Application of GPS for Aerial Triangulation

Abstract

The paper reviews the actual status of GPS *camera positioning for aerial triangulation applications. The special strategy of allowing linear* GPS *drift errors and correcting them subsequently in a combined block adjustment is presented, and its operational features are discussed. Theoretical and empirical results of combined block adjustment show the high accuracy potential of* GPS*-supported aerial triangulation. A review of practical projects confirms the economic benefit of the method and demonstrates that it is ready for practical application.*

Recalling Status and Performance of Kinematic GPS

GPS in Photogrammetry

The NAVSTAR Global Positioning System (GPS) has become generally available and can be considered operational on a world wide basis. GPS already exercises a revolutionary impact in various disciplines which are concerned with navigation, movement monitoring, and positioning. It is thoroughly changing geodetic positioning methods in particular. Also aerial photogrammetry has started to use GPS widely, and similarly revolutionary effects are envisaged.

There are three main fields in photogrammetry in which the application of GPS is going to have great impact:

- GPS-controlled survey flight navigation;
- High precision camera positioning, especially for aerial triangulation; and
- Positioning of other airborne sensors.

This paper does not deal with GPS flight navigation. It may suffice to say that quite a number of systems are in practical operation today, based on C/A-code pseudorange measurements and data processing in real time. This paper is particularly concerned with high precision relative kinematic camera positioning by differential GPS for aerial triangulation purposes.

Operational Conditions

The well known principles of GPS positioning are not reviewed here. We recall briefly that the GPS L1/L2 carrier waves with the C/A-code and P-code (now SPS- and PPS-code) modulations, allow basically two kinds of distance measurements: Pseudo-ranging by C/A- or P-code signals with real time capability, and the more precise phase measurements on the carrier waves. The latter have the problem of initial phase ambiguity, and normally require post-processing. The C/A-code signals and the carrier waves can be degraded by selective availability (SA) perturbations. Camera positioning for aerial triangulation normally uses phase observations in differential mode, for accuracy reasons.

The accuracy potential of differential phase observations is very high. Standard deviations for range observations of < 2 mm have been confirmed and are regularly obtained. Calculating positions from ranging is essentially a straight forward procedure of least-squares distance intersection. The positioning precision depends primarily on the geometry of the satellite constellation. Today, the space segment being almost complete, constellations of at least six simultaneously visible GPS satellites can be observed almost anywhere and at any time, giving PDOP values ≤ 6. Thus, except for systematic errors, the internal positioning precision is expected to be in the order of 1 to 2 cm (standard coordinate errors). It is difficult to empirically check and verify such high precision airborne positioning. But there are test results from the Flevoland aerial triangulation (Frieß, 1991) which empirically confirm a precision of roughly 2 cm for kinematic aerial camera positioning. According to available experience, the inherent precision of differential carrier phase observations are little affected by SA.

GPS hardware will not be discussed here. Suffice it to say that quite a number of GPS antennae, receivers, and recording equipment are commercially available which fully serve the purpose. At present, most receivers used for aerial photogrammetry record only the L1 carrier phase, together with C/A-code pseudoranges and the satellite message. Such standards are sufficient for photogrammetric applications, although the current development on dual frequency receivers, P-code, Y-code, and more sophisticated processing, will improve conditions in the future. Normally photogrammetric requirements do not ask for camera positioning precision to a few centimetres. Standard errors in the order of 0.1 m to 1 metres are often sufficient, depending on the applied map scales.

The operational circumstances of photogrammetric GPS applications are not quite favorable. Usually only one stationary receiver on the ground and one receiver in the airplane are used, which has prevented, up to now, any sophisticated error modeling, like in geodetic networks with multistation GPS. The distance between the stationary GPS receiver, preferably located at the airport or a home base, and the mission area can be large, i.e., up to several hundred kilometres. And the duration of a flight mission may extend over several hours.

The recording rate of the GPS observations must be as high as 1 Hertz or more, as the speed of a survey aircraft is on the order of 50 to 100 metres per second. GPS positions at such intervals must be interpolated onto the camera stations in between.

Aerial triangulation does not require the immediate availability of GPS data. Therefore, the GPS observations are only recorded for post-processing. Nevertheless, quick GPS processing is desirable, for immediate acceptance verification.

Ambiguity Solution and Drift Errors

Carrier phase observations have the problem that at the beginning the total number of integer cycles the signal has trav-

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eled through from the satellite to the receiver is unknown. The required initial ambiguity solution can be obtained by starting from a known GPS-determined baseline. The determination of a GPS baseline, before take-off, used to take at least one hour of stationary recordings to the same satellites by both receivers. That situation has been eased, recently, as the GPS observing windows got larger and more sophisticated processing algorithms have reduced the recording time to perhaps 10 minutes or less. After the initial ambiguity solution, the GPS receivers stay locked onto the carrier waves, and the kinematic positioning can continuously track the flight trajectory of the moving GPS antenna, until an interruption or loss of signal occurs. Any interruption means that the ambiguity solution is lost and must be restored, if the trajectory is to be continued. Kinematic GPS positioning is quite sensitive to inaccurate ambiguity solutions. Erroneous solutions lead to systematic drift errors in the subsequent GPS positioning. Errors of 1 or 2 cycles only (1 cycle ≈ 19 cm on L1 carrier) can generate positioning errors of 0.5 metres after perhaps 15 minutes of flying, depending on the distance between the receivers (Schade, 1992).

There are other causes of drift errors or systematic errors in differential kinematic GPS positioning, due to incomplete cancellation of the originally very large error effects by the differential mode. Of special concern are the ionospheric effects, if just single frequency receivers are used, and orbital errors, if the distance between the receivers is large and if the flying times are long.

The mentioned error effects are likely to increase linearly at the beginning, but may turn into nonlinearity after some time. In practically all empirical investigations, constant or linear GPS drift errors have been observed. Thus, some drift errors have to be generally expected in kinematic GPS camera positioning, under the described circumstances. The real question is, therefore, how to suppress them or compensate for them or how to deal with them if they cannot be avoided. A special case is to keep them small enough to be linear, which will be of particular importance for aerial triangulation.

Some More Problems

There are a number of additional problems which have to be handled in GPS positioning for aerial triangulation. Trivial ones are the eccentricity between GPS antenna and camera and the time offsets between the GPS recordings and the actual exposures of the camera. The GPS positions, referring to the antenna phase center, have to be reduced onto the camera perspective center (external node). The coordinate corrections should relate to the external coordinate system. They depend, therefore, on the attitude of the aircraft. In combination with aerial triangulation, there is an easy solution, as the camera orientation parameters can be estimated from an initial block adjustment run. For that case, it is advisable to directly measure the spatial offset in the airplane on the ground with respect to the image coordinate system of the camera. The camera would then have to be kept in zero position during the flight, or the settings, especially the crab angle, should be recorded. The corrections can be neglected (2 m / 1° ≈ 3.5 cm), except for a constant vertical offset, if the antenna is mounted vertically above the camera. Before the spatial eccentricity of the GPS antenna is corrected, its position at the moment of camera exposure must be interpolated from the neighboring antenna positions. The time relation must be accurate to about 1 msec or better (1 msec ≈ 6 cm at 60 m/sec). The deviation of the actual flight path from the linear (or other) interpolation remains an unmodeled GPS position error.

A more serious source of problems are GPS signal disruptions or loss of signals during the flight, which cause a loss of lock and spoil the previous ambiguity solutions. Such events caused by cycle slips or constellation changes happen from time to time. They can be detected and corrected during data processing. Loss of signals due to obstruction by wings or fuselage of an aircraft flying a turn can, however, last for tens of seconds. In such cases it is practically impossible to precisely reconnect the interrupted trajectory by new ambiguity solutions.

Finally, the datum problem has to be mentioned. GPS positioning generally refers to the WGS 84 reference, an Earth-centered Cartesian coordinate system. Results of the aerial triangulation and of subsequent mapping, however, are normally given in a national or local reference system. There is a transformation problem, as the relations between the systems are not sufficiently constant. Also, the geodetic map projections (like UTM) are non linear, and the geoid as the vertical datum poses a particular problem.

GPS for Aerial Triangulation

The application of GPS for aerial triangulation means simply the determination of camera station coordinates by high precision differential GPS positioning and the use of such data in a combined block adjustment. The aim is to improve efficiency by avoiding ground control points almost completely. The problem areas as identified in the introductory chapter will now be discussed in more detail with special regard to aerial triangulation.

GPS Signal Interruptions

The ideal scenario of GPS camera positioning for aerial triangulation would be complete and uninterrupted tracking of the flight trajectory throughout the flight mission, after initial ambiguity solution by a stationary baseline. In addition, any errors, especially systematic errors, should be small enough to be negligible or, at the most, to be sufficiently captured via closing errors by a post mission stationary baseline determination. Unfortunately, that ideal scenario has its problems. There is especially the risk of signal interruptions during flight turns, although short interruptions, cycle slips, or constellation changes can be bridged and remedied by software methods and do not constitute major problems. Serious signal interruptions can be handled in three ways:

- The common approach tries to avoid major signal interruptions by carefully slipping the aircraft through the turns with low banking angles. It is hoped, in this way, that no interruptions occur or that a sufficient number of satellites can be tracked through to maintain the trajectory. That method is not favored by pilots. The turns take considerably longer than normal, using up valuable flying time, and signal interruptions may happen despite all precautions taken.
- There is considerable research and development in this field, with the aim of computationally restoring absolute ambiguity solutions which were lost through shorter or longer signal interruptions. The methods are known as *"fast ambiguity solution"* or *"ambiguity solution on the fly,"* also referred to as OTF methods. They start from precise pseudorange solutions and search numerically for the actual ambiguity solution by intelligent trial and error techniques. Such methods are likely to represent the future solution to the problem. However, the initial pseudorange solutions must be accurate to around 2 m or better, preferably 0.5 m. Thus, dual frequency receivers will be required, with P-code capability or Y-code code correlation techniques. Whether also C/A-code data can be pushed that far remains to be seen.
- There is a third much safer and highly pragmatic method for re-establishing ambiguity solutions. It can, however, be ap-

plied in connection with aerial triangulation only. After signal interruption, any new ambiguity solution is reassessed by locking on new coarse GPS positions derived from C/A-code pseudoranges. Because of the inaccuracy of C/A-code pseudorange positioning, the new ambiguity solutions are biased. Biased ambiguity solutions generate GPS drift errors as described earlier. The biased solutions are corrected later on, in connection with the aerial triangulation.

Acceptance and Correction of Systematic GPS Errors

The described principle of simplified ambiguity solutions does not seem to yield any advantage, because of the resulting systematic GPS positioning errors. However, the picture is changing, if the systematic GPS errors can be assessed and corrected later, as is the case in combination with aerial triangulation. That approach to GPS-supported aerial triangulation will be given special attention in the following sections of this paper. It is considered highly practical, and has been thoroughly and successfully tested in practice. Also it is ready for practical application. It is characterized by simplified and possibly biased ambiguity solutions, based on straightforward C/A-code pseudorange positioning, and temporary acceptance of potential linear drift errors which will be corrected in the combined block adjustment.

The approach has the side effect that the correction of other unmodeled systematic GPS errors is implied. There is only the condition that the systematic drift errors of GPS positioning are small enough to be linear. That condition can safely be met if the ambiguity solutions are assessed for each photo strip separately, independent of whether signal interruptions have occurred or not. The flying times for single photo strips extend normally over 5 to 15 minutes only. All available empirical evidence confirms that the drift errors of differential GPS positioning are generally quite linear over such short time intervals.

The acceptance of linear GPS drift corrections per photostrip has the consequence that the distance between the stationary receiver and the mission area can be quite large. In a recent case a second stationary receiver was placed at a distance of more than 400 km. And it was empirically confirmed that there was no noticeable accuracy deterioration after the application of linear drift corrections. We conclude, therefore, that the stationary receiver may be placed 500 km away or more, and will still give highly accurate results with the described method. The possibility of placing the stationary receiver at a great distance can be of great economic importance in practice.

There are some operational advantages related to the acceptance of GPS signal interruptions and the application of subsequent drift corrections. First, we do not ask for any stationary baseline determination before take off, nor any initial ambiguity solution. The GPS receivers need only be switched on when the mission area is reached. And as described, the stationary receiver may be far away. Also, the pilot can fly as usual, with fast turns with steep banking angles, as loss of phase lock need not be avoided. It is evident that the described method will only be stabilized and simplified, if the OTF methods will reach a stage of reliable practical performance, sooner or later. Drift corrections may then be reduced to constant offset corrections or to one set of linear parameters for the entire block.

Datum and Ground Control

The datum transformation from WGS 84 to a national horizontal and vertical reference system is a geodetic problem, but GPS aerial triangulation is directly concerned with it. It is possible to execute the GPS aerial triangulation in the GPS reference system WGS 84 and apply, if required, the (UTM) map projection and the vertical datum transformation thereafter. Unfortunately, a direct transformation employing an absolute transformation formula is not possible because the WGS 84, or rather its transfer through the satellite orbits, is not stable enough. (Permanent GPS stations, as scheduled in some countries, may solve the problem in future.) Therefore, the datum transformation must rely on some ground control points for the time being.

It is possible, in principle, to carry out the GPS aerial triangulation without any photo control within the block area, i.e., to rely on outside control for the datum transformation. Such control points must be given in both the WGS 84 and the national reference system. It may be sufficient even to use only one control point on which the GPS ground receiver would be stationed, as all other transformation parameters might be derived precisely enough from the given geographical location. There is the condition, however, that the complete GPS flight trajectory be recorded continously without any interruption or be restored to that state. Under such conditions the photo block is determined and tied to the GPS camera air stations. In that case, one faces the problem of unmodeled errors of interior camera orientation, the effects of which can be quite considerable and are fully imposed on the points of the block. The possibilities for system calibration by GPS test fields are not considered here any further. We are not convinced of the practicality of the method, as the calibration and the flight mission are usually not performed under identical conditions.

We present hereafter an alternative approach to GPS aerial triangulation, the execution of which is simple and robust and which has been shown to work very well in practice, under standard operational conditions. The method does, however, require and rely on some conventional ground control points. The standard suggestion is four control points located more or less in the corners of a block or of a strip, in case single strips are used. The control points must be given in the national horizontal and vertical coordinate system and be measured in the aerial photographs. They are not required to be given in the GPS WGS 84 system. The use of some few ground control points is considered practical and demandable. It solves the datum transformation conveniently and circumvents the interior orientation problem.

Finally, there is the problem of the geoid as vertical datum. It constitutes a general problem with GPS, as well as in geodetic applications. In our suggested scenario of GPS blocks with four ground control points, it is easy to define and fit a spherical or ellipsoidal surface as vertical datum to the control points. If, however, the geoid is to be taken into account, it must be given explicitly, in order to be superimposed as an actual vertical reference surface. In conventional aerial triangulation, the problem has been less serious, as the geoid was implicitly represented by the usually large number of vertical control points. The high accuracy performance of GPS aerial triangulation for large-scale applications requires high precision geoids at the 10-cm level for vertical accuracy.

Combined Block Adjustment

Input

According to the basic concept of GPS aerial triangulation, the GPS camera station positions represent observations to the unknown perspective centers of the aerial photographs and are treated in a combined block adjustment together with the conventionally measured photogrammetric observations (image coordinates). In the approach described and ap-

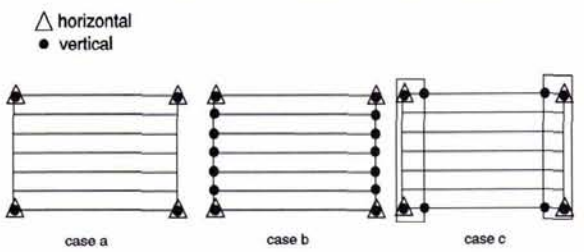

Figure 1. Control scenarios for GPS blocks.

plied here, the unknown parameters for stripwise linear GPS drift corrections are introduced into the combined block adjustment and solved for, together with all other unknowns.

The differential GPS observations are assumed to be processed to preliminary WGS 84 camera station coordinates and rigorously transformed into an approximate national or local reference frame (Cartesian or non-Cartesian). The GPS camera stations are given and will be treated separately per photo strip (or sub-strip if necessary), referring to separate ambiguity solutions. The spatial offset reductions (from GPS antenna to camera) are applied in the course of the adjustment iterations, which provide the required attitude parameters. Such corrected and approximately transformed GPS camera position coordinates, adequately weighted, constitute the GPS input to the final run of the combined adjustment.

As in conventional analytical aerial triangulation, the input from the photogrammetric side is reduced image coordinates. It is emphasized that the photogrammetric aerial triangulation is identical in all respects with the conventional procedure, especially as far as number, distribution, and transfer of tie points is concerned. However, GPS blocks contain normally a few ground control points only. They complete the data input to the combined block adjustment. The ground control coordinates refer to the national horizontal and vertical reference systems. They may be treated as constants or, preferably, as weighted observations.

Block Adjustment

The observation equations for the photogrammetric image coordinate observations are exactly the same as in conventional analytical aerial triangulation. The GPS camera station coordi-nates are brought into the adjustment by additional observation equations which relate directly to the unknown perspective center coordinates through which both groups of observations are linked together. The special approach with linear GPS drift corrections at the block adjustment stage is realized by adding correction terms accordingly, which leads to the following type of GPS observation equations for an exposure station j:

$$\begin{bmatrix} X_j^{GPS} \\ Y_j^{GPS} \\ Z_j^{GPS} \end{bmatrix} + \begin{bmatrix} \nu_x^{GPS} \\ \nu_y^{GPS} \\ \nu_z^{GPS} \end{bmatrix} = \begin{bmatrix} X_j^{PC} \\ Y_j^{PC} \\ Z_j^{PC} \end{bmatrix} + \mathbf{R}\,(\omega, \varphi, \kappa) \cdot \begin{bmatrix} d_x \\ d_y \\ d_z \end{bmatrix}$$

$$+ \left(\begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix} + \begin{bmatrix} b_x \\ b_y \\ b_z \end{bmatrix} \ast (t - t_0) \right)_k$$

The equations contain the observed GPS camera station coordinates and their least-squares corrections, the unknown coordinates of the perspective centers (PC) and the drift corrections with the unknown parameters a_x, a_y, ..., b_z which are common for all observation equations of a photo strip k. Further, the spatial offset between camera and antenna [dx, dy, dz] has to be considered and transformed with the rotations around ω, φ, and κ.

The structure of the combined observation equation matrix is quite clear, and processing to equally structured least-squares normal equations is straightforward. The standard block adjustment programs need only be extended to incorporate the additional unknown drift parameters. Well known numerical solution techniques can be applied accordingly.

The remaining datum transformation from the transformed WGS 84 into the national reference system, as defined by the ground control points of the block, is accomplished by a linearized seven-parameter transformation, the coefficients of which are also treated and solved for as unknowns in the combined block adjustment. The separate formulation and solution of the datum transfer parameters can be omitted, however, if the approach with linear drift parameters per strip is used, the corrections automatically include the datum transformation.

The numerical least-squares solution of all unknowns of the combined block adjustment is straightforward, provided there are no singularities (see next section). Statistical parameters are also derived which allow a quality assessment of the results. These are, in particular, the residuals ν of all observations, the variance factor σ_0^2, and the standard errors (or the complete correlation matrix) of all adjusted coordinates and orientation parameters. The photogrammetric block adjustment part is the same as in conventional aerial triangulation, in every respect. This includes that both the analytical bundle method, as described here, and the independent model method can be used for the combined adjustment. It also means that block adjustment with self-calibration parameters can be applied, as well as gross error detection programs, extended to GPS observations. Here the term combined block adjustment is consistently used. It does not exclude, however, the GPS-supported aerial triangulation of single strips. A strip is a special case of a block. It does not require special consideration, except for slightly modified accuracy features.

Geometric Stability of Blocks

It is easy to formally introduce unknown drift parameters per strip into the combined block adjustment. However, their numerical solution can become difficult, if the block has standard overlap and if only four control points are used. In that case, the least-squares solution can run into singularities or near singularities of the normal equation matrix and the adjustment could break down numerically. The explanation is that the system is geometrically weak or indetermined. Consequently, the normal equation matrix can have poor condition or rank deficiencies.

The system can be made geometrically and numerically stable and solvable for all unknowns in three ways:

- if the block has 60 percent side overlap (or double stereo coverage with crossed flight lines);
- by two chains of vertical control points, running across the front ends of the block (see Figure 1, case b); or
- by two cross strips at the front ends of the block (see Figure 1, case c).

For reasons of autonomy and economy, the case with two cross strips is considered the standard case and is generally recommended for GPS aerial triangulation. In that case, the minimum configuration of four control points can be maintained, although it is suggested, for reasons of reliability and for stronger stabilization of the block, that pairs of points or

TABLE 1. EXAMPLE THEORETICAL ACCURACY OF A GPS BLOCK - STANDARD ERRORS [CM] OF ADJUSTED TIE POINTS

free datum transformation
$\sigma_0 = 10\mu m = 30cm = \sigma_{GPS}$

block size 7*13
q=60% side overlap
4 control points
1:30000

rows/ columns of points		2	3	4	5	6	7	8	∘∘∘∘	strip
1	X	28	37	41	45	47	47	47		
1	Y	28	55	62	62	61	60	60		①
1	Z	29	76	82	81	78	76	75		
2	X	39	33	34	37	38	39	39		
2	Y	41	40	44	45	45	44	44		②
2	Z	65	59	61	60	58	56	56		
3	X	40	34	33	34	35	35	35		
3	Y	41	39	40	41	41	41	41		③
3	Z	58	53	53	51	49	48	48		
4	X	41	35	34	34	34	34	34		
4	Y	44	39	39	40	40	40	40		④
4	Z	54	50	48	47	45	44	44		
5	X	42	36	34	33	34	34	34		
5	Y	45	40	39	40	40	40	40		
5	Z	53	48	47	45	44	43	43	∘∘∘∘	
⋮		⋮						⋮		

R.m.s. accuracy of block

[cm]			$[\sigma_0]$		
μ_x	μ_y	μ_z	μ_x	μ_y	μ_z
37	45	58	1.2	1.5	1.9
41		58	1.4		1.9

TABLE 2. ACCURACY OF ADJUSTED GPS-BUNDLE-BLOCKS

Control cases: a (4 CP)
b (4 CP + 2 chains of vertical CP)
c (4 CP + 4 vertical CP + 2 cross-strips)

If

$$\sigma_{CP'} = \sigma_0 \;,\; \sigma_{CP} \le \bar{\sigma}_0 = \sigma_0 \cdot s \;,\; \sigma_{GPS} \le \bar{\sigma}_0 = \sigma_0 \cdot s \quad (s = \text{photo scale number})$$

then

$\mu_{x,y} \le 1.0\bar{\sigma}_0$	$\mu_z \le 1.6\bar{\sigma}_0$	(c, no drift parameters)
$1.0\bar{\sigma}_0$	$1.6\bar{\sigma}_0$	(a, no drift parameters)
$1.7\bar{\sigma}_0$	$2.3\bar{\sigma}_0$	(c, drift parameters per block)
$1.7\bar{\sigma}_0$	$1.7\bar{\sigma}_0$	(a, drift parameters per block)
$2.1\bar{\sigma}_0$	$2.3\bar{\sigma}_0$	(a, drift parameters per strip)
$1.5\bar{\sigma}_0$	$2.0\bar{\sigma}_0$	(b, drift parameters per strip)

block-size: 6 * 21 photographs

point triples at the respective locations in the cross strips be used. This holds especially for vertical control points.

It is evident that cross strips, in order to be effective, must be strongly connected to all strips they cover, by transferring and measuring all mutual tie-points, in all combinations. Also, the ground control points should be measured in all images in which they appear, for the same reason.

Blocks often have quite irregular shapes. In such cases, it may be necessary to fly more than two cross strips and also to have ground control in more than four standard locations. Nevertheless, the reduction of ground control points still remains quite substantial as compared to conventional aerial triangulation without GPS.

Accuracy Performance

GPS blocks have highly favorable accuracy features, as can be anticipated from the general scenario. They are effectively controlled by the GPS air stations, which act practically as control points. In such densely controlled blocks, there is little propagation of errors, and the accuracy distribution in the block is highly uniform. It is evident, too, that the accuracy will depend very little on block size and that it is close to the sheer intersection accuracy of the rays, i.e., it is determined essentially by the photogrammetric measuring accuracy (σ_0). It can be further concluded that ground control points are no longer required for controlling the accuracy of the blocks, as was the case in conventional aerial triangulation. They have to provide the datum transformation, for which only a few ground control points are sufficient. The free drift and datum parameters will weaken the geometric stability somewhat, but the basically favorable accuracy structure is essentially maintained.

Such general accuracy features have been confirmed by extended theoretical accuracy studies based on the inversion of the normal equation matrices. The example of Table 1 shows, for the top left quarter of a GPS block, the standard coordinate errors at the tie points, after combined block adjustment with four ground control points and only one set of free datum parameters. The quite uniform accuracy distribution is demonstrated. The overall RMS accuracies amount to 1.4 σ_0 (horizontal) and 1.9 σ_0 (vertical), in this case.

Quite a number of such theoretical GPS blocks have been analyzed, with various scenarios for ground control, overlap, and drift correction parameters. General accuracy relationships have been derived and published in Ackermann (1992).

The general relationships confirm, as expected, that the block accuracy deteriorates if the GPS camera positioning accuracy gets poorer. But the relationship is relatively weak. GPS blocks do not strongly depend on high GPS camera positioning accuracy, except for very large scale high precision blocks. As the GPS accuracy is actually rather high, in the decimetre order of magnitude, the general accuracy relationships can be simplified and summarized as shown in Table 2. There it is assumed that the standard coordinate errors of the GPS camera positions (σ_{GPS}) and of ground control points (σ_{CP}) are not larger than the photogrammetric image coordinate accuracy (σ_0), projected into terrain scale (e.g., $\sigma_{GPS} = \sigma_0 * s$, with s = image scale). Below that set threshold for σ_{GPS} the geometric scenario of control points, overlap, and datum and drift parameters determines the result. The actual GPS accuracy has little influence on the adjustment results in that case. We are here particularly concerned with the case of two cross strips, four 4 by 2 ground control points, respectively, and free linear drift parameters per strip. That case is summarized in Table 2 by the simple relations $\mu_{X,Y} = 1.5\ \sigma_0$ and $\mu_Z = 2.0\ \sigma_0$ for the horizontal and vertical accuracy of

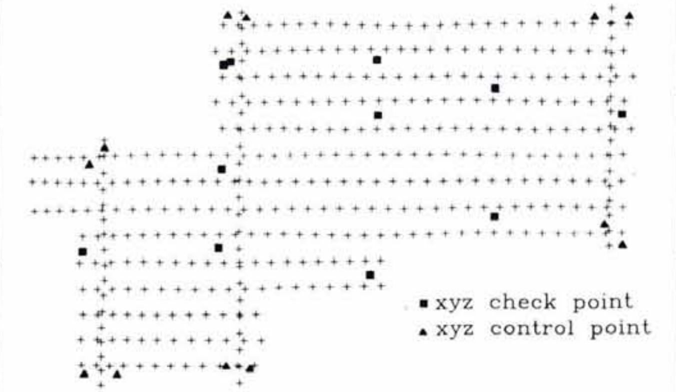

Figure 2. Index map of testblock USA I.

the adjusted GPS blocks. They are approximately valid for any block size, the variation of which affects the results by less than 10 percent.

The above accuracy relations may be illustrated with an example. Assume a photo scale of 1:20,000, and a standard deviation σ_0 of the image coordinates of 10 micrometres (μm), which gives $\sigma_{GPS,CP}$ = 20 cm. As long as σ_{GPS} < 20 cm and σ_{CP} < 20 cm, the combined block adjustment, based on two cross strips and linear drift correction parameters per strip, is expected to give RMS errors of 30 cm in the horizontal and 40 cm in the vertical coordinates ($=0.13‰h$) for the adjusted points of the block. That illustrates the previous statement that the resulting block accuracies are close to the photogrammetric measuring accuracies in planimetry and height. It is evident that the conventional aerial triangulation would require quite a number of ground control points in order to obtain the same accuracy result.

Summarizing our theoretical accuracy investigations, it can be stated that GPS blocks are expected to have a very high accuracy potential and favorable accuracy features. With only few ground control points, the accuracy of adjustment comes close to the photogrammetric intersection accuracy of rays, as the propagation of errors within the blocks is effectively suppressed by the GPS camera positioning. That result is valid for the full range of photo scales as they are applied in aerial photogrammetry for mapping purposes.

Practical Application of GPS Aerial Triangulation

Software, Execution of Combined Block Adjustment

In this section, the actual block adjustments which the GPS teams in Stuttgart have carried out since 1991 are reviewed. The software for GPS trajectory computation, including all transformations, ambiguity solutions, and reductions, has its origin at the Institute for Photogrammetry at the University of Stuttgart. Commercially available software for GPS positioning and for combined GPS block adjustment has been developed at INPHO GmbH. The actual adjustments partly refer to test blocks and to real photogrammetric mapping projects. The block adjustments have been carried out by the Institute for Photogrammetry and by INPHO GmbH, respectively. In all cases, cross strips were applied. The GPS camera position computations were based on L1 phase and C/A-code observations by two receivers, one stationary near the mission area and one in the aircraft. No initial GPS base lines were determined, as the ambiguities were derived from C/A-code pseudorange positioning of the first camera station of each photogrammetric strip. Thus, they were considerably biased, in all cases. The block adjustment always applied linear drift corrections per strip. In some cases, more than two cross strips were flown, because of block shape and size, as Figures 2 and 3 demonstrate. Accordingly, the number of ground control points had to be increased. The photogrammetric measurements, i.e., point transfer, image coordinate observations, and data reduction, had been executed by the photogrammetric companies in almost all cases.

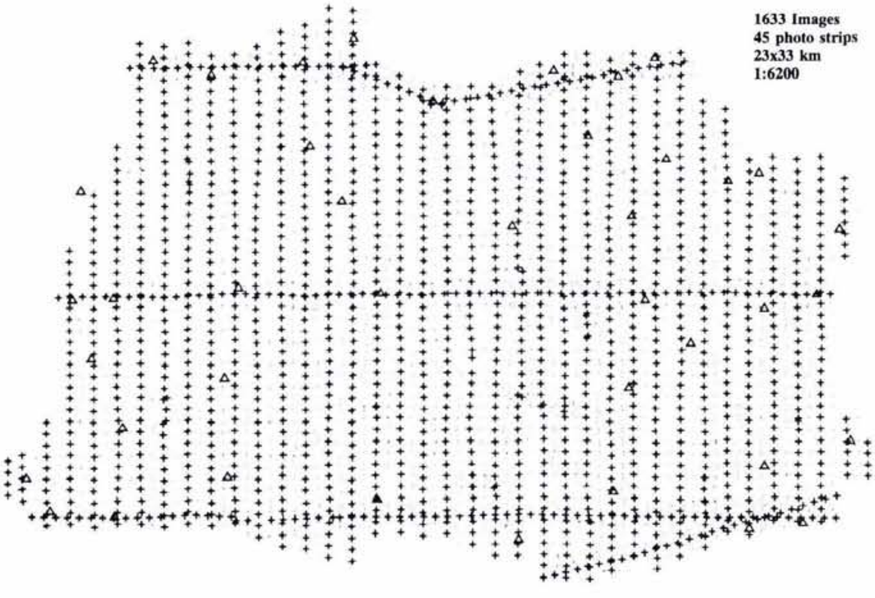

Figure 3. Index map of testblock Germany V.

TABLE 3. SUMMARY OF PHOTOGRAMMETRIC GPS PROJECTS

	Project	Year	Image scale	Number of Images	Number of Photo strips	Number of control points	area of photo coverage
1	Guinea	1991	1:30000	346	15 (11+4)	29	45×90 km²
2	Germany I	1991	1:8000	90	7 (5+2)	4	6×8 km²
3	Germany II	1992	1:7500	50	6 (4+2)	4	5×6 km²
4	Germany III	1992	1:7500	70	8 (6+2)	4	7×7 km²
5	Germany IV	1992	1:7500	55	6 (4+2)	4	6×5.5 km²
6	USA I	1992	1:8700	415	17 (14+3)	12	16×25 km²
7	Germany V	1992	1:6200	1633	45 (39+6)	34	23×33 km²
8	USA II	1992	1:42000	78	6	4	35×35 km²
9	USA III	1992	1:34000	106	7	4	35×40 km²
10	USA IV	1992	1:41000	65	5	4	35×30 km²
11	Canada	1992	1:6100	159	10 (8+2)	12	7.6×7.6 km²
12	Arabia	1992	1:28000	249	12 (10+2)	38	38×25 km²
13	Germany VI	92/93	1:4000	44	7 (4+3)	4	2×2 km²
14	Nepal	92/93	1:50000	(3000)	—	—	—

TABLE 4. THEORETICAL AND EMPIRICAL ACCURACY RESULTS OF GPS BLOCKS

	Project	Flying time per strip [min]	Data rate [sec]	Overlap	σ_0 [μm]	Theoretical accuracy in units of [σ_0] horizontal/vertical	checked RMS accuracy in units of [$\hat{\sigma}_0$] horizontal/vertical
1	Guinea	2 - 16.5	1.0	60/20	9.0	1.2/2.0	-/-
2	Germany I	1 - 2	0.5	60/20	6.3	-/-	1.3/2.2
3	Germany II	1 - 2	0.6	60/20	4.8	1.3/2.5	1.8/2.3
4	Germany III	1 - 2	0.6	60/20	4.5	1.2/2.6	1.6/3.0
5	Germany IV	1-2	0.6	60/20	4.0	1.2/2.6	1.6/2.6
6	USA I	2 - 8.2	1.0	60/20	6.4	1.1/1.8	1.8/2.6
7	Germany V	1 - 8.5	1.0	60/20	7.8	1.3/3.0	-/-
8	USA II	7.5	1.0	60/20	7.5	1.2/2.3	-/-
9	USA III	7.5	1.0	60/20	8.0	1.6/2.2	-/-
10	USA IV	7.5	1.0	60/20	8.0	1.2/2.7	-/-
11	Canada	1.6	1.0	60/20	6.9	0.9/1.6	1.8/1.9
12	Arabia	3.3	1.0	60/20	9.1	0.9/1.8	-/-
13	Germany VI	0.7	0.5	60/20	4.8	0.7/1.8	0.44/1.8
14	Nepal		0.6	60/20	—	-/-	-/-

Table 3 summarizes the GPS blocks which have been completed so far. A wide range of photo scales is covered, as well as a wide range of block sizes. The largest block which was adjusted as a unit had 39 strips and six cross strips, 34 control points, and contained altogether 1633 photographs. It is not the occasion here to discuss individual cases. It should be obvious, however, that the method of combined GPS block adjustments has been applied operationally. There were no serious difficulties encountered which would have prevented successful completion of the adjustments, i.e., there were no failures of adjustment projects.

Results

The accuracy results obtained are summarized in Table 4. The generally good photogrammetric measuring quality is reflected in the magnitudes of the standard image coordinate errors (σ_0) which range from 4.0 to 4.8 μm for signalized tie points (blocks 3,4,5,13) and from 6.3 to 9.1 μm for artificially marked or natural tie points. The second to last column shows the theoretical RMS accuracy of the adjusted tie points, as derived from the least-squares adjustment by matrix inversion. The overall theoretical RMS errors amount to 1.15 $_0$ horizontally and 2.23 $_0$ vertically. Those summarized theoretical values as well as the individual ones may be compared with the simplified theoretical relations of Table 2, showing that the individual geometrical conditions of a block do cause considerable deviations from the schematic standard of Table 2. Thus, the simplified rules of Table 2 can serve only for general planning purposes. They cannot represent precisely the theoretical accuracy expectation of individual blocks. Nevertheless, the results of Table 4 show that the order of magnitude is reasonably maintained.

In a number of blocks some or many given terrain points had been withheld at the block adjustment and used for check purposes. The RMS errors at check points represent an independent and unbiased assessment of the average block accuracy. They can be directly compared to the respective theoretical estimates. The last column in Table 3 displays the empirical accuracy assessments from available check points. The results show that there is always a certain variation against the theoretical expectations. Nevertheless, the total RMS average of 1.6 σ_0 (horizontal) and 2.3 σ_0 (vertical) confirm that the empirical accuracies agree approximately with the theoretical accuracy models. In particular, they confirm that the approach with linear drift corrections functions and performs about as expected.

Operational Status

The above examples sufficiently demonstrate that the method of combined GPS block adjustment for aerial triangulation, here in the special form of cross strips and linear drift corrections, is operational and ready for practical application. The hardware and software tools are readily available, having reached a mature and robust state of development.

The operational conditions at the flight missions are not restrictive at all and allow practical application virtually anywhere and for any case. Also, the theoretically assessed accuracy potential and the empirical accuracy results make the practical application highly recommendable and economic. The method performs in practice about as expected. Similar accuracy results would be obtained with conventional aerial triangulation only by the use of many more ground control points.

It is equally important to state that at least the described method of GPS aerial triangulation is directly applicable today. There is no critical need, with regard to the aerial triangulation application, to wait for refined future hardware and conceptional developments or extended research, which will certainly come in due time and improve the system.

The main point of applying GPS aerial triangulation is the considerable reduction of ground control points, as compared to conventional aerial triangulation. Thus, the motivation for GPS supported block adjustment is essentially economic. The savings in ground control are so dominant that the higher efforts in aerial triangulation (GPS receivers, cross strips, GPS data processing) are vastly overcompensated, unless ground control points are available at little costs. In addition, the photogrammetric operations become more and more autonomous with GPS, which can be highly decisive in many applications, especially in foreign country projects. In our opinion, the advantages and the state of performance of GPS-supported aerial triangulation are convincing enough to recommend the use of relative kinematic GPS camera positioning as standard procedure, to be applied regularly.

Ceterum censeo disponibilitatem delectam (SA) esse delendam.

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