PEER-REVIEWED ARTICLE

PRACTICAL PAPER

A Data Reduction Approach Using the Collinearity Model from Non-Metric Photography

Abstract

A new computation procedure for calculation of both the interior and exterior orientation elements from a single non-metric photograph based on the collinearity equation and "an algorithm for least-squares estimation of nonlinear parameters" (Marquardt, 1963) has been investigated in detail. In this procedure, the traditional intermediate step of transforming image coordinates from a comparator system to a photo-coordinate system is by-passed. An experimental verification of the proposed technique has been carried out. The experimental results support the conclusion that the developed calculation procedure offers a theoretical and practical alternative to the existing models for the on-the-job calibration of non-metric cameras.

Introduction

In close-range and industrial applications of non-topographic photogrammetry, metric as well as non-metric cameras are used as photographic data acquisition systems. Since the early 1970s, concentrated research and development has resulted in the development of a number of methods for data reduction particularly suitable for use with photographs from non-metric cameras. These methods are based on highly sophisticated analytical techniques which combine, in most cases, the calibration and evaluation phases (Karara, 1980). Because of the lack of fiducial marks in non-metric cameras, special techniques had to be devised. One such method, the Direct Linear Transformation (DLT), was developed at the University of Illinois (Abdel-Aziz and Karara, 1971, 1974; Karara and Abdel-Aziz, 1974, 1974). The solution establishes a direct linear relationship between coordinates of image points and the corresponding object space coordinates. This linear approach for the calibration of a camera does not require fiducial marks on the photographs. In 1978, Bopp and Kraus developed an exact solution to the DLT basic equations, which leads to a least-squares adjustment with linear fractional observation equations and non-linear constraints, treated as additional observation equations with zero variances. The term "11-parameters solution" is used for this method. In contrast to the *DLT*, the "11-parameter solution" can also be applied to cases where the known interior orientation should be held (Bopp and Krauss, 1978).

In contrast to the DLT and "the 11-parameter solution," the presented solution established a direct nonlinear relationship between coordinates of image points and the corresponding object space coordinates. This nonlinear approach for the calibration of a camera does not require fiducial marks or reseau points on the photographs. Based on "Algorithm for Least-Squares Estimation of Nonlinear Parameters" developed by D.W. Marquardt in 1963, a reliable solution for the orientation and calibration elements is derived, which performs an optimum interpolation between the Taylor series (Gauss-Newton) method and the gradient method.

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Theoretical Consideration

Modified Collinearity Equations

A perspective or central projection is one in which all points are projected on to the reference plane through one point called the perspective center. The position of the perspective center with respect to the image coordinate system represents the geometric elements of interior orientation. The position of the reference plane is defined by the object space coordinates of the perspective center X_0, Y_0, Z_0. The orientation, which describes the attitude of the camera at the moment of exposure, refers to the spatial relationship between the object coordinate system and the image coordinate system. The relationship between the image and object coordinate systems is expressed by a 3 by 3 orthogonal matrix (Moffitt and Mikhail, 1980).

The perspective center, the image point, and the object point lie on a straight line. This fundamental relationship is basic to all procedures in metric and non-metric cameras. However, in non-metric cameras, because of the lack of fiducial marks or reseau points, establishing the photocoordinate system and obtaining the parameters for correcting linear film deformation, lens distortion, and comparator errors are not possible. Therefore, the solution cannot be obtained using the collinearty equations. However, without loss of generality, the photocoordinate system may be assumed parallel to the comparator coordinate system (Abdel-Aziz and Karara, 1971, 1974; Marzan and Karara, 1975), and the coordinates of the pseudo-position of the principal point (x_o, y_o) which are in the comparator coordinate system for non-metric photography are used. The measured comparator coordinates (x_c and y_c) will take the following form (Müftüoğlu, 1980, 1984):

$$
\begin{aligned}
x_c &= x_o - c\frac{m_{11}(X - X_o) + m_{12}(Y - Y_o) + m_{13}(Z - Z_o)}{m_{31}(X - X_o) + m_{32}(Y - Y_o) + m_{33}(Z - Z_o)} \\
y_c &= y_o - c\frac{m_{21}(X - X_o) + m_{22}(Y - Y_o) + m_{23}(Z - Z_o)}{m_{31}(X - X_o) + m_{32}(Y - Y_o) + m_{33}(Z - Z_o)}
\end{aligned} \tag{1}
$$

Also, the measured values in these equations contain systematic and random errors.

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Thus, the intermediate step of transforming image coordinates from a comparator system to a photocoordinate system is by-passed and a direct nonlinear relationship between coordinates of image points and the corresponding object space coordinates is derived. Fiducial marks are not required on the photographs.

When the elements of the transformation matrix are written in terms of the variables ω, ϕ, κ, there are nine unknown parameters in Equation 1. These are the exterior orientation elements X_o, Y_o, Z_o, ω, ϕ, κ, and the interior orientation elements x_o, y_o, and c. The interior orientation parameters of each photograph in the stereoscopic model are independent variables; thus, non-stability of the interior orientation parameters will have no effect on the accuracy of the solution. Therefore, a single photograph is sufficient for the solution of the interior orientation elements.

Statement of Problem

Let the model to be fitted to the data be

$$E(y) = f(x_1, x_2, \ldots, x_n;\ b_1, b_2, \ldots, b_k) = f(\mathbf{x}, \mathbf{b}) \quad (2)$$

where

- $E(y)$: dependent variable predicted by the equation,
- x_n: independent variables of the equation, and
- b_k: parameters of the equation.

Let the data points be denoted by

$$(YO_i, X_{1i}, X_{2i}, \ldots, X_{mi}) \qquad i = 1,2,\ldots,n \quad (3)$$

where YO_i is the vector of measured comparator coordinates of imaged control points. The problem is to compute those estimates of the parameters which will minimize

$$\Phi = \sum_{i=1}^{n} (YO_i - YP_i)^2 \quad (4)$$

where YP_i is the value of y predicted by Equation 2 at the ith data point. According to Marquardt (1963), it is well known that when $f(\mathbf{x},\mathbf{b})$ is a linear function of $\mathbf{b}$'s, the contours of constant Φ are ellipsoids, while if $f(\mathbf{x},\mathbf{b})$ is nonlinear, the contours are distorted according to the severity of the nonlinearty. Even with nonlinear models, however, the contours are nearly elliptical in the immediate vicinity of the minimum of Φ. Typically, the contour surface of Φ is greatly attenuated in some directions and elongated in others so that the minimum lies at the bottom of a long curving trough.

Methods in Photogrammetric Use

Least-squares estimation of the nonlinear collinearity condition model is based upon "An Algorithm for Least-Squares Estimation of Nonlinear Parameters" (Marquardt, 1963). This algorithm performs an optimum interpolation between the Gauss-Newton method and the gradient method. The interpolation is based upon the maximum neighborhood in which the truncated Taylor series gives an adequate representation of the nonlinear model. The Taylor series expansion for the collinearty equations is

$$\langle YP(X_i, Y_i, Z_i, \mathbf{b} + \boldsymbol{\delta}_t)\rangle = f(X_i, Y_i, Z_i, \mathbf{b}) + \sum_{j=1}^{9} \left(\frac{\partial E(y_i)}{\partial b_j}\right) (\boldsymbol{\delta}_t)_j \quad (5)$$

where object space coordinates of the control point X,Y,Z are independent variables at the ith data point and elements of $\mathbf{b}$ correspond to the unknown orientation parameters $x_o, y_o, c, \omega, \phi, \kappa, X_o, Y_o, Z_o$, respectively. The vector $\boldsymbol{\delta}_t$ contains a small correction to $\mathbf{b}$, with the subscript t used to designate $\boldsymbol{\delta}$ as calculated by the Taylor series method. The brackets < > are used to distinguish predictions based upon the linearized model from those determined from the actual nonlinear model. Thus, the value of Φ predicted by Equation 5 is

$$\langle\Phi\rangle = \sum_{i=1}^{n} (YO_i - \langle YP_i\rangle)^2 \quad [6]$$

$\boldsymbol{\delta}_t$ appears linearly in Equation 5, and can therefore be found by the standard least-squares method of setting $\partial\langle\Phi\rangle/\partial\delta_j = 0$, for all j. Thus, $\boldsymbol{\delta}_t$ is found by solving

$$\mathbf{A}\boldsymbol{\delta}_t = \mathbf{g} \quad (7)$$

where

$$\mathbf{A}_{9,9} = \mathbf{P}^T_{9,n}\,\mathbf{P}_{n,9} \quad (8)$$

$$\mathbf{P}_{n,9} = \left(\frac{\partial E(y_i)}{\partial b_j}\right) \quad i = 1,2,\ldots n;\ \ j = 1,2,\ldots,9 \quad (9)$$

$$\mathbf{g}_{9,1} = \left(\sum_{i=1}^{n} (YO_i - E(y)\,\frac{\partial E(y_i)}{\partial b_j}\right) \qquad j = 1,2,\ldots,9 \quad (10)$$

The gradient methods, by contrast, simply step off from the current trial value in the direction of the negative gradient of Φ. Thus,

$$\boldsymbol{\delta}_g = -\left(\frac{\partial\Phi}{\partial x_o}, \frac{\partial\Phi}{\partial y_o}, \frac{\partial\Phi}{\partial c}, \frac{\partial\Phi}{\partial\omega}, \frac{\partial\Phi}{\partial\phi}, \frac{\partial\Phi}{\partial\kappa}, \frac{\partial\Phi}{\partial X_o}, \frac{\partial\Phi}{\partial Y_o}, \frac{\partial\Phi}{\partial Z_o}\right)^T \quad (11)$$

According to Marquardt (1963), any proper method must result in a correction vector whose direction is within 90° of the negative gradient of Φ. Otherwise, the values of Φ can be expected to be larger rather than smaller at points along the correction vector. Also, because of the severe elongation of the Φ surface in most problems, $\boldsymbol{\delta}_t$ is usually almost 90° away from $\boldsymbol{\delta}_g$ (this angle, γ, usually falls in the range $80° < \gamma < 90°$).

The theoretical basis for the algorithm for least-squares estimation of nonlinear parameters is contained in several theorems (Marquardt, 1963).

THEOREM 1. Let $\lambda > 0$ be arbitrary and let $\boldsymbol{\delta}_o$ satisfy the equation

$$(\mathbf{A} - \lambda\mathbf{E})\boldsymbol{\delta}_o = \mathbf{g} \quad (12)$$

Then $\boldsymbol{\delta}_o$ minimizes Φ on the sphere whose radius $\|\boldsymbol{\delta}\|$ satisfies

$$\|\boldsymbol{\delta}\|^2 = \|\boldsymbol{\delta}_o\|^2$$

THEOREM 2. Let $\boldsymbol{\delta}(\lambda)$ be the solution of Equation 12 for a given value of λ. Then $\|\boldsymbol{\delta}(\lambda)\|^2$ is a continuous decreasing function of λ, such that as $\lambda \to \infty, \|\boldsymbol{\delta}(\lambda)\|^2 \to 0$.

THEOREM 3. Let γ be the angle between $\boldsymbol{\delta}_o$ and $\boldsymbol{\delta}_g$. Then λ is a continuous monotone decreasing function of λ such that as $\lambda \to \infty$, $\gamma \to 0$. Because $\boldsymbol{\delta}_g$ is independent of λ, it follows that $\boldsymbol{\delta}_o$ rotates toward $\boldsymbol{\delta}_g$ as $\lambda \to \infty$.

The relevant properties of the solution, $\boldsymbol{\delta}_t$, of Equation 7 are invariant under linear transformations of the $\mathbf{b}$-space. However, the properties of the gradient methods are not scale invariant. It becomes necessary, then, to scale the $\mathbf{b}$-space in the collinearity condition.

Thus, there can be defined a scaled matrix $\mathbf{A}^*$, and a scaled vector $\mathbf{g}^*$:

$$\mathbf{A}^* = (a^*_{jj'}) = \left[\frac{a_{jj'}}{\sqrt{a_{jj}}\sqrt{a_{j'j'}}}\right] \tag{13}$$

$$\mathbf{g}^* = (g^*_j) = \left[\frac{g_j}{\sqrt{a_{jj}}}\right] \tag{14}$$

At the rth iteration the equation

$$(\mathbf{A}^{*(r)} + \lambda^{(r)}\mathbf{E})\ \boldsymbol{\delta}^{*(r)} = \mathbf{g}^{*(r)} \tag{15}$$

is constructed. This equation is then solved for $\boldsymbol{\delta}^{*(r)}$. Then

$$\boldsymbol{\delta}_j = \boldsymbol{\delta}^*_j/\sqrt{a_{jj}} \tag{16}$$

is used to obtain $\boldsymbol{\delta}^{(r)}$. The new trial vector

$$\mathbf{b}^{(r+1)} = \mathbf{b}^{(r)} + \boldsymbol{\delta}^{(r)} \tag{17}$$

will lead to a new sum of squares $\Phi^{(r+1)}$. It is essential to select $\lambda^{(r)}$ such that

$$\Phi^{(r+1)} < \Phi^{(r)} \tag{18}$$

Let $\lambda^{(r-1)}$denote the value of from the previous iteration. Initially let $\lambda^{(o)} = 10^{-7}$.

Test Data

For the calibration procedure, a three-dimensional control field has been prepared, as shown in Figure 1. Horizontal and vertical control points were placed at 25-cm intervals. The test field consisted of 121 target points on different spatial planes. The bars which were used as control targets were all circular in shape, painted black, and had diameters of 5 and 10 mm. On the ends of these bars were circular, white targets 1 mm in diameter, painted white with a black dot on them, indicating the center of targets (Müftüoğlu, 1980). A similar control field was used by Wong and Ho (1986). Precise theodolite surveys were carried out to determine the object space coordinates of the control points. The mean standard error of X,Y,Z coordinates was 0.096 mm. Thus, coordinates of these fixed target points were assumed to be without error. This fact allows one to use these coordinate values as independent variables without error in the solution.

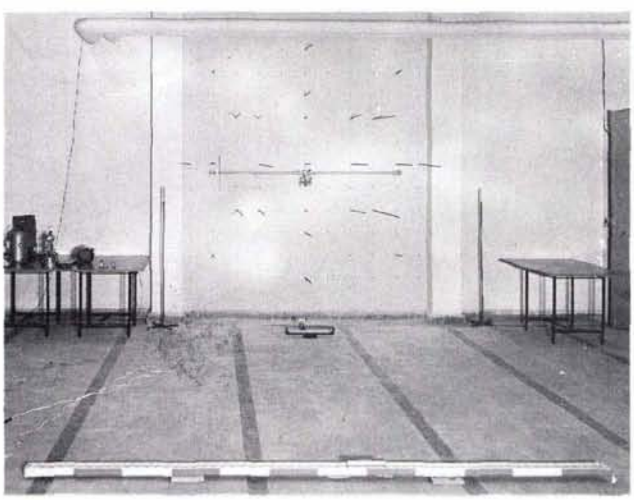

Figure 1. General Layout of the Test Field.

Digital image data were generated using a Hasselblad 500 C/M non-metric camera equipped with an 80-mm f/2.8 lens. The photographic materials were OR WO NP 20 negative film and Kodak Microdol-X developing. The photography was taken with an exposure time of 1/60 sec at f/5.6 by using a special system developed by Müftüoğlu (1986).

This system is designed to take photographs in the normal case of terrestrial photogrammetry, that is, capable at sliding on two steel bars ranging from 1 mm to 400 mm. Also, it overcomes the problems related to camera position, camera-object distance, and the orientation of the non-metric camera image plane. The resulting photographs were taken with a 400-mm base length and the camera was focused at 8.0 m.

Measurements of comparator coordinates for images of target points were performed while viewing the pair of negative films stereoscopically on a Zeiss PSK2 stereocomparator. Then, comparator coordinates were separately obtained for the left and right negatives, but only one of these was used as dependent variables in the proposed solution procedure. The measured comparator coordinates of the target points contain systematic and random errors.

In this solution procedure, the program iteratively ad-

Table 1. Sample Output

	Comparator Coordinates		Object Space Coordinates		
No.	x_o	y_o	X	Y	Z
1	496.673	518.251	10616.189	14128.544	11375.714
2	503.357	518.271	11115.013	14137.301	11375.581
3	506.674	518.283	11363.578	14130.116	11373.795
4	513.774	519.396	11878.082	13657.068	11348.848
5	520.048	518.300	12362.246	14137.532	11376.443
6	530.035	518.230	13114.475	14139.879	13374.401
7	499.731	515.242	10874.691	13910.576	11108.136
8	506.695	514.944	11364.860	14139.020	11125.336
9	513.351	514.959	11861.325	14132.241	11124.782
10	520.076	514.918	12363.477	14136.655	11122.639
11	496.665	511.616	10616.918	14128.145	10877.547
12	499.977	511.562	10863.057	14134.697	10873.498
13	506.700	511.612	11365.504	14134.138	10874.441
14	523.425	511.521	12615.403	14138.726	10870.868
15	530.107	511.546	13114.698	14139.633	10870.867
16	502.076	509.314	11124.474	13156.539	10597.228
17	513.446	508.539	11860.404	13922.871	10626.087
18	520.910	509.159	12336.210	13413.275	10614.300
19	526.815	508.202	12867.679	14144.968	10621.030
20	496.673	504.907	10618.171	14134.035	10377.530
21	503.348	504.840	11115.554	14133.262	10370.930
22	510.078	504.837	11615.250	14133.206	10368.738
23	520.077	504.844	12363.401	14130.328	10366.801
24	526.736	504.869	12864.068	14146.190	10370.815
25	530.095	504.920	13112.085	14138.771	10373.606
26	493.752	501.540	10606.137	13149.764	10111.347
27	503.343	501.547	11113.641	14134.693	10125.977
28	506.923	501.165	11430.420	13153.201	10085.140
29	513.594	501.604	11862.072	13671.645	10121.621
30	523.502	501.555	12614.546	14138.263	10123.373
31	503.381	498.127	11116.287	14147.524	9870.220
32	510.073	498.141	11615.503	14146.797	9871.195
33	516.809	498.099	12115.626	14133.097	9867.268
34	520.191	498.071	12365.879	14131.843	9865.313
35	526.859	498.208	12866.149	14129.808	9873.245
36	530.205	498.153	13115.549	14125.232	9868.723
37	496.733	494.834	10619.253	14127.352	9627.042
38	513.537	494.533	11865.258	13926.112	9616.636
39	526.415	493.366	12642.864	13139.209	9599.158
40	520.195	494.859	13114.255	14127.090	9623.583

TABLE 2. SAMPLE OUTPUT

No. iterations	using 20 points 7	using 30 points 6	using 40 points 6
x_o	511.39060 ∓0.26734	511.45138 ∓0.18011	511.37882 ∓0.14611
y_o	502.09081 ∓0.38480	501.49579 ∓0.20240	501.50907 ∓0.18314
c	82.23882 ∓0.62729	81.76552 ∓0.44764	81.57414 ∓0.29503
ϕ	−0.00618694 ∓0.00272	−0.00665851 ∓0.00225	−0.00552819 ∓0.00213
ω	−0.02157788 ∓0.00447	−0.01400837 ∓0.00225	−0.01411448 ∓0.00213
κ	−0.00169970 ∓0.00045	−0.00186204 ∓0.00025	−0.00177197 ∓0.00022
X_o	11675.377 ∓5.59503	11676.890 ∓5.18782	11678.695 ∓3.39567
Y_o	8012.096 ∓45.50060	8038.237 ∓32.71853	8052.654 ∓21.03495
Z_o	10033.035 ∓7.68775	10035.1263 ∓5.15481	10035.692 ∓3.45035
Standard Error of Estimate	0.01588	0.01762	0.01752

justs the interior and exterior orientation elements to minimize the function (Equation 6). The new comparator coordinates of the target points, minimized with respect to the function (Equation 6) and containing random and systematic errors, will be calculated in the next or final phase of iteration.

Results and Discussions

The nonlinear least-squares estimation routine contained in the program was written in FORTRAN IV language (Clements and Schnelle, 1969). This program has been adapted to personal computers by using standard FORTRAN language.

Real data obtained from the test field are given in Table 1. The interior and exterior orinetation elements calculated by the suggested computation procedure are presented in Table 2 using different numbers of control points. Note that 20 control points are good enough to estimate the orientation elements, when the standard errors of estimated parameters are compared.

In order to compare the presented procedure to the well known DLT method, an example prepared using the data of Marzan and Karara (1975) and the results of this simulation are given on Table 3. The comparison of the two methods for the interior orinetation elements is shown on Table 3. Because the exterior orientation elements were not presented in the Marzan and Karara (1975) studies, no comparison is possible for these elements.

Conclusion

In this paper, a computation procedure for calculation of both the interior and exterior orientation elements from a single photograph, through the use of the collinearity condition equations and a three-dimensional test field, is presented. This model can work very well for non-metric cameras for the following reasons:

- The fiducial marks are not required on the photographs.
- The computation time for the algorithm is short, and the values of the parameters converge to their true values in a few iterations, provided that suitable initial estimates are utilized. However, the use of very close estimates is not necessary for the convergence of the algorithm.
- Although the procedure has been developed for non-metric cameras, it can be applied to metric cameras for checking the interior orientation elements.

TABLE 3. COMPARISON TO THE DLT METHOD

Parameters	Suggested calibration procedure	DLT Method
x_o	1.06425 ∓0.07097	1.0337
y_o	0.98251 ∓0.04922	1.0125
c	100.49292 ∓0.07596	100.5368
ϕ	0.03555882	
φ	−0.03468423	
κ	−0.00001737	
X_o	9304.245	
Y_o	8095.382	
Z_o	15001.101	
Standard Error of Estimate	0.00451	

In architectural photogrammetry, interior and exterior orinetation elements calculated for the photographs by using this procedure can be used in preliminary studies for restoration and improvement, in inventory work, and in the study of the history of art.

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