Description of Terrain as a Fractal Surface, and Application to Digital Elevation Model Quality Assessment

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ABSTRACT: Fractional Brownian motion is a surface model through which the topographic surface may be described. This model provides a simple method for estimating the fractal dimension of a digital elevation model. By computing the fractal dimension value at different scales and in different directions, interpolation artifacts may be revealed, contributing to digital elevation model quality assessment.

INTRODUCTION

A TOPOGRAPHIC MAP is generally required to describe the terrain surface as faithfully as possible, but the smallest shapes escape this requirement, and in most cases their representation depends on the surveyor's appreciation or even on his imagination. Indeed, compared to classical geometrical forms (plane, sphere, . . .), natural terrain is such a chaotic surface that we cannot completely represent it but only provide an approximate image, disregarding the smallest details. Nevertheless, despite their random appearance, terrain forms seem to follow an underlying order. It is not true that each part of terrain is a copy of a larger part including it. However, we can frequently observe a stochastic self-similarity, because terrain seems to conserve the same statistical characteristics over a wide range of scales.

This property of self-similarity is called self-affinity for vertical profiles or for the terrain surface itself (Vicsek, 1989). Beyond its intuitive obviousness, it has been confirmed by Richardson's experimental results (in Mandelbrot, 1982). Indeed, by measuring coastline lengths on maps, Richardson observed that the measured length varies in a regular way as the length unit varies: that variation, in log-log coordinates, is always represented by an almost straight line for length units ranging from a few kilometres to several thousands of kilometres. Håkanson (1978) obtained similar results. The self-affinity of terrain has been discussed by Goodchild (1980), Mark and Aronson (1984), and Andrle and Abrahams (1989). Mandelbrot (1982) gave a mathematical expression to the self-affinity phenomenon by introducing the fractal dimension concept. In the case of a surface, we can intuitively say that fractal dimension — extension of the Euclidean dimension — is a non-integer value which ranges from 2 to 3 and increases when the surface progressively changes from a plane ($D=2$) to a surface so folded that it would fill a volume ($D=3$). The physical significance of fractal dimension for landscapes has been analyzed by Mandelbrot (1982), Pentland (1984), and Goodchild (1988). Fractal geometry has been applied in three-dimensional (3D) landscape synthesis and resampling (Voss, 1985; Miller, 1986; Barnsley, 1988) and in terrain shape analysis (Curl, 1986; Yokoya *et al.*, 1989).

Our purpose is to use the fractal dimension concept for raster digital elevation model quality assessment. Most DEM evaluation techniques, which consist in estimating a global accuracy with regard to a reference (Strory and Congalton, 1986), do not detect artifacts such as those caused by digitizing and resampling. The aim of this article is to reveal DEM interpolation artifacts using fractal dimension. In the first section we present a technique for computing fractal dimension based upon the fractional Brownian motion model. In the second section, we use this model to reveal some artifacts in digital elevation models. Finally, in the third section, the contribution of this technique to DEM quality assessment is discussed.

FRACTIONAL BROWNIAN MOTION

The fractional Brownian motion model is a continuous, non-differentiable surface model which is generally fit for describing chaotic surfaces such as terrestrial relief (Mandelbrot, 1982; Fournier *et al.*, 1982). If we call $z(x,y)$ this surface model, we have the following relationship between the horizontal distance and the expected elevation variation over this distance:

$$E[|z(x+\Delta x,y+\Delta y)-z(x,y)|] = \sigma\sqrt{2/\pi}\,\sqrt{\Delta x^2+\Delta y^2}^{H} \quad (1)$$

where σ and H are constant parameters for a given landscape. σ is a vertical scaling factor, related to local slope (Yokoya *et al.*, 1989). In the case of terrestrial relief, σ takes low values in plain regions, and high values in mountainous regions where height variations over a given distance are important. H is an indicator of the surface complexity (Burrough, 1981). This parameter may take values between 0 and 1. Its physical meaning is developed in Vicsek (1989) by considering two consecutive displacements along a profile in any direction, and over the same horizontal distance:

- if $H<1/2$, then the two height variations are likely to have opposite signs;
- if $H=1/2$, then the two height variations are independent; and
- if $H>1/2$, then the two height variations are likely to have the same sign.

H is related to the fractal dimension of the surface. Indeed, Falconer (1990) shows that

$$D=3-H. \quad (2)$$

The smaller H, the larger D and the more irregular the surface. On the contrary, the larger H, the smaller D and the smoother the surface (Goodchild 1980).

The fractional Brownian motion model allows one to estimate the fractal dimension of a digital elevation model in a very sim-

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ple way. Indeed, writing Equation 1 in the log-log space results in a linear relationship between the expected height difference and the horizontal distance, the increment rate of which is H: i.e.,

$$\log E[|z(x+\Delta x, y+\Delta y) - z(x,y)|] = \log(\sigma\sqrt{2/\pi}) + H \log \sqrt{\Delta x^2 + \Delta y^2}. \quad (3)$$

In a DEM, the average elevation difference $E[|z(x+\Delta x, y+\Delta y) - z(x,y)|]$ may be plotted against the horizontal distance $\sqrt{\Delta x^2 + \Delta y^2}$ for all integer values of Δx and Δy in any direction, yielding the fractal plots in log-log coordinates. H is obtained from the fractal plots as the slope of the least-squares straight line, and D is derived from Equation 2. Other techniques for computing D from a DEM have also been developed (Clarke, 1986). In fact, D is computed for a given distance interval by selecting Δx and Δy so that the resulting horizontal distance belongs to the required interval. Indeed, Mark and Aronson (1984) show that the fractal plots of most terrestrial landscapes present slope variations corresponding to typical lengths of the landscape.

This fractal dimension value is not meaningful unless the DEM "fractalness," i.e., the linearity of the fractal plots over the considered distance interval, has been previously checked. This is easily achieved by computing the correlation coefficient ρ between $\log E[|z(x+\Delta x, y+\Delta y) - z(x,y)|]$ and $\log \sqrt{\Delta x^2 + \Delta y^2}$: ρ should be close to 1. Other "fractalness" indices have been defined (Yokoya *et al.*, 1989).

The properties of fractional Brownian motion have been extensively studied in literature (Mandelbrot, 1982; Voss, 1985; Falconer, 1990). Two of these properties—self-affinity and isotropy—are considered here. They will be used in the next section.

Intuitively, self-affinity means that each part of the surface is similar to the whole surface. Indeed, fractional Brownian motion is statistically scale-independent, i.e., its statistical properties remain unchanged by a similarity. In the case of terrestrial relief, the self-affinity property means that, according to this model, each terrain shape would be a set of similar smaller shapes, the concentration of which would be characterized by the fractal dimension (Mandelbrot, 1982).

Falconer (1990) shows that the intersection of the fractional Brownian surface with a vertical plane, i.e., a vertical profile, has the fractal dimension $2-H$, whatever the direction is. In other words, this surface is statistically direction-independent, i.e., its statistical properties remain unchanged by a rotation. Consequently, fractional Brownian motion is an isotropic surface model.

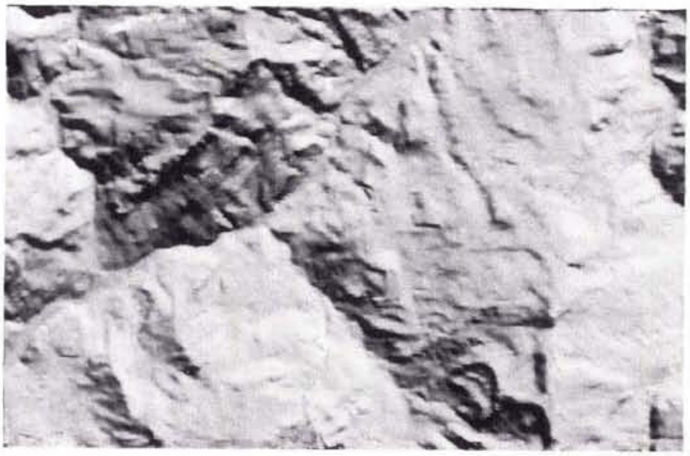

FIG. 1. DEM of Colombia.

FIG. 2. Locator map of the test-site.

APPLICATION TO REVEALING ARTIFACTS IN DIGITAL ELEVATION MODELS

In this section we show that fractal dimension measurements may contribute to digital elevation model quality assessment by revealing interpolation artifacts. To do so, we must assume that the real topographic surface has the same statistical properties as the fractional Brownian motion model, namely, self-affinity and isotropy (this hypothesis will be discussed at the end of this section). Then we check whether the DEM satisfies these conditions or not.

This experiment was carried out with a DEM over a 10- by 15-km test-site in Colombia, in the Paz del Río region (see DEM on Figure 1 and locator map on Figure 2). This DEM was generated for geomorphological pattern recognition (Chorowicz *et al.*, 1989) but limitations due to resolution had to be taken into account and a quality evaluation was required.

The DEM was obtained from a 1:25,000-scale topographic map by interpolating a 40-metre interval raster grid between the digitized contour lines. The interpolation process we used consisted in searching known points in two directions (N-S and W-E). Because the contour lines, of 50-metre vertical interval, were rather far apart on the map sheet compared to the pixel size, the resampling process had a great incidence on the data quality. By computing the DEM fractal dimension for different distance intervals and for different directions, we revealed two interpolation artifacts: excessive smoothness and directional tendency.

EXCESSIVE SMOOTHNESS

The DEM fractal dimension may be computed at different scales, i.e., over different distance intervals. Table 1 shows the fractal dimension values obtained over short distances (typically 1 to 5 pixels, i.e., below 200 metres) and over large distances (typ-

TABLE 1. DEM FRACTAL DIMENSION FOR TWO DIFFERENT DISTANCE INTERVALS.

distance interval (pixels)	fractal dimension
1 - 5	2.07
10 - 30	2.25

ically 10 to 30 pixels i.e., from 400 to 1200 metres). Those fractal dimensions are obtained from Equations 2 and 3 by computing average height variations corresponding to horizontal displacements in all directions, i.e., the four main directions N-S, NW-SE, W-E and NE-SW.

The two distance intervals were chosen in order to discriminate between distances over which the interpolation process is relevant (the shorter ones) and those over which it is not (the largest ones). Indeed, the horizontal interval between the map contour lines was generally larger than 200 metres (5 pixels) and shorter than 400 metres (10 pixels). Therefore, the contour lines were not close enough to depict the terrain shapes smaller than 200 metres, while they allowed the depiction of shapes of some 400 metres or more.

We can observe that D has a small value, close to 2, for short distances, which means that the DEM is locally planar. On the contrary, for larger distances, D has a higher value. The discrepancy between small-scale and large-scale fractal dimensions may be interpreted as a consequence of the smoothing interpolation process.

DIRECTIONAL TENDENCY

The DEM fractal dimension may be computed in different directions by measuring height differences along particular profiles. We did so over some 800 square windows uniformly distributed over the DEM. Table 2 shows the average D values obtained along the N-S and W-E directions on the one hand, and along the complementary NW-SE and NE-SW directions on the other hand, for two window sizes: 5- by 5-pixel windows, over which the interpolation process has an important effect, and 30- by 30-pixel windows, over which it has not. In each case, the fractal plots are obtained by averaging elevation variations in the selected directions, and for distances ranging from 1 pixel to the window size. The standard deviation of the fractal dimension distribution is also indicated, in order to evaluate the homogeneity of this parameter.

For short distances, over which the interpolation process is relevant, we observe an important discrepancy between the two direction categories: D is smaller in the N-S and W-E directions, which means that the DEM is smoother in the directions in which the interpolation is done. Besides, the standard deviation is very small, i.e., the fractal dimension is almost constant over the DEM. On the contrary, for larger distances, over which the interpolation process is less relevant, we do not observe such a difference, which was the expected result because interpolation has no effect at that scale. Moreover, the standard deviation is rather large in all directions, which means that D varies from one area to another, depending on the landscape characteristics. At that scale, a directional tendency could exist, but it would be due to some anisotropic large terrain feature and not to the interpolation process.

The variations of fractal dimension with direction for short distances may be interpreted as a consequence of the anisotropic interpolation process.

DISCUSSION OF THE SELF-AFFINITY/ISOTROPY HYPOTHESIS

A DEM is required to provide a representation of the topographic surface, and the purpose of DEM quality assessment is to detect discrepancies between the DEM and the real terrain.

TABLE 2. COMPARISON OF FRACTAL DIMENSION STATISTICS BETWEEN INTERPOLATION DIRECTIONS (N-S AND W-E) AND COMPLEMENTARY DIRECTIONS (NW-SE AND NE-SW) FOR TWO DIFFERENT WINDOW SIZES.

	mean	standard deviation
Fractal dimension over 5 by 5 windows		
N-S and W-E	2.01	0.01
NW-SE and NE-SW	2.05	0.14
Fractal dimension over 30 by 30 windows		
N-S and W-E	2.12	0.10
NW-SE and NE-SW	2.13	0.11

So far, however, we have only revealed discrepancies between the DEM and a theoretical surface model, namely, fractional Brownian motion. To what extent can these discrepancies be interpreted as consequences of interpolation artifacts? In other words, can the smoothing and anisotropic interpolation process be held as responsible for the non self-affinity and anisotropy of the DEM?

First, can the non self-affinity of the DEM be interpreted as an artifact? On the one hand, although it is intuitively true that terrain is self-affine within limited intervals only, Mark and Aronson (1984) observe that the limits of self-affinity intervals generally correspond to some typical terrain feature in the landscape, while the limit distance observed here (between 5 and 10 pixels) does not coincide with any dominant terrain feature in the area. On the other hand, this limit distance coincides with the approximate horizontal distance between contour lines. This coincidence shows that the interpolation process has an effect on the DEM fractal dimension over very short distances.

Second, can the anisotropy of the DEM be interpreted as an artifact? Although terrain may be anisotropic for large distances due to the main ridges and valleys, it is intuitively isotropic over short distances. Besides, the anisotropy observed here coincides with the interpolation directionality, and this coincidence shows that the DEM anisotropy is a consequence of the interpolation process.

Finally, because the discrepancies between the DEM and the theoretical surface model have a correlation with the resampling technique peculiarities, the self-affinity / isotropy hypothesis may be reasonably accepted.

DISCUSSION

Modeling the terrain surface with fractional Brownian motion can contribute to digital elevation model quality assessment by revealing some artifacts, which are generally due to interpolation. In fact, the technique presented in the previous section aims at assessing the interpolation process. On the one hand, an excessive smoothing generally means that the grid interval is too small compared to the horizontal distance between contour lines, so that the kind of curve chosen for interpolating (often spline or polynomial) has a great incidence on the DEM. On the other hand, a directional tendency means that the known points have been searched in too few directions. It can then be useful to compute fractal dimensions for revealing such interpolation artifacts, all the more so as they may be undetectable by a mere visual analysis on a shadow or perspective image, although they can jeopardize the DEM reliability for many applications.

This quality assessment technique presents an important practical advantage, insofar as it does not require a reference DEM. Indeed, as mentioned by Carter (1988), most DEMs are produced in areas where no better data are available, so that they cannot be compared to a reference. Besides, even when a reference is available, the two grids are not always geometrically compatible, so that the comparison process may require one of the DEMs to be resampled. In this case, the interpolation assessment would become meaningless. Therefore, computing fractal dimensions provides an intrinsic quality assessment technique, and the use of *D* as a numerical parameter allows one to define quantitative quality indicators.

Obviously, this quality assessment technique is not sufficient for a complete data evaluation. Indeed, as well as the techniques based upon transfer function analysis (Elghazali and Hassan, 1986), it can only reveal high-frequency systematic artifacts like those mentioned earlier. In other words, it can only reveal interpolation artifacts. It cannot be used for checking the global agreement of the DEM with the real landscape, or for revealing global errors such as those caused by rotations or translations, which may be detected more easily by comparing the DEM to a reference data set. The most classical quality assessment method, which consists in computing an RMS error, i.e., the standard deviation of height discrepancies between the evaluated DEM and a reference DEM or set of control points (Torlegård, 1986), is not sufficient either because it is not sensible to interpolation artifacts. Consequently, both quality assessment techniques should be applied together to a resampled DEM for a more complete evaluation.

CONCLUSION

Generally, digital elevation model quality assessment is carried out by comparing the DEM to a reference, and an RMS error is computed in order to estimate the DEM accuracy. However, this kind of evaluation is not sensitive to interpolation artifacts, like excessive smoothness and directional tendency. Such artifacts may be detected by computing fractal dimensions at different scales and in different directions. Therefore, fractal geometry can contribute to digital elevation model quality assessment through a simple technique which does not require a reference DEM.

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