PRACTICAL PAPER

Some Equations for the Space Oblique Mercator Projection

Abstract

The Space Oblique Mercator (SOM) projection is found most suitable for continuous mapping of satellite imagery. Simple working equations with modification in the solution of λ'' and constants for the Clarke and Everest spheroids are produced here for facilitating calculations and improving working accessibility to the general user.

INTRODUCTION

The Space Oblique Mercator (SOM) projection, developed by John P. Snyder in 1978 and later modified in 1987 (Snyder, 1978; Snyder, 1987), is the only map projection suitable for continuous mapping of satellite imagery. Although over a decade has elapsed since the evaluation of the SOM mathematical formulation, the general user still does not have access to simplified working equations. Despite the SOM projection's usefulness, conventional conformal map projections such as the Lambert and UTM are generally used to map satellite images, primarily because conventional projections are used for surveying purposes and compilation of topographical maps. Thus, a need arises to provide simple working equations to cartographers and thematic resource scientists to encourage the use of the SOM projection to yield results of optimum accuracy.

Snyder's equations, with a modified approach to calculate the first estimate of λ'' (longitude from the ascending node), are presented here. Use of a few of these equations may be avoided by directly using the constant parameters, given in Tables 1 and 2 for Landsat, SPOT, and IRS-1 satellites for the Clarke and Everest spheroides, respectively.

FCC film products of Landsat TM and IRS-1A LISS-II sensors are also evaluated for geometric fidelity against rectangular coordinates of Ground Control Points (GCPs) in the Lambert conical orthomorphic, UTM, and SOM projection systems.

Notations

- ϕ, λ Geodetic latitude and longitude of the point
- ϕ'' Transformed latitude measured from the ground track, positive eastward
- λ'' Transformed longitude measured from the ascending node
- λ_t Satellite apparent longitude
- λ_0 Geodetic longitude of the ascending node
- i Inclination of the satellite orbit
- e Eccentricity of the spheroid used
- P_2 Temporal resolution of satellite
- P_1 Number of revolutions on P_2 days
- p satellite path number

Photogrammetric Engineering & Remote Sensing,
Vol. 59, No. 5, May 1993, pp. 667–669.

0099-1112/93/5905–667$03.00/0

TABLE 1. CONSTANTS FOR THE CLARKE (1866) SPHEROID

constant	Landsat 1, 2, 3	Landsat 4, 5	SPOT-1	IRS-1
B	1.00579814	1.00456031	1.00524785	1.00570466
A_2	−0.00109791	−0.00094250	−0.00103350	−0.00109558
A_4	−0.00000130	−0.00000127	−0.00000129	−0.00000130
C_1	0.14344099	0.13759268	0.14104768	0.14335910
C_3	0.00002851	0.00002995	0.00002915	0.00002855
P_2/P_1	18/251	16/233	26/369	22/307
J	<-------- 0.97983115 -------->			

TABLE 2. CONSTANTS FOR THE EVEREST SPHEROID

constant	Landsat 1, 2, 3	Landsat 4, 5	SPOT-1	IRS-1
B	1.00583237	1.00459449	1.00528207	1.00573890
A_2	−0.00111308	−0.00095780	−0.00104872	−0.00111075
A_4	−0.00000125	−0.00000123	−0.00000125	−0.00000125
C_1	0.14341591	0.13756864	0.14102302	0.14333404
C_3	0.00002737	0.00002885	0.00002802	0.00002741
P_2/P_1	18/251	16/233	26/369	22/307
J	<-------- 0.98021835 -------->			

Simplified Working Equations

The SOM projection is the only rigorous near conformal projection which provides continuous mapping of satellite imagery of the rotating Earth, true to scale along the ground track. The scale 1° away from ground track averages 0.015 percent greater than that at the ground track. It differs from the Oblique Mercator projection in that the central line (the ground track of the orbiting satellite) is slightly curved. It appears as a nearly sinusoidal curve crossing the X axis at an angle of about 8° (Snyder, 1981; Snyder, 1987).

The geodetic coordinates of Ground Control Points (GCPs) are converted into the SOM system by Snyder's equations (Snyder, 1987), which are presented here, together with some constant parameters for the Clarke (Table 1) and Everest (Table 2) spheroids.

$$x/a = B\lambda'' + A_2 \sin(2\lambda'') + A_4 \sin(4\lambda'') - [S/(J^2 + S^2)^{0.5}] \ln \tan(\pi/4 + \phi''/2) \tag{1}$$

$$y/a = C_1 \sin \lambda'' + C_3 \sin(3\lambda'') + [J/(J^2 + S^2)^{0.5}] \ln \tan(\pi/4 + \phi''/2) \tag{2}$$

where

$$\lambda'' = \arctan[\cos i \tan \lambda_t + (1 - e^2)\sin i \tan \phi/\cos \lambda_t] \tag{3}$$

$$\lambda_t = \lambda - \lambda_0 + (P_2/P_1)\lambda'' \tag{4}$$

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TABLE 3. STANDARD ERRORS OF POSITIONAL DISCREPANCIES (GROUND METRES)

GCPs	Check points	Conformal			Affine			Polynomial		
		Lamb.	UTM	SOM	Lamb.	UTM	SOM	Lamb.	UTM	SOM
TM FCC 10	36	102	108	98	30	30	29	19	20	19
LISS-2 FCC 09	23	99	103	101	41	40	40	27	29	28

$$\phi'' = \arc \sin\{[(1 - e^2)\cos i \sin \phi - \sin i \cos \phi \sin \lambda_t]/(1 - e^2 \sin^2\phi)^{0.5}\} \quad (5)$$

$$S = (P_2/P_1)\sin i \cos \lambda''[(1 + T \sin^2\lambda'')/\{(1 + W \sin^2\lambda'')(1 + Q \sin^2\lambda'')\}]^{0.5} \quad (6)$$

$$W = [(1 - e^2 \cos^2 i)^2/(1 - e^2)^2] - 1 \quad (7)$$

$$Q = e^2 \sin^2 i/(1 - e^2) \quad (8)$$

$$T = e^2 \sin^2 i(2 - e^2)/(1 - e^2)^2 \quad (9)$$

$$B = (2/\pi) \int_0^{\pi/2} [(HJ - S^2)/(J^2 + S^2)^{0.5}]d\lambda'' \quad (10)$$

$$A_n = [4/(\pi n)] \int_0^{\pi/2} [(HJ - S^2)/(J^2 + S^2)^{0.5}]\cos n\lambda''\, d\lambda'' \quad (11)$$

$$C_n = [4/(\pi n)] \int_0^{\pi/2} [S(H + J)/(J^2 + S^2)^{0.5}]\cos n\lambda''\, d\lambda'' \quad (12)$$

$$J = (1 - e^2)^3 \quad (13)$$

$$H = [(1 + Q \sin^2\lambda'')/(1 + W \sin^2\lambda'')]^{0.5} [\{(1 + W \sin^2\lambda'')/(1 + Q \sin^2\lambda'')^2\} - (P_2/P_1)\cos i] \quad (14)$$

and

$$\begin{aligned} \lambda_0 &= 128.87 - (360°/251)p \text{ (Landsat 1,2,3 only)} \\ \lambda_0 &= 129.30 - (360°/233)p \text{ (Landsat 4,5 only)} \\ \lambda_0 &= 161.95 - (360°/369)p \text{ (SPOT only)} \\ \lambda_0 &= 295.90 - (360°/307)p \text{ (IRS-1 only)} \end{aligned} \quad (15)$$

Equations 3 and 4 are to be iterated together to get the final value of λ''. In remote sensing, while dealing with the data of Landsat, SPOT, and IRS-1 a satellites, images of the Earth's surface are acquired during day time satellite passes only. The value of λ'' will be either in the second or third quadrant, depending upon the geodetic latitude of the point, being positive (northern hemisphere) or negative (southern hemisphere). Hence, the general approach for calculating a first estimate of λ'' (Snyder, 1987) may be replaced by taking the first estimate equal to $(180 - \phi)$ degrees, where ϕ is the geodetic latitude of the point (+ve in northern and −ve in southern hemispheres), whose rectangular coordinates have to be converted into the SOM system. It saves the iterations, needed in solving Equations 3 and 4, by one. In addition, it is good from a conceptual standpoint in understanding the meaning and calculation of λ''. Because the computer normally calculates arctan as an angle between −90° and +90°, a quadrant adjustment is applied to λ'' by adding 180° to the calculated value of λ'', and this λ'' is used as next trial λ''. Normally, three to four iterations are required for seven decimal place accuracy.

The constants B, A_n, and C_n need to be computed only once for a given satellite. Hence, Equations 10 to 14 are replaced by their constant values, using the Clarke 1866 ellipsoid (a = 6,378,206.4 m and e^2 = 0.00676866) and Everest ellipsoid (a = 6,377,301.24 m and e^2 = 0.006637847). These constants, for Landsat, SPOT, and IRS-1 satellites, are given in Tables 1 and 2 for the Clarke (1866) and Everest ellipsoids, respectively. The B values are for the λ'' in radians.

Analysis and Results

Landsat TM and IRS LISS-1 FCC film products were analyzed for cartographic accuracy using the GCPs, whose rectangular coordinates were derived in the Lambert conical orthomorphic, UTM, and SOM projection systems. The standard parallel for the Lambert projection and the central meridian for the UTM projection, were selected such that the true scale is maintained within the central portion of the imagery. Standard techniques of two-dimensional conformal and affine (Wolf, 1983) and second-order polynomial (Kennie, 1990) transformations were used for the analysis.

Initially, 50 control points on TM FCC and 35 on LISS-II FCC were selected by correlating enlarged views, using a Wild Heerbrugg APT-1 stereoscopic instrument, with existing 1:50,000-scale topographic maps. Points selected were unmistakeably identifiable sharp linear features, such as junctions of canals, roadways, and railways. Image coordinates of all control points, so selected, were measured on a comparator.

Ground coordinates, in terms of geodetic latitude and longitude, of all control points were derived from existing 1:50,000-scale topographic maps. These were then converted to rectangular coordinates in the Lambert, UTM, and SOM projection systems.

Ten control points for Landsat TM and nine for IRS-1 LISS-II were used as GCPs to determine transformation coefficients of the conformal, affine, and polynomial transformations. The rest of the points were treated as check points to compare ground coordinates calculated by using the transformation coefficients, against their map derived coordinates. The standard error (s.e.) of the positional discrepancies at check points, shown in Table 3, was calculated by rejecting the points showing errors more than three times the standard error.

Results using SOM rectangular coordinates are comparable to those obtained with Lambert and UTM coordinates. Because the SOM projection maintains the scale along ground track, it can efficiently be used for geometric corrections of satellite data of complete path as a whole.

Conclusion

The present equations, with the modification to calculate the first estimate of λ'', may be helpful to the general user in working with the SOM projection. Constant values on the Everest spheroid may be useful to resource scientists of the Indian sub-continent.

Acknowledgment

The author is thankful to Prof. Snyder for his cooperation and acceptance of the change in calculating the first estimate of λ'', without which this work could not have been possible.

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(Received 4 May 1992; revised and accepted 18 November, 1992)