order of the autoregressive process is represented by p, and q represents the order of the moving average process. The majority of the data sets tested exhibited behavior characteristic of an ARMA (1,0) process, or simply AR(1). This was determined by the pattern produced by plots of the autocorrelations and partial autocorrelations using methods described by Box and Jenkins (1976). The data were then fit to an AR(1) model and plots of the residual autocorrelations and residual partial autocorrelations were examined. The correlation of the residuals after fitting the model was statistically indistinguishable from that of white noise, a result which confirms the choice of an AR(1) model over some more complicated ARMA process. Chi-square tests also showed the residuals to be random, or white noise. Therefore, most of the autocorrelation present in the digitizing error data sets was explained using an AR(1) model. The serial correlation coefficient, ϕ, was also estimated for each of the 80 data sets. Most of the coefficients were in the 0.65 to 0.85 range which indicated a strong positive correlation process in digitizing error. This empirical distribution was used later in the simulation process to select values for ϕ.

EVALUATION OF ERROR EFFECT

Simulation Design

The results of the foregoing empirical model development were used in a simulation which was designed to evaluate the effect of digitizing error on map accuracy. The simulator was a computer program that produced an error-influenced stream of points representative of a standard digitized feature (line or polygon). The digitizing errors were generated according to an AR(1) model whose parameters were set at values that would ensure adherence to a stipulated digitizing accuracy standard. A range of digitizing standards was examined. For lines, the object of interest was the deviation from estimated (measured) length that was induced by digitizing error, while nonetheless maintaining a predetermined standard. Similarly, for polygons the object of interest was the deviation from measured area.

The manner in which a stream of digitizing errors can be simulated according to an AR(1) process and a specific digitizing standard deserves explanation. An AR(1) process is one in which the current observation (error, event, etc.) is correlated with the preceding observation but also consists of white noise: i.e.,

$$E_i = \phi E_{i-1} + V_i \quad (4)$$

where

E_i = the error of the i^{th} (current point,
ϕ = The serial correlation coefficient, and
V_i = the "white noise" component of error.

Alternatively Equation 4 can be written as an infinite moving average process, but for our simulation purposes it was more straightforward to use the lag-one autocorrelation formulation. A number of algorithms exist which can generate white noise errors that are normally distributed with zero mean and a variance of σ_v^2 for any stipulated σ_v^2; see, for example, the International Mathematics & Statistics Library, Version 1.0. Given V_i that are $N(0,\sigma_v^2)$ and given a stipulated value of ϕ, then it can be shown (Theil 1971, p. 251) that E_i is normally distributed with mean zero and variance $\sigma_e^2 = \sigma_v^2/(1-\phi^2)$.

For sake of simulation, then, it is necessary to stipulate values of ϕ and σ_e. From our empirical studies we had determined that ϕ ranged from 0.65 to 0.85, and so we randomly chose from among this range for each simulation run. The other value is determined by the map accuracy standard. Recall that a digitizing standard stipulates a distance within which a certain percentage of digitized points must be located. For example, an accuracy standard of $W = 0.01$ inches at 95 percent means that 95 percent of the digitized points are within 0.01 map inches of their position on the source map, on average. As a probability statement, a digitizing standard can be expressed as

$$\text{Prob}[|E| \leq W] = 1 - \alpha \quad (5)$$

where

E = error from true location,
W = allowable distance on either side of a digitized point, and
α = 1 - (stipulated proportion).

From statistical theory we know that if $E_i \sim N(0,\sigma_e^2)$, then

$$\text{Prob}\ (|E_i| \leq \sigma_e K_\alpha) = 1 - \alpha \quad (6)$$

where

K_α = 1 - α quantile of the standard normal distribution.[2]

Equating Equation 5 to Equation 6 yields

$$\sigma_e = W/K_\alpha.$$

Therefore, by stipulating a digitizing standard, σ_e is determined; and σ_e combined with ϕ serves to stipulate σ_v, so that the generating model (Equation 4) is completely parameterized. Further details are provided in Keefer (1988). Note here, however, that as α increases, K_α decreases; for a stipulated distance, W, increasing α causes σ_e to increase, too.

The mechanics of simulating the digitizing error were very similar to reversing the process of measuring digitizing error. The simulation used the accuracy standard to set the bounds on the size of error to be created. Using the accuracy standards and the ARMA model, a set of errors was generated at the same scale as the feature being simulated. For every point on the standard feature, a corresponding simulated "error coordinate" was created by turning perpendicular to the standard line and moving the distance of the generated error. Once error coordinates were calculated for every point of the standard feature, the line length or polygon area for a generated feature was calculated. Line length was calculated as the sum of all the straight line lengths between each pair of coordinates. Polygon area was calculated using a standard algorithm for calculating the area of irregular shapes using Cartesian coordinates (Stolk and Ettershank, 1987). The length or area was stored in an array, and the process was repeated for the next simulation by generating a new array of error distances. After all the specified simulations had been conducted for a particular standard feature, the program calculated a statistical summary. The statistical summary included the area or length measurement of the standard feature, the mean simulated area or length, the bias (standard − mean simulated), variance, standard deviation, root-mean-squared error, the maximum error distance, and the average absolute error distance.

Twenty-five polygons and 25 lines were constructed and carefully digitized to serve as standard features. These features covered a range of sizes and shapes: polygons ranged in size from 0.5 square map inches to 25 square map inches, and lines ranged in length from 3 map inches to 12 map inches. The features included curvy and nonlinear features as well as straighter, more nearly linear features. The features were digitized and then entered into the simulation program. Four error distances, selected on the basis of currently used map accuracy standards, were examined: W = 0.005, 0.01, 0.02, and 0.04 map inches. At each value of W, four values of α were used: α = 0.025, 0.05, 0.10, and 0.20. In combination, these error distances and α-levels gave rise to 16 different accuracy standards that were studied. Error distances of 0.01 inches and 0.02 inches were judged to be reasonable, based on the authors' experience. The error distances of 0.005 inches and 0.04 inches are at the extremes of reasonable accuracy. Each feature was simulated 200 times. This provided 200 different error influenced versions of each particular feature.[3]

SIMULATION RESULTS

Results were summarized on the basis of bias and standard deviation. The latter is a commonly used measure of precision, i.e., the spread around the mean value. In the present context, the mean value is the arithmetic average length or area that was recorded for a particular feature over the 200 simulations. Bias is a customary measure of the deviation of the mean value from the value, which is often used as a measure of accuracy. From the simulation results, bias was calculable because the value for each feature was known. We report results in feet or acres (based on a map scale of 1:24,000) and also as a percentage of the targeted value. By reporting results in relative terms, the effect of size is removed. Relative (or percent) standard deviation is also known as coefficient of variation.

The bias and standard deviation was computed as explained above for each of the 50 standard features at each of the 16 digitizing accuracy standards. Because the standard features that we used are not of inherent interest, we averaged the estimates of bias across the 25 polygon features and across the 25 line

[2] For example if $\alpha = 0.05$, $K_\alpha = 1.965$.

[3] This number of simulations was selected after running the simulation program for an increasing number of simulations and observing where the variance in polygon size or line length began to stabilize. One hundred simulations appeared to provide sufficient stability of variance, and this number was doubled to further ensure that simulation (sampling) error would be negligible.

features.[4] Likewise, we obtained the average standard deviation, which estimates the mean of the distribution of the within-feature standard deviations.

Table 1 shows the average bias of polygon area. The bias was essentially zero for all α levels and accuracy standards. No trend in bias was apparent as error distance or α levels were changed. Evidently, there was a compensating effect on area estimates when digitizing polygons. That is, the increase in area caused by errors on one side of the line was approximately equal in magnitude to the decrease in area caused by errors on the other side of the line. Regardless of the digitizing accuracy standard, there were slight differences between digitized polygon area and the source map polygon area in the long run.

The average standard deviations for each error distance and α-level are presented in Table 2. The standard deviations were not very large, with the values ranging from 0.118 percent of actual area to 1.677 percent of actual area (Table 2b). Clearly, changing the digitizing accuracy standards affected the amount of variation, or precision, in area estimates, and error distance had more of an impact than α level. The estimate of area for a particular polygon was more variable under less stringent standards. Doubling the allowable error distance caused an almost exact doubling of the standard deviation. As the α level doubled, the standard deviation increased by a factor of 1.15 to 1.3. Despite lack of bias, it is evident that a less stringent accuracy standard results in lower precision, i.e., larger standard deviation.

Unlike polygon area error, the estimate of line length appeared to be biased (Table 3). The biases changed dramatically as the accuracy standards were varied. The most stringent accuracy standard produced a bias of −0.066 percent of actual line length while the loosest standard had a bias of −11.119 percent of actual line length. Doubling the allowable error distance caused a quadrupling of the bias. Doubling the alpha level only increased bias by a factor of 1.5, approximately.

[4] Had the line and polygon features themselves been of inherent interest, then it would have been appropriate and informative to apply a significance test or construct confidence intervals for differences among the features. Moreover, if the between-feature variation of length or area estimates had been of interest, then pooling the within-feature variance would have been appropriate.

The phenomenon of the quadrupling bias can be explained mathematically. At a distance $S < L$ the error in line length is $L - L_S$, as shown in Figure 4. Using a binomial series expansion of $L_1 = L\left(1 + \frac{S^2}{L^2}\right)^{1/2}$ we get $L - L_1 \approx S^2/2L$, to a first order approximation. Similarly $L - L_2 \approx 2S^2/L$ to the same order of approximation. Thus, the ratio $(L - L_2)/(L - L_1) \approx 4$.

The bias of line length was always negative, indicating that the error influenced version of the line was always longer than the actual length. This was an expected result because deviations from the correct line tend to increase cumulative length except in rare circumstances.

Table 4 shows the average standard deviations for line length in stream mode simulation. Values range from 0.089 percent of actual length to 4.670 percent of actual length. The precision of line length was substantially less at the less stringent accuracy standards. Furthermore, the trend across accuracy standards was not as clear as in previous cases. At the smaller accuracy standards, a doubling of the accuracy standard distance produced an increase in standard deviation of approximately 2.5 times, but at the larger end of the accuracy standards, a doubling produced an increase in standard deviation of three times. Standard deviation increased by approximately 1.2 to 1.4 times as α level doubled. It was expected that standard deviation of line length would double as accuracy standard distance doubled as in the case of polygon area. However, another factor, such as line complexity, may have influenced the standard deviation.

SUMMARY AND CONCLUSION

Digitized maps get more expensive to produce and purchase as accuracy standards become more demanding, and the user should be able to judge whether the cost of increased accuracy is worthwhile. A method is needed for evaluating digitizing standards in terms of how they affect the accuracy and precision of map measurements, such as polygon area and line length.

The objectives of this study were to statistically model stream-mode digitizing error, and to evaluate the effect of digitizing error upon polygon size and line length at varying map accuracy standards. Data were collected from several experienced

TABLE 1. AVERAGE BIAS OF POLYGON-AREA ESTIMATES. IN (A), BIAS IS SHOWN IN TERMS OF ACTUAL ACRES REPRESENTED. IN (B), BIAS IS SHOWN IN RELATIVE TERMS AS A PERCENTAGE OF TRUE AREA.

Band width (map inches)	(a) Average bias (acres)				(b) Average bias (%)			
	α=0.025	0.05	0.10	0.20	α=0.025	0.05	0.10	0.20
0.005	0.003	0.004	0.001	0.015	0.005	0.000	−0.001	0.006
0.010	0.025	−0.021	−0.039	0.017	0.003	−0.003	−0.013	−0.012
0.020	0.024	0.030	−0.099	−0.097	−0.007	0.013	−0.017	−0.052
0.040	−0.094	0.078	−0.038	−0.256	−0.038	−0.004	0.054	−0.080

TABLE 2. AVERAGE STANDARD DEVIATION OF POLYGON-AREA ESTIMATES. IN (A) STANDARD DEVIATION IS SHOWN IN TERMS OF ACTUAL ACRES REPRESENTED. IN (B), STANDARD DEVIATION IS SHOWN IN RELATIVE TERMS AS A PERCENTAGE OF THE TRUE AREA.

Band width (map inches)	(a) Average standard deviation (acres)				(b) Average standard deviation (%)			
	α=0.025	0.05	0.10	0.20	α=0.025	0.05	0.10	0.20
0.005	0.470	0.543	0.638	0.813	0.118	0.137	0.162	0.207
0.010	0.940	1.067	1.283	1.658	0.235	0.269	0.324	0.418
0.020	1.891	2.127	2.541	3.321	0.492	0.533	0.639	0.832
0.040	3.746	4.232	6.116	5.564	0.938	1.077	1.284	1.677

TABLE 3. AVERAGE BIAS OF LINE-LENGTH ESTIMATES. IN (A), BIAS IS SHOWN IN TERMS OF ACTUAL LENGTH REPRESENTED. IN (B), BIAS IS SHOWN IN RELATIVE TERMS AS A PERCENTAGE OF THE TRUE LENGTH.

Band width (map inches)	(a) Average bias (feet)				(b) Average bias (%)			
	$\alpha=0.025$	0.05	0.10	0.20	$\alpha=0.025$	0.05	0.10	0.20
0.005	−9.1	−12.3	−17.5	−28.7	−0.066	−0.086	−0.124	−0.206
0.010	−36.9	−48.4	−68.3	−113.5	−0.264	−0.348	−0.486	−0.813
0.020	−145.9	−188.9	−264.1	−431.5	−1.039	−1.353	−1.896	−3.077
0.040	−562.1	−720.9	−986.6	−1558.9	−3.988	−5.153	−7.079	−11.119

TABLE 4. AVERAGE STANDARD DEVIATION OF LINE-LENGTH ESTIMATES. IN (A), STANDARD DEVIATION IS SHOWN IN TERMS OF ACTUAL LENGTH REPRESENTED. IN (B), STANDARD DEVIATION IS SHOWN IN RELATIVE TERMS AS A PERCENTAGE OF THE TRUE LENGTH.

Band width (map inches)	(a) Average standard deviation (acres)				(b) Average standard deviation (%)			
	$\alpha=0.025$	0.05	0.10	0.20	$\alpha=0.025$	0.05	0.10	0.20
0.005	11.8	14.0	16.8	23.8	0.089	0.105	0.127	0.179
0.010	27.8	33.6	43.9	65.1	0.208	0.249	0.326	0.477
0.020	79.9	99.0	132.5	205.7	0.588	0.720	0.965	1.498
0.040	258.4	319.3	432.7	650.5	1.869	2.322	3.115	4.670

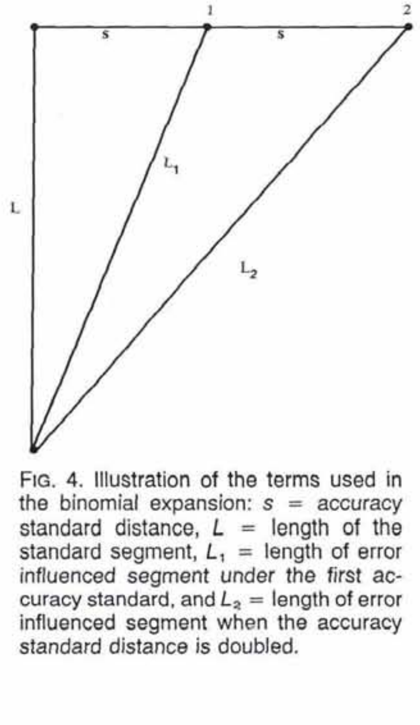

FIG. 4. Illustration of the terms used in the binomial expansion: s = accuracy standard distance, L = length of the standard segment, L_1 = length of error influenced segment under the first accuracy standard, and L_2 = length of error influenced segment when the accuracy standard distance is doubled.

digitizing operators in order to develop a model of digitizing error. Stream-mode digitizing error was modeled using an ARMA (1,0) procedure from time-series analysis. The stream-mode digitizing errors exhibited all of the characteristics of a time series which included serial correlation of the errors and regularly spaced data in time or distance. The model fit the data extremely well, and indicated a strong autocorrelation among the stream-mode errors.

The effect of digitizing errors on map measurements was explored using simulation techniques and the ARMA (1,0) model. Essentially, no bias in area estimation was created by digitizing polygons because of the compensating effect of digitizing error. But the variation in polygon area increased as the accuracy standards became less stringent; roughly, a doubling of standard deviation occurred as accuracy standard distance doubled. Even at the loosest accuracy standard, polygon area varied only by 1.677 percent of total area. Bias and variance of line length both increased as the accuracy standard increased. A doubling of the accuracy standard distance caused a quadrupling of the line-length bias and a doubling to tripling of the line-length standard deviation. For stream-mode digitizing, the reasonable accuracy standards discussed above created a line-length bias of less than 2 percent of total length, and a standard deviation of less than 1 percent of total length.

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