A Modified Fractal Dimension as a Measure of Landscape Diversity

Abstract
Fractals have been used recently to describe spatial patterns in many landscape-level applications. One such application has been to measure the geometric complexity of landscape features. This paper describes a modified fractal dimension to be used as a measure of distribution of landscape diversity in a classified GIS image. The resulting modified fractal dimension calculation consistently describes diversity for the landscape, accounting not only for patch shape, but also for patch juxtaposition and evenness.

Introduction
Recently, researchers have turned to fractals to describe spatial patterns in a variety of landscape-level applications (Burrough, 1986; O'Neill *et al.*, 1988; De Cola, 1989; Lam, 1990; Milne, 1990; Rex and Malason, 1990; Turner, 1990; Polidori *et al.*, 1991; Ripple *et al.*, 1991; Baker and Cai, 1992). Most of this work used fractals to characterize the shape of features within a landscape. Generally, fractal dimension was calculated by computing the slope of a regression line between the natural logarithm of perimeter and area pairs calculated from the feature(s) of interest. For instance, Burrough (1986) used the natural log of one-fourth the perimeter against the natural log of the area. This technique requires the presence of a fairly large number of landscape patches in order to generate enough perimeter and area pairs to accurately calculate the regression. In other work, the fractal dimension of individual landscape features was calculated as an aid in land-cover classification (De Cola, 1989). Lam (1990) used fractals to discuss the spatial complexity of three land types. Milne (1990) applied fractals to estimate the probability of locating a landscape patch within a larger landscape. These researchers used fractals to estimate landscape complexity as a function of patch shape; in most cases, a single fractal value was determined for the entire landscape for selected cover types or for all cover types (Turner, 1990).

Natural resource managers and researchers working at the landscape level need to understand the spatial dynamics of diversity within a given landscape, not just the overall diversity of a landscape. Our research attempts to address this need by examining how to evaluate the diversity of landscapes at various scales (e.g., with various changes of extent, and/or grain size) and how to evaluate the distribution of diversity over a landscape (i.e., what areas within a given landscape are more diverse than others).

As shown by past work in the remote sensing, GIS, and landscape ecology literature (Mandelbrot, 1983; O'Neill *et al.*, 1988; Peitgen and Saupe, 1988; De Cola, 1989; Turner *et al.*, 1989; Lam, 1990; Milne, 1990; Rex and Malason, 1990; Ripple, 1991), fractals can be applied to a variety of landscape ecology problems because they conveniently describe many of the irregular, fragmented patterns found in nature (Mandelbrot, 1983). While the applications and calculation of fractals vary, our use of fractals is limited to describing the degree to which the area of a landscape patch (a continuous grouping of grid cells representing the same landscape feature) is related to its edge and how this measure can be modified to address diversity. By determining the fractal relationship of patch area to patch edge for a given landscape, measures of the geometric diversity of that landscape and, therefore, the complexity of patch interaction within it can be determined. However, as Rex and Malason (1990) have shown, the shape of a landscape patch is not the only factor which affects ecological processes within a landscape; they show that the juxtaposition of a patch to other patches can also have significant effects. Furthermore, a complete definition of diversity needs to include patch richness (e.g., number of different patches) and patch evenness (distribution of patches across the landscape) as well (Shannon and Weaver, 1962).

Past research has also demonstrated that as the extent of a given landscape changes, so do the various landscape indices, including diversity (Turner *et al.*, 1989). Given this, the estimation of landscape diversity is dependent on defining the extent of the landscape. In order to determine the distribution of diversity within a landscape, reduced areas need to be examined. Unfortunately, using the regression method for computing a fractal index for small landscapes is particularly problematic. As the landscape extent is reduced, the number of patches present in the landscape is also reduced. With the lower number of patches, fewer perimeter and area pairs are generated for computing the slope of the regression line, meaning that the computation is based on too few data points and is, therefore, inaccurate.

The diversity index proposed here combines patch complexity, as calculated by fractals, with richness and evenness of patches within a landscape. This index combines work by Patton (1975), who addressed patch edge diversity, and work by Shannon and Weaver (1962), who evaluated species richness and evenness. By combining the definitions of diversity of Patton (1975), Shannon and Weaver (1962), and Rex and Malason (1990), a definition for landscape diversity — which is a function of the number and types of patches, their distribution (juxtaposition), and their shape — was obtained. Fractal indices have been used in many cases as a diversity index, but unfortunately can only account for diversity arising from patch shape; therefore, a pure fractal index (based solely on patch shape) is only a partial measure of landscape diversity. The methodology presented here modifies the fractal dimension measured by accounting for both the geometry

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of patches and their juxtaposition with other patches. This technique calculates a modified fractal index which better represents landscape diversity.

Methodology

In general, fractal dimensions described in the literature are derived by calculating the area and perimeter of a single landscape feature (i.e., a patch) (De Cola, 1989). As stated previously, this technique accounts for only patch shape and is generally only applied to large landscapes. In a following section, a nonregression technique is used for calculating the fractal dimension of a single landscape patch, much like that presented by Gardner (1986). Next, an example of computing the fractal dimension for a small landscape and some of the deficiencies which arise from failing to account for patch juxtaposition are shown. A modified fractal dimension which resolves the problem of patch juxtaposition is then presented.

In the last section, portions of a classified Landsat Thematic Mapper (TM) image (using an unsupervised minimum-distance-to-the-means algorithm available in the ERDAS™® image processing and grid based GIS software) are presented as test "actual" landscapes. Five spectral clusters were used to represent a distribution of patch patterns across a 5- by 5-km landscape. The diversity measure presented here is designed to work with landscapes of any size. However, to make the test visually understandable, the resulting classification was subset from the original 5- by 5-km landscape into six 10- by 10-cell (300- by 300-m) sub-landscapes representing various levels of diversity. These 100-cell (9-ha) landscapes were large enough to capture various levels of diversity and small enough to allow easy visual evaluation of diversity.

Note that this technique is intended for use with a classified image in a grid-based GIS. Also, note that the terms "small landscape" or "sub-landscape" refer to a sampled area from a larger grid-based landscape.

Fractal Dimension of a Single Landscape Patch

Noting that the regression techniques described in the literature were better suited to large landscapes (and our interests are in smaller ones), we turned to the basic definition of the fractal relationship between patch area and perimeter as described in Peitgen and Saupe (1988). Their equation (Equation 1.131 on p. 62) became the basis of our technique: i.e.,

$$P \propto A^{D/2} \tag{1}$$

where

A = area of the object of interest,
P = perimeter of the object of interest, and
D = fractal dimension.

To avoid using the regression technique, the constant of proportionality for Equation 1 needed to be calculated. Because the landscapes in this technique are grid based, consisting of square cells, and having a square shape, Equation 1 was modified to deal specifically with squares by calculating the constant of proportionality between area and perimeter for a single cell: i.e.,

$$P = k * A^{D/2} \tag{2}$$

where

k = constant of proportionality for cell,
A = 1 cell area,
P = 4 cell lengths, and
D = 1.0 (a single cell is our simplest case).

Rearranging Equation 2 and solving for k yielded

$$k = 4.$$

Therefore,

$$P = 4 * A^{D/2}$$

and

$$D = 2 * \ln (P / 4) / \ln (A) \tag{4}$$

where

A = total patch area and
P = total patch perimeter.

This became the equation for calculating fractal dimension for individual patches within a sub-landscape.

Fractal Dimension of a Landscape

However, the objective here is the fractal dimension of the entire sub-landscape and not single patches. To accomplish this, each patch within the landscape was identified and the fractal dimension was calculated using Equation 4. Once the fractal dimension for each patch within the landscape was determined, the average landscape fractal is calculated by weighting each patch fractal by the ratio of the patch area to the landscape area. This weighted average addresses patch evenness within the sub-landscape.

Minor problems occur when the landscape contains a patch which is represented by a single cell. These cells have an area of one (1) which yields an infinite result using Equation 4. Because a single cell represents the simplest shape possible, we set $D = 1.0$, by definition, for all single cell patches (i.e., those with $A = 1$).

Equation 4 was geometrically acceptable for small landscapes, but it could not account for diversity due to variation in patch type. If the geometry of a landscape was unchanged, so was the fractal dimension (which was desired as a measure of diversity using both geometry and patch type). The example in Figure 1 shows two distinctly different cases which were indistinguishable by this technique.

Adding Patch Type to Fractal Dimension

One problem with using pure fractal dimension to measure landscape diversity is that it only deals with geometric diversity. The class variability shown in Figure 1 is not due to geometry. In this case, the fractal dimension does not distinguish the patch variability resulting from patch classification. Referring to Figure 1, in Case I the background patch (blank areas) is adjacent to patches of only one other class (a); in Case II the background class patch is adjacent to patches of four different classes (a, b, c, and d). The patch variability and edge interaction of Case II results in a more complex landscape. Therefore, a diversity index needs to include the variability of patch juxtaposition in the calculations.

To include patch type in the calculation, a modification was made in the way the perimeter of a patch was counted. First, the number of outer cell sides on a patch are counted, as before, and then the perimeter count is modified based on the number of other classes adjacent to the patch: i.e.,

$$P_m = P + P_c \tag{5}$$

where

P = perimeter count, based solely on geometry, and

Case (I):

a	a		a	a
a	a		a	a
a	a		a	a
a	a		a	a

Case (II):

a	a		b	b
a	a		b	b
c	c		d	d
c	c		d	d

Patch	A	P	D-patch
a	4	8	1.0000
b	4	8	1.0000
c	4	8	1.0000
d	4	8	1.0000
bkgnd	9	20	1.4650

Landscape

D = 1.1674

Figure 1. Two landscapes that are indistinguishable using the simple fractal calculation from Equation 4. These represent 5- by 5-cell landscapes. Case (I) has cells of only two GIS classes (the background and a). Case (II) has cells of five GIS classes (the background, a, b, c, and d). Both cases yield the same fractal dimension because their geometry is identical (only the calculation for case (II) is shown because its patches are easily distinguished).

P_c = perimeter class modification.

The perimeter class modification was calculated as follows:

$$P_c = Q * C / (C_t - 1) \tag{6}$$

where

Q = perimeter reduction (discussed below),
C = count of classes adjacent to patch (not diagonally), and
C_t = total number of classes in entire landscape image.

The perimeter reduction, Q, requires some explanation. Note that, in the grid-based landscape, patches of more than one cell can not possibly reach the potential theoretical maximum perimeter assumed by the definition of fractal dimension. For instance, a patch with area of 4 has a theoretical maximum perimeter (P_t) of 16, but a grid imposed maximum perimeter (P_g) of 10 (see Table 1 for other patch sizes). The theoretical maximum perimeter comes from the definition for the fractal relationship between area and perimeter of Equation 3. In other words, a patch of a given area which has a perimeter equal to the theoretical maximum will have a fractal dimension of $D = 2.0$ (i.e., the theoretical maximum perimeter is four times the patch area). The lower grid-imposed maximum perimeter results from the nature of the gridded landscape. Each cell has an area of one and all cells within a patch must be adjacent to one full cell side (the patch shape, in a grid, with the highest perimeter is a "string" in which the two end cells are adjacent to only one other cell and the interior cells are adjacent to two other cells). Therefore, the maximum possible perimeter for a patch in a grid system is reduced from the theoretical maximum by the amount Q: i.e.,

$$Q = P_t - P_g, \tag{7}$$

more easily computed as

$$Q = 2 * (A - 1). \tag{8}$$

Table 1. Patch Maximum Perimeters: Theoretical (P_t) and Grid-Imposed (P_g).

Patch Area	P_t	P_g	$Q = P_t - P_g$
1	4	4	0
2	8	6	2
3	12	8	4
4	16	10	6
5	20	12	8

The value Q, the unused portion of perimeter, is used to incorporate class diversity. The reasoning behind this was that, in order for a landscape to reach the theoretical maximum diversity of 2.0, the landscape must contain all possible patch types within its boundary. In addition, each patch must attain its maximum perimeter and be adjacent to patches of all other types. Therefore, for the most compact (square) patch, occupying the entire sampled portion of the landscape (representing the simplest situation), $D = 1.0$; for the patch with the greatest perimeter allowed by grid constraints and surrounded by patches of all other classes in the image (representing the most complex situation), $D = 2.0$. In summation, incorporating Equation 8 into Equation 6, the modified perimeter count became

$$P_m = P + (2 * (A - 1) * C / (C_t - 1)) \tag{9}$$

and the modified fractal dimension became

$$D_m = 2 * \ln (P_m / 4) / \ln (A) \tag{10}$$

where

A = area of patch within sample landscape,
P = perimeter of patch within sample landscape,
C = count of classes adjacent to patch within the landscape
C_t = total number of classes in entire landscape,
P_m = modified perimeter count, and
D_m = modified fractal dimension.

For a sample landscape, as before, area weighted averages of all patches in the sample landscape were made. Also, for a single cell patch, again $D_m = 1.0$. For an example calculation, see Figure 2.

Results and Discussion

This modified fractal dimension combines the number of landscape patches, their distribution, and shape into an overall measure of landscape diversity. To test the fractal dimension diversity measure of Equation 10 on an actual landscape, we used the six portions of the classified TM image described above. The modified fractal dimension, D_m, was calculated for each portion. Figure 3 shows each landscape: The associated modified fractal dimension is contained in Table 2. In general, as the sub-landscapes progress from 1 to 6, they visually appear more diverse. However, landscape 6 is less diverse than landscapes 4 or 5, as measured by the modified fractal dimension. An examination of the distribution and shape of landscape 6 more closely re-

a	a	c	b
a	a	c	b
c	c	b	b
c	c	g	b

Patch	Class	A	P	C	Pm	D	Dm
#1	a	4	8	1	8.86	1.0000	1.1468
#2	c	2	6	2	6.57	1.1699	1.4324
#3	b	5	12	2	14.29	1.3652	1.5819
#4	c	4	8	3	10.57	1.0000	1.4021
#5	g	1	4	2	-	1.0000	1.0000
Weighted Avg. for Landscape:						1.1354	1.3731

Figure 2. Example calculation comparing un-modified and modified techniques. Note that Patches #1 and #4 have the same D, but different D_m; the increase of diversity is added by the different number of classes to which Patch #4 is adjacent. Also, note that we assume that the total number of classes in our landscape was $C_t = 8$ (this 4 by 4 cell represents a subarea for which we were measuring local diversity within the landscape).

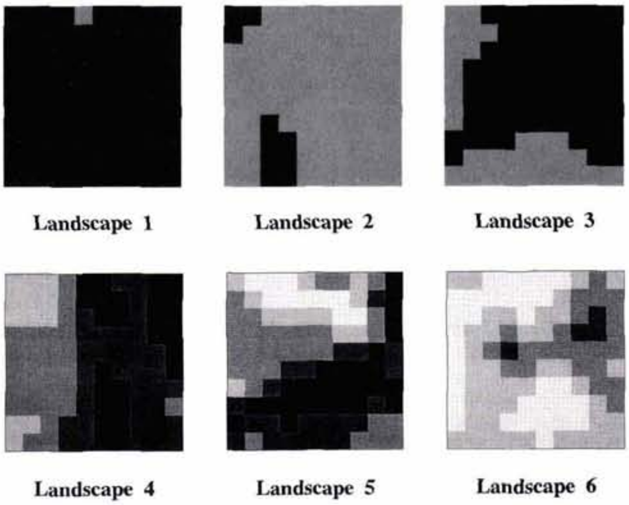

Figure 3. Sample landscapes from a classified TM Landsat Image and their corresponding fractal diversity.

TABLE 2. FRACTAL DIVERSITY RESULTS FOR A SUBSET TM IMAGE.

Landscape	D_m
Landscape 1	1.3563
Landscape 2	1.3958
Landscape 3	1.4277
Landscape 4	1.5917
Landscape 5	1.6092
Landscape 6	1.4831

veals that most of the patches appear to be more compact than landscapes 4 or 5. Through casual observation, landscape 6 appears to be more diverse than 4 but less than 5. This may lead to an interesting discussion on the ability of an individual to detect various levels of diversity through visual interpretation.

Conclusion

By adjusting the fractal dimension with the class adjacency correction, an index that better measures landscape diversity is calculated because it incorporates not only the shape of patches, but also patch juxtaposition, richness, and evenness. Further, by moving away from a regression technique of calculating the fractal dimension of landscape patches, smaller landscapes can be evaluated. The use of sub-landscapes allows researchers and managers to evaluate the distribution of diversity within the context of a larger landscape.

While more work is needed to compare this index with those of others, we can see by this exercise that a modified fractal index can provide a simple measure of diversity. This can be used within a GIS framework to identify areas of high (or low) diversity which can be related to land management practices, wildlife/habitat interactions, and biodiversity.

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