Reducing the Registration Time for Photographs with Non-Intersecting Cross-Arm Fiducials on the Analytical Plotter

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ABSTRACT: Interior orientation is the most frequently required of all the orientations needed for mapping with the analytical plotter, particularly when using standard format photographs. Although elementary, the routine nature of the process is sometimes unpleasant, especially during digitizing. In practice, the registration of the diapositives on the analytical plotter is the main part of the process which has to be repeated as often as photographs are changed. The measurements and the computations needed for this registration are often time consuming, especially when remeasurements have to be made to ensure proper fit.

Although some modern cameras have intersecting cross or single dot fiducial marks which clearly indicate the exact location of each mark, a large number of photographs are equipped with the non-intersecting cross fiducial marks. This increases the registration time because four measurements have to be made at each fiducial location to achieve a fit. A program implementation, designed to be non-interactive beyond allowing the operator to position the mark and to terminate the process, is developed and compared with the usual iterative implementation to determine how much time reduction is achievable. Single measurements are made to the geometric center instead of the four normally made to the arms of each fiducial mark; and the Givens updating algorithm is used to provide real time indication of goodness of fit during remeasurement. Tests show that a less interactive operation generally reduces the orientation time. When combined with single measurements to the center of the fiducial marks, this approach reduces the registration time for diapositives equipped with the non-intersecting cross-fiducial marks by about 50 percent.

INTRODUCTION

BASIC TO THE PRECISE METRIC EVALUATION of photographic images is the interior orientation which recreates the geometry of the bundle of image forming rays at the instant of exposure. When the camera used for photography has been properly calibrated, the image space coordinate system is well established (Figure 1). The objective of interior orientation is to relate this coordinate system as accurately as possible to the camera projection center for spatial evaluation purposes.

On the analytical plotter, this operation is achieved by entering the known camera parameters into a file and registering the diapositives to the instrument stages with the aid of the fiducial marks. The plate registration process involves the measurement of the fiducial locations and the computation of a least-squares regression to determine the stage-to-fiducial transformation parameters. Normally, the instrument drives to the vicinity of each fiducial location, and the operator makes one or four measurements at each of the corner fiducials depending on its type. After all points are measured, the estimation is carried out, and the least- squares residuals are displayed, with the point having the largest residual highlighted. The operator then decides whether to remeasure individual points or to repeat the whole operation, depending on the size of the residuals. In the case of the Kern DSR-11, the operator is generally advised to ensure that all residuals are less than 10 micrometres (Kern unpublished documentation, 1988). If a remeasurement is necessary, the analytical plotter drives to the point in question and allows the operator to remeasure it. The new measurement is then substituted for the old one, and the estimation is repeated. This cycle continues until a good fit is achieved.

For conventional mapping where the same camera is used for the entire block of photographs, the camera parameters are entered only once at the beginning of the restitution process. However, the measurements and computations for the registration have to be repeated as often as photographs are replaced on the instrument stages. The bulk of the time and effort for interior orientation is therefore spent at the registration stage, particularly when using diapositives equipped with non-intersecting cross fiducial marks (see Figure 2).

To register this type of diapositive (Figure 2b), the operator has to make four measurements at each fiducial location as opposed to only one (Figure 2a). This significantly increases the registration time, especially when combined with the time required for the operator to assess the residuals for possible re-

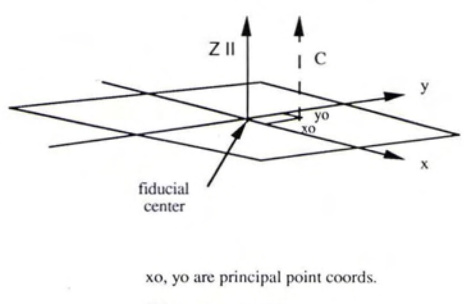

FIG. 1. Image space system and its orientation.

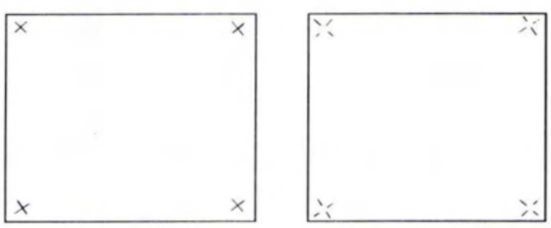

FIG. 2. (a) Crossing fiducial marks. (b) Non-crossing fiducial marks.

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measurements. This time may appear insignificant when only a few photographs are involved, but when blocks of photographs are being serially and routinely registered, particularly when collecting data from stereo models, the effect of this time becomes appreciable. Although, the use of correlator attachments for plate registration has removed these problems, not many organizations can afford them; thus, it is common to have this operation performed by the operator assisted by a software module.

In developing an interior orientation scheme designed to reduce the registration time for the type of diapositive shown in Figure 2b, the following criteria were considered: (a) because with experience an operator can judge visually the approximate center of each fiducial mark, only a single measurement to its center is to be made; (b) a sequential processing of each measurement is carried out so that, if a remeasurement is needed for any point, the quality of any new measurement may be assessed in real time and displayed for the operator; (c) the program should assess the residuals and initiate any remeasurement; this operation, usually performed by the operator, can be more effectively initiated by the program, and this means less interactiveness; and (d) the program should terminate either when all measurement tolerances are met or when commanded by the operator.

Basic to a program which fulfills these criteria is a sequential data processing algorithm which, in addition to processing the data one at a time, can give a real time indication of the least-squares criterion (LSC), which provides an immediate and useful check on any new measurement. Moreover, by relieving the operator of the task of visually assessing the residuals to decide whether and what point to remeasure before entering into a dialog in order to initiate the remeasurement, the program is less interactive and thus reduces the turn-around time during remeasurement.

The classical sequential least-squares algorithm is well known and has been used to solve estimation problems in mapping applications (Krakiwsky, 1975). Given an *a priori* covariance estimate, this algorithm can process data one at a time. However, to update the sum-of-squares of the residuals, the estimates of the parameters must be computed first. On the other hand, the orthogonal decomposition method based on the square-root free Givens transformation has been known to be equally suitable for regression updating (Gentleman, 1973; Lawson and Hanson, 1974; Blais, 1983). In addition to its advantages of numerical stability and ability to process measurements one row at a time, it is capable of providing the least-squares criterion without having to compute the estimates of the parameters (Bierman, 1977). For the process of interior orientation, the operator uses only the residuals to indicate the goodness of the measurements. When these residuals are small, it is usually assumed that the parameter estimates are good and the process is terminated. Thus, during remeasurement, an immediate availability of the least-squares criterion reduces the turn around time for a particular point. The Givens algorithm was therefore chosen for this implementation.

MATHEMATICAL MODELS

In practice, plate registration during analytical interior orientation is performed through the transformation of the measured machine coordinates to the fiducial system. Among the most commonly used mathematical models are the following (Ziemann, 1971; Moffitt and Mikhail, 1980):

Similarity transformation equations

$$\begin{aligned} x_f &= a_0 + a_1 x_m - b_1 y_m \\ y_f &= b_0 + b_1 x_m + a_1 y_m \end{aligned} \tag{1}$$

Six parameter affine transformation

$$\begin{aligned} x_f &= a_0 + a_1 x_m + a_2 y_m \\ y_f &= b_0 + b_1 x_m + b_2 y_m \end{aligned} \tag{2}$$

Projective transformation

$$\begin{aligned} x_f &= \frac{a_0 + a_1 x_m + a_2 y_m}{1 + c_1 x_m + c_2 y_m} \\ y_f &= \frac{b_0 + b_1 x_m + b_2 y_m}{1 + c_1 x_m + c_2 y_m} \end{aligned} \tag{3}$$

Bilinear transformation

$$\begin{aligned} x_f &= a_0 + a_1 x_m + a_2 y_m + a_3 x_m y_m \\ y_f &= b_0 + b_1 x_m + b_2 y_m + b_3 x_m y_m \end{aligned} \tag{4}$$

where x_f ,y_f are known coordinates, and x_m, y_m are measured stage coordinates.

Although, the linear affine transformation is most commonly used, most analytical plotter software modules for interior orientation include more than one of these equations to enable the user to select the best function for particular situations, for example, to correct for film deformation. Only the affine transformation was used in our experiment.

LEAST-SQUARES ESTIMATION FOR INTERIOR ORIENTATION

Given

$\mathbf{L} = \begin{bmatrix} x_f \\ y_f \end{bmatrix}$ as the vector of known fiducial coordinates,

$\mathbf{x}^t = [a_0\ a_1\ a_2\ b_0\ b_1\ b_2]$ the vector of transformation parameters,

$\mathbf{A} = \begin{bmatrix} 1 & x_m & y_m & 0 & 0 & 0 \\ 0 & 0 & 1 & x_m & y_m \end{bmatrix}$ the coefficient matrix,

where $[x_m\ y_m]$ are measured stage coordinates, and

$\mathbf{W}$ the diagonal weight matrix;

then the method of least-squares leads to an estimate of the unknown parameters $\mathbf{x}$ (Vanicek and Krakiwsky, 1986) and their cofactor matrix $\mathbf{Q}_x$

$$\begin{aligned} \mathbf{x} &= (\mathbf{A}^t\mathbf{W}\mathbf{A})^{-1}\,\mathbf{A}^t\mathbf{W}\mathbf{L} \\ \mathbf{Q}_x &= (\mathbf{A}^t\mathbf{W}\mathbf{A})^{-1} \\ \mathbf{v} &= \mathbf{L} - \mathbf{A}\mathbf{x} \\ e &= \mathbf{v}^t\mathbf{W}\mathbf{v} \end{aligned} \tag{5}$$

where e is the least-squares criterion (LSC).

These matrix equations are normally used for a simultaneous adjustment of all data. However, the same rigorous results are obtainable from a sequential processing of the data, and often this form is preferable in real-time applications. As stated earlier, the square-root free Givens transformation is one such scheme whose use in mapping applications is becoming increasingly popular (Blais, 1983).

THE SQUARE-ROOT FREE GIVENS ALGORITHM

This method does not form normal equations but decomposes the design matrix into a triangular form. Being square-root free, it allows the use of weights for a sequential addition and deletion of data.

Considering the estimation process discussed above, and assuming for simplicity that the measurements are equally weighted, then the normal equations (first line, Equation 5) become:

$$\mathbf{A}^t\mathbf{A}\mathbf{x} = \mathbf{A}^t\mathbf{L} \tag{6}$$

and using the Givens transformation, the matrix **A** is decomposed into the product **QR**; i.e., **A** = **QR** where **Q** is an m by n matrix whose columns are orthonormal, and **R** is an n by n upper triangular matrix. The normal equations can be rewritten as

$$\begin{aligned} \mathbf{A}^t\mathbf{A}\mathbf{x} &= \mathbf{A}^t\mathbf{L} \\ \mathbf{R}^t\mathbf{Q}^t\mathbf{Q}\mathbf{R}\mathbf{x} &= \mathbf{R}^t\mathbf{Q}^t\mathbf{L} \\ \mathbf{R}^t\mathbf{R}\mathbf{x} &= \mathbf{R}^t\mathbf{Q}^t\mathbf{L} \end{aligned} \tag{7}$$

and because **R** is nonsingular whenever $\mathbf{A}^t\mathbf{A}$ is, then

$$\begin{aligned} \mathbf{R}\mathbf{x} &= \mathbf{Q}^t\mathbf{L} \\ \mathbf{R}\mathbf{x} &= \mathbf{b} \end{aligned} \tag{8}$$

where $\mathbf{b} = \mathbf{Q}^t\mathbf{L}$. This triangular system can be solved for **X** without ever forming the normal equations.

For sequential addition and deletion of data using this decomposition method, we are interested in constructing a least-squares solution to the *a priori* data expressed in matrix form as

$$\begin{bmatrix} \mathbf{R}_{j-1} & \mathbf{b}_{j-1} \\ 0 & \mathbf{e}_{j-1} \end{bmatrix}. \tag{9}$$

Given the new row of data formed from the new observation as

$$[\mathbf{A}_j \quad \mathbf{L}_j],$$

the decomposition becomes

$$\mathbf{Q}_j \begin{bmatrix} \mathbf{R}_{j-1} & \mathbf{b}_{j-1} \\ 0 & \mathbf{e}_{j-1} \\ \mathbf{A}_j & \mathbf{L}_j \end{bmatrix} = \begin{bmatrix} \mathbf{R}_j & \mathbf{b}_j \\ 0 & \mathbf{e}_j \end{bmatrix} \tag{10}$$

where, as stated earlier, $\mathbf{A}_j$ and $\mathbf{L}_j$ are the elements of the row formed from the new data, and $\mathbf{Q}_j$ is the orthogonal matrix for the jth decompostion.

For the Givens transformation without square root, the procedure is to update

$$\begin{bmatrix} \mathbf{D}_{j-1}^{1/2}\overline{\mathbf{R}}_{j-1} & \overline{\mathbf{b}}_{j-1} \\ 0 & \mathbf{e}_{j-1} \end{bmatrix} \tag{11}$$

with the row of data $[\mathbf{W}_j\mathbf{A}_j \quad \mathbf{W}_j\mathbf{L}_j]$; i.e.,

$$\mathbf{Q}_j \begin{bmatrix} \mathbf{D}_{j-1}^{1/2}\overline{\mathbf{R}}_{j-1} & \overline{\mathbf{b}}_{j-1} \\ 0 & \overline{\mathbf{e}}_{j-1} \\ \mathbf{W}_j^{1/2}\,\mathbf{A}_j & \mathbf{W}_j^{1/2}\,\mathbf{L}_j \end{bmatrix} = \begin{bmatrix} \mathbf{D}_j^{1/2}\overline{\mathbf{R}}_j & \overline{\mathbf{b}}_j \\ 0 & \overline{\mathbf{e}}_j \end{bmatrix} \tag{12}$$

where **D** is a diagonal scale matrix.

Bierman (1977, pages 71-75) has shown that the quantity e_j in Equation 12 is the sum of weighted squares of the residuals (LSC) of all the observations up to the *jth*, a quantity which indicates the goodness of fit or the agreement between the observations and the mathematical model. It is obtained here without having to estimate the parameters.

The computational formulas used for the square root-free scheme, as derived and implemented by Gentleman (1973), are summarized below:

Given a row of *a priori* data and new data from Equation 12 above as

$$\begin{aligned} &0 \ldots 0 \;\; \sqrt{d} \ldots \sqrt{d}\,\bar{r}_k \ldots \\ &0 \ldots 0 \;\; \sqrt{W}\,a_i \ldots \sqrt{W}\,a_k \ldots \end{aligned} \tag{13}$$

one obtains

$$\begin{aligned} &0 \ldots 0 \;\; \sqrt{d^*} \ldots \sqrt{d^*}\,\bar{r}^*_k \ldots \\ &0 \ldots 0 \;\; 0 \ldots \sqrt{W^*}\,a^*_k \ldots \end{aligned} \tag{14}$$

where

$$\begin{aligned} d^* &= d + Wa_i^2 \\ W^* &= dW/(d+Wa_i^2) = dW/d^* \\ \bar{c} &= d/(d+Wa_i^2) = d/d^* \\ \bar{s} &= Wa_i/(d+Wa_i^2) = Wa_i/d^* \\ a_k^* &= a_k - a_i\bar{r}_k \\ \bar{r}_k^* &= \overline{c}\bar{r}_k + \bar{s}a_k \end{aligned} \tag{15}$$

However, Gentleman's implementation of this algorithm has been further modified to recursively compute the residuals (Farebrother, 1976; Griffith and Hill, 1985). This modified version was adapted for use in the new program for interior orientation.

IMPLEMENTATION

This algorithm has been used in a program for interior orientation on the Kern DSR-11 analytical plotter, hosted by the MicroVax-II mini computer. The following psuedocode gives the general outline of this implementation:

```
0 DECLARATIONS
   .LSC is the sum of squares of residuals (Least Squares Criterion)
   .Tolerance1 is the max. limit set for the LSC (default is 140
      times number of fiducial points)
   .Tolerance2 is the max. allowable value for the absolute value
      of any residual (default is 8 micrometres)
1 BEGIN
   For point going from 1 to number of fiducial marks
      a. drive to the point and allow operator to take measurement
      b. update the solution and the LSC with new measurement
   NEXT POINT
3 back-substitute for parameters, compute residuals, and norms
   of residuals; sort the norms in descending order with point
   IDs (the residuals at this stage are computed from V = L -
   Ax for simplicity)
4 okay = LSC less than tolerance1 and MaxResid less than
      tolerance2
5 if okay then stop   (exit from interior orientation)
6 if not okay then   (remeasurement is needed)
      a. drive to point having largest residual norm
      b. remove effect of old measurement for the point from
            solution by a negative updating
      c. LOOP UNTIL OPERATOR PRESSES FOOT-PEDAL
            -make a copy of solution for real time updating
            -get plate coordinates of current location automatically
            -update the LSC with this measurement
            -display old LSC, new LSC, difference
            -get status of foot pedal
         END LOOP
      d. include the operator measured coordinates in the solution
      e. goto step 3
7 END
```

TESTS AND RESULTS

This implementation (which for easy reference is called scheme1) has been used in an experiment of repeated registration in order to compare its performance with another implementation which uses both single and cross-arm measurements in the usual interactive mode. The single measurement mode of this other package is referred to here as scheme2 and the cross-arm mode is called scheme3. The three schemes were used for the orientation of a diapositive of the type shown in Figure 2b. To reduce the influence of pointing errors on this experiment, six points were selected on a photograph and their stage coordinates were measured and held fixed thus only the errors of pointing at the fiducial locations inevitably remain. Each of the three schemes was used for the registration of the diapositive, and the parameters of the transformation obtained were used to compute the fiducial coordinates of these points. The time taken for each registration was obtained through calls to a system time routine at the start and at the end of each orientation process. The difference represents the elapsed time for the process.

The computed fiducial coordinates and the time for each scheme are given in the Appendix. However, Table 1 shows the average time per registration and the discrepancy in the position of a point averaged over ten repetitions. Figure 3 gives a visual illustration of the data from Table 1. Table 2 shows the progression of the time taken for interior orientation using each scheme when different numbers of fiducial points are revisited for measurement while Figure 4 presents the time variation graphically for the three schemes.

Comparing the values for schemes 1 and 2 in Table 1, it can be observed that, while both achieve the same precision, scheme1 which is less interactive has smaller average registration time than scheme2 which uses the usual interactive approach. Moreover, comparing Table 2 (or Figure 4) for these two schemes, it can be seen that the time advantage for scheme1 ranges from about 4 seconds when no point is repeated, to about two-thirds of a minute when three points are remeasured. Because the two schemes use single measurements made to the centers of the marks, the implication is that, for registration with any type of fiducial marks, a time saving is achievable if the program is less interactive than is the case with most programs currently used.

Comparing scheme1 with scheme3 which measures to the cross-arms and offers the usual interactive environment, it is seen that scheme3 achieves twice the precision of scheme1 and takes more than twice the time for the registration. Furthermore, from Table 2 it is seen that the time reduction using scheme1 as compared to scheme3 ranges from about 50 percent when no points are remeasured to about 65 percent when three points are revisited. In addition, by being less interactive, scheme1 has an improved level of automation, thus reducing the operator's mental exercise and fatigue.

CONCLUSIONS AND RECOMMENDATIONS

From the results of this experiment, the following conclusions are derived:

- Making the interior orientation program less interactive to a certain degree generally reduces the time for the registration of diapositives on the analytical plotter.
- Although simple reasoning suggests that making one measurement to the center instead of four to the cross-arms naturally reduces the registration time, this test has shown that a further time reduction can be achieved by processing the single measurements serially, allowing the program to assess the goodness of fit and to initiate remeasurements. This leads to a time reduction of between 50 and 65 percent, depending on the number of points that require remeasurement.
- There is a slightly better precision of computed fiducial coordinates when the cross-arms are used than when a single measurement to the unmarked center is used for registration.
- This experiment has demonstrated the applicability of the Givens updating algorithm to even as small a real time photogrammetric estimation problem as interior orientation registration. Its ability to compute the residuals recursively enables an assessment of each fiducial measurement as soon as it is made, a feature which has been used to automate the remeasurement process.

While this paper has focused on the problem of using non-crossing fiducial marks for registration on the analytical plotter, we note that the less interactive method we have used here is just one possible approach. Having the mapping photographs marked with easily measurable fiducial marks will significantly reduce the registration time. As an alternative, special measur-

TABLE 1. AVERAGE TIME (SEC) AND AVERAGE POSITIONAL ERROR (µM)

	Average Time (sec)	Average Positional Error (µm)
scheme 1	93.6	5.1
scheme 2	115.2	5.5
scheme 3	243.6	2.4

TABLE 2. VARIATION OF ORIENTATION TIME (SECONDS) WITH NUMBER OF REMEASURED POINTS

		Points Remeasured			
		0	1	2	3
scheme 1	Time in Seconds	73.8	93.0	111.6	135.0
scheme 2		77.4	102.6	129.6	172.8
scheme 3		151.8	209.4	300.0	409.8

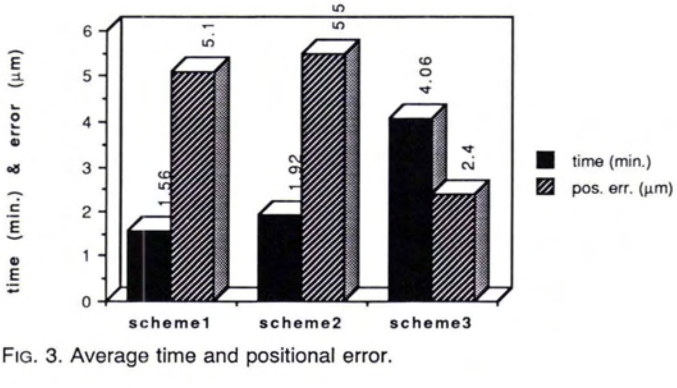

FIG. 3. Average time and positional error.

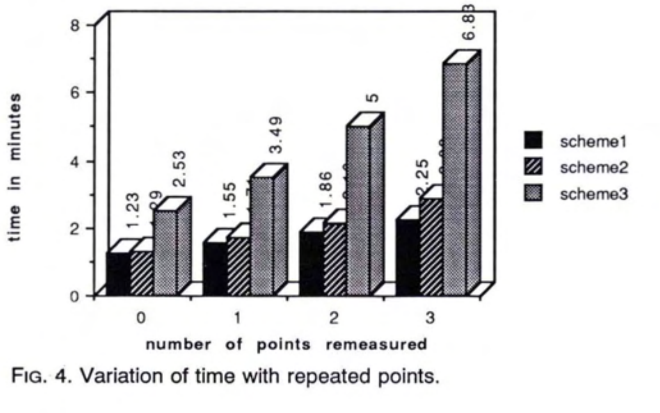

FIG. 4. Variation of time with repeated points.

ing marks could be devised in the plotter especially for measuring the non-crossing fiducial marks.

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APPENDIX

TABLE A1. DATA FOR SCHEME 1

			computed fiducial coordinates (μm)											
			point 1		point 2		point 3		point 4		point 5		point 6	
Ori. Srn	pts rept	Time (min)	x	y	x	y	x	y	x	y	x	y	x	y
1	3	2.25	−2155	−367	7985	−90059	105622	−85052	99521	13281	100564	91726	5535	98799
2	2	1.84	−2156	−366	7977	−90058	105628	−85052	99519	13270	100566	91721	5525	98793
3	0	1.36	−2160	−371	7978	−90059	105624	−85049	99521	13273	100559	91721	5533	98808
4	0	1.31	−2158	−367	7981	−90052	105620	−85050	99527	13272	100563	91721	5528	98798
5	0	1.19	−2154	−370	7987	−90061	105628	−85046	99517	13265	100561	91728	5527	98791
6	2	1.87	−2155	−365	7982	−90052	105627	−85045	99519	13267	100567	91719	5529	98792
7	1	1.55	−2155	−371	7981	−90058	105625	−85054	99529	13263	100567	91723	5526	98789
8	0	1.17	−2159	−369	7982	−90060	105628	−85045	99526	13263	100562	91723	5528	98799
9	2	1.88	−2159	−369	7982	−90056	105621	−85047	99519	13262	100561	91719	5529	98787
10	0	1.13	−2160	−366	7981	−90059	105628	−85046	99524	13266	100563	91726	5525	98788
	mea	1.56	−2157.1	−368.1	7981.6	−90057.4	105625.1	−85048.6	99522.2	13268.2	100563.3	91722.7	5528.5	98794.4
	stdv		2.3	2.2	2.9	3.1	3.2	3.3	4.0	5.9	2.7	3.1	3.3	6.5
	p stdv		3.193744		4.279148		4.5619197		7.158523		4.1123121		7.309811	

TABLE A2. DATA FOR SCHEME 2

			computed fiducial coordinates (μm)											
			point 1		point 2		point 3		point 4		point 5		point 6	
Ori. Srn	pts rept	Time (min)	x	y	x	y	x	y	x	y	x	y	x	y
1	0	1.29	−2157	−367	7989	−90059	105625	−85055	99520	13275	100559	91730	5538	98794
2	3	2.77	−2158	−367	7977	−90059	105632	−85055	99519	13265	100562	91723	5527	98789
3	2	2.12	−2163	−373	7979	−90059	105628	−85052	99520	13268	100554	91724	5536	98802
4	0	1.14	−2161	−367	7983	−90051	105623	−85053	99526	13267	100559	91723	5530	98794
5	0	1.25	−2155	−371	7991	−90062	105632	−85048	99517	13261	100556	91732	5529	98787
6	1	2.71	−2157	−366	7984	−90052	105631	−85047	99518	13263	100562	91721	5531	98788
7	0	1.30	−2157	−372	7983	−90058	105629	−85057	99528	13259	100562	91726	5528	98785
8	3	2.98	−2162	−370	7985	−90061	105632	−85047	99525	13259	100557	91726	5531	98795
9	0	1.46	−2162	−370	7984	−90056	105625	−85049	99519	13258	100556	91721	5531	98783
10	2	2.19	−2163	−367	7983	−90060	105632	−85048	99523	13262	100558	91731	5527	98784
	mea	1.92	−2159.5	−369	7983.8	−90057.7	105628.9	−85051.1	99521.5	13263.7	100558.5	91725.7	5530.8	98790.1
	stdv		3.0	2.5	4.1	3.7	3.5	3.8	3.7	5.2	2.8	4.1	3.6	6.0
	p stdv		3.89444		5.494442		5.1185936		6.434283		4.9508697		7.027723	

TABLE A3. DATA FOR SCHEME 3

			computed fiducial coordinates (μm)											
			point 1		point 2		point 3		point 4		point 5		point 6	
Ori. Srn	pts rept	Time (min)	x	y	x	y	x	y	x	y	x	y	x	y
1	4	7.27	−2159	−368	7986	−90058	105625	−85052	99520	13271	100562	91727	5535	98797
2	3	6.83	−2159	−368	7983	−90058	105628	−85052	99520	13266	100564	91723	5530	98794
3	1	3.50	−2161	−370	7983	−90058	105626	−85050	99520	13267	100559	91723	5534	98802
4	2	5.00	−2160	−368	7984	−90056	105624	−85051	99522	13267	100562	91723	5532	98797
5	0	2.53	−2158	−369	7986	−90058	105627	−85049	99519	13264	100561	91728	5531	98793
6	1	3.57	−2159	−367	7984	−90056	105627	−85048	99520	13265	100564	91721	5532	98794
7	1	3.41	−2159	−370	7984	−90058	105626	−85053	99523	13263	100564	91725	5531	98792
8	0	2.49	−2160	−369	7985	−90058	105627	−85049	99522	13264	100561	91724	5532	98798
9	1	3.45	−2160	−369	7984	−90057	105625	−85049	99520	13263	100561	91721	5532	98791
10	0	2.58	−2161	−368	7984	−90058	105627	−85049	99521	13265	100562	91727	5530	98792
	mea	4.06	−2159.6	−368.6	7984.21	−90057.5	105626.2	−85050.1	99520.7	13265.5	100562.0	91724.2	5531.9	98795.0
	stdv		1.0	1.0	1.0	0.8	1.2	1.5	1.3	2.4	1.6	2.5	1.6	3.4
	p stdv		1.36626		1.331207		1.9419635		2.727412		2.9739611		3.751207	