Digital Analysis of Viewshed Inclusion and Topographic Features on Digital Elevation Models

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Abstract
This short paper formally defines and documents how pixels of digital elevation models dominate each other in visibility with respect to their topographic characteristics. Two pixels are defined as mutually visible if a straight line that connects these two pixels can be constructed without intersecting any other parts of the topographic surface. Dominance occurs when all visible pixels from a viewing pixel are also visible from another viewing pixel. Pixels of various topographic characteristics examined here include pixels classified as peaks and pits, and those pixels on ravines and ridges. Visibility dominance among pixels of digital elevation models may be used to enhance and speed up visibility analyses such as watchtower siting or viewshed assessment.

Introduction
Traditional applications using visibility information derived from digital elevation models (DEMs) are limited to the delineation of viewsheds or to line-of-sight analyses. Recently, a number of new applications using visibility information have been further explored and developed. These include those in civil engineering, orientation and navigation (Burrough, 1986), scenic planning (Dietrich et al., 1988), and landscape planning (Litton, 1973). Furthermore, there are other applications taking one step further in using visibility information from DEMs; these include military surveillance (U.S. Military Academy, 1988) and site selection for watchtowers or radio wave transmission stations - collectively called the analysis of visibility sites in Lee (1991).

Computing visibility information from DEMs is usually very time-consuming and requires larger memory spaces, particularly when using DEMs of high resolutions. However, we suggest that a great deal of computer resources can be saved in visibility analyses if the analyses utilize the relationships of how DEM pixels relate to each other in terms of visibility such that only those DEM pixels of critical importance need be processed. Visibility dominance is one such relationship one can examine among DEM pixels for more efficient visibility analyses.

In this paper, we will define and examine the relationships of visibility dominance among DEM pixels. First, DEM pixels will be classified into four categories of peaks, pits, and those on ravines and ridges. Second, formal definitions of visibility dominance will be introduced and then applied to a sample DEM for more detailed examination of the relationships between visibility dominance and topographic features of DEM pixels.

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Definitions of Visibility Information and Topographic Features
A DEM, P_{ij}, $i=1,r$, $j=1,c$, where r is the number of rows and c is the number of columns, usually takes the form of a rectangular matrix and typically contains rc pixels. For simplicity, E_P is used to represent the elevation of the pixel P.

Definition 1 (Visibility): Two pixels, P and P', are mutually visible if there exists a straight line that connects P and P' without intersecting any part of the surface between those two pixels. With respect to a viewpoint, VP, one can construct a visibility matrix, V, in which $V_{ij}=1$ when P_{ij} is visible from VP, and $V_{ij}=0$ otherwise. Furthermore, the visibility regions of VP, ΣV_P, will be the pixels that are visible from VP.

A pixel *dominates* another pixel in visibility if and only if its visible regions include all visible regions of the other pixel. An opposite relationship can be defined for situations where a pixel *is dominated* by another pixel in visibility if and only if its visible regions are entirely included in the visible regions of the other pixel.

Definition 2 (Visibility Dominance): There exists a visibility dominance, $D_{PP}'=1$, if all visible pixels from P' are also visible from P. More specifically, D is a set of measures of visibility dominance, $\{D \mid D_{PP'} = 1 \text{ if } V_{ij} \geq V'_{ij} \text{ for all } i, j; \text{ Otherwise, } D_{PP'} = 0\}$. Note that a pixel may dominate more than one pixel in visibility. Also, it may be dominated by more than one pixel in visibility.

To classify topographic characteristics of DEM pixels, Peucker and Douglas (1975) outlined a set of methods for recognizing peaks, pits, ridges, ravines, and other topographic features. Their approach is based on the patterns of elevation changes between a pixel's neighboring pixels.

Definition 3 (Topographic Features): Using a 3 by 3 window in a DEM, let

- n = number of grid neighbors, *i.e.*, $n=8$;
- Δi = the difference in elevation between a pixel and its ith neighbor, $i=1,2,...,n$, in either clockwise or counter-clockwise order;
- Δ_+ = the sum of all positive differences in Δi;

Photogrammetric Engineering & Remote Sensing,
Vol. 60, No. 4, April 1994, pp. 451–456.

0099-1112/94/6004–451$03.00/0

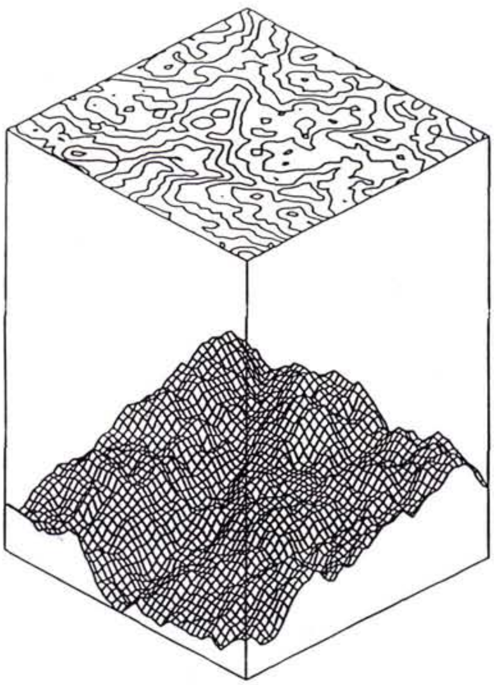

Figure 1. The sample digital elevation model.

Δ_- = the sum of all negative differences in Δi;
N_c = the number of sign changes in Δi; and
L_c = the number of pixels between two sign changes in Δi.

The number of sign changes, N_c, refers to the number of times elevation differences between every two consecutive grid neighbors changed from positive to negative or vice versa. The number of pixels between two sign changes, L_c, refers to the number of consecutive grid neighbors between a change of sign of elevation differences.

The following definitions are adopted from Peucker and Douglas (1975) for pixel classifications that are relevant to this study:

Peak *(pk)*: $\Delta_+ = 0, \Delta_- > tp, N_c = 0;$
Pit *(pt)*: $\Delta_+ > tp, \Delta_- = 0, N_c = 0;$
Ridge *(rg)*: $\Delta_- \text{-} \Delta_+ > tr, L_c \neq n/2, N_c = 2;$
Ravine *(rv)*: $\Delta_+ \text{-} \Delta_- > tr, L_c \neq n/2, N_c = 2.$

Note: tp and tr are thresholds that may be defined according to users' need. For simplicity, we define $tp = tr = 0$.

Research Propositions

Several speculations are raised here to explore how elevations and the topographic characteristics of DEM pixels are related to visibility dominance:

Proposition 1 (Viewpoint Height): There is a positive correlation between the areas of visibility regions of DEM pixels and the elevations of the pixels. A pixel of higher elevation has larger visible regions than that of a pixel of lower elevation. That is: $\Sigma V_P \geq \Sigma V_{P'}$ *if* $E_P \geq E_{P'}$.

Terrain surfaces rarely exhibit regular trends in relief changes. Therefore, visibility regions portrayed here may not always be continuous in space. For a viewpoint, its visibility regions may consist of a number of disconnected polygons on the digital terrain surface. The combined size of these disconnected polygons is the size of the viewpoint's visibility regions.

Proposition 2 (Visibility Dominance): There is a positive correlation between visibility dominance and the heights of pixels. A pixel of higher elevation is more likely to have visibility dominance over those pixels of lower elevations; that is: $D_{PP'} = 1$ when $E_P \geq E_{P'}$.

Proposition 3 (Topographic Features): There is a tendency for pixels of peaks and ridge lines to have larger visibility regions than those of the pixels of pits or ravines. For example, $\Sigma V_{pk} \geq V_{pt}$; $\Sigma V_{rg} \geq \Sigma V_{rv}$; *etc.*

Proposition 4 (Visibility Dominance): There is a tendency for pixels of peaks and along ridge lines to have more visibility dominance than those pixels of pits or ravines; therefore, pixels of peaks and along ridge lines may be better candidates for visibility sites; or $D_{pk,pt} = 1$ or $D_{rg,rv} = 1$, *etc.*

In short, the initial motivations for these investigations are (1) to see if elevations of DEM pixels control visibility; (2) to see if visibility dominance and elevations are related; (3) to see if topographic characteristics account for visibility; and, finally, (4) to see if pixels of some topographic features have greater visibility dominance over others. A sample DEM is created and used to run computer codes developed for the above investigations. Summarizing the tested results by descriptive statistics should provide us with the understanding of the relationships being examined.

Sample Digital Elevation Model

A sample DEM of the area of Welchland, Tennessee was created by first digitizing the contours from a 7.5-minute quadrangle topographic map of the area. With a simple technique of the inverse distance interpolation, the contours are then interpolated to generate a grid DEM with 50 rows by 50 columns. See Figure 1 for a contour map and a three-dimensional diagram which describe the topography of the sample DEM. A mountainous area is chosen because it provides the required complexity in visibility patterns. The size of 50 by 50 is due to the limited computing device available to the author.

The resultant sample DEM has elevations ranging from 264.12 metres to 322.67 metres above the sea level with a mean of 295.81 metres and a standard deviation of 12.47 metres. A main valley in the middle of the DEM characterizes the DEM as a mountainous area which has rugged ridge lines running in various directions, serving as a good testing sample for the study.

The sample DEM serves as a demonstration and a verification of the visibility dominance among DEM pixels. We use this simple dataset to explore the pattern of visibility dominance and its related properties concerning topographic features. Although this simple dataset may not represent all types of landscape at all possible scales, the findings of the properties and patterns of visibility dominance should be applicable to most cases because the algorithms for computing visibility information and topographic features of pixels can be applied to all DEMs.

The level of visibility dominance in a DEM is expected to be dependent on the degree of ruggedness of landscape described by the DEM. The more ruggedness a terrain surface has, the less visibility dominance is expected for the surface because the visibility regions of DEM pixels tend to be more fragmentary and less overlapping. In turn, a gentle landscape is expected to have more visibility dominance because visibility regions of its pixels tend to be more connected and are more likely to overlap. Direct comparisons of the levels of visibility dominance from various types of landscape would be difficult because a scale-independent classification structure of all landscapes would be needed before conducting such studies.

There are no short cuts in computing the visibility dominance among DEM pixels. They must be searched for among pixels and, therefore, the computations tend to be relatively time-consuming. Faster solutions can be reached if there are less potential visibility sites to be evaluated. By knowing the properties and patterns of visibility dominance and how they are related to the topographic characteristics of DEM pixels, a great number of pixels can be eliminated from being further evaluated if they are not likely to be good candidates for visibility sites. In this fashion, the entire process of analyses can be speeded up dramatically.

Algorithms for Visibility and Visibility Dominance

Many algorithms have been developed to compute hidden-line and hidden-surface removal in computer graphics (see, for examples, Sutherland *et al.* (1974), Griffiths (1978), Foley and van Dam (1983)). These algorithms for computing hidden-line and hidden-surface removal are designed primarily for displaying three-dimensional objects on two-dimensional computer screens or paper output. For our purpose, these algorithms do not have a usable data structure for keeping track of visible/non-visible surfaces from given viewpoints. In addition, using these algorithms requires that viewpoints be specifically located away from the depicted objects whereas our need is to be able to locate the viewpoints ***within*** the checked surface. Consequently, we have to explore another approach to computing the inter-visibility on raster DEMs by modifying the line-of-sight algorithm.

Following our definition of visibility between two DEM pixels, a and b, we know that a and b are mutually visible ($V_{ab} = 1$) *if and only if* a straight line can be constructed to connect a and b without intersecting any other part of the surface between a and b. This translates to a checking routine where

- a straight line is first constructed to connect a and b;
- all DEM pixels which fall on that straight line are compared to see if their elevations are any higher than the elevations interpolated from the straight line at their locations; and
- for a and b to be mutually visible, none of the intermediate pixels can be higher than the interpolated elevation from the straight line.

To decide the locations of the intermediate pixels, we use an algorithm known as the simple digital differential analyzer (DDA) (see Newman and Sproull (1979), page 24, for a detailed discussion of the algorithm) which is very accurate and has been widely adapted as a software line generator:

```
{(ra, ca), (rb, cb): two end pixels}
{r, c: row # and column # of the intermediate pixels}
PROCEDURE DDA(ra, ca, rb, cb: integer)
    VAR length, i: integer; r, c, Δr, Δc: real;
BEGIN
  length := abs(cb − ca);
  IF abs(rb − ra) > length THEN length := abs(rb − ra);
  Δr := ra + 0.5;
  Δc := ca + 0.5;
  FOR i := 1 TO length DO
  BEGIN
    keep(trunc(r), trunc(c));
    r := r + Δr;
    c := c + Δc;
  END
END
```

With this algorithm for computing visibility between a given pair of pixels, all DEM pixels are tested against all other DEM pixels to obtain visibility regions associated with each pixel. Furthermore, visibility regions of DEM pixels are compared to check for possible inclusions. As defined previously, a pixel is dominated in visibility by another pixel if the visibility regions of the former are entirely enclosed by the visibility regions of the later.

For visibility information computed from DEMs of low resolution, one would be skeptical as to the inclusion of those intermediate pixels. This is because four corners do not form a plane when their elevations are not co-linear. In this paper, we use centroids of pixels to represent the pixels. It should be noted that the validity of the resultant visibility information is, of course, highly dependent upon the resolutions of the DEMs used. When computing resources are available, higher resolutions are likely to improve the precision of the analysis.

Elevations and Visibility

To examine the relationship between elevation of the DEM pixels (E_P's) and the sizes of their visible regions (ΣV_P's), we computed, for every pixel in the DEM, the number of visible pixels as the size of its visibility regions. Repeating this process for all pixels in the DEM, the resultant visibility regions range from 6 pixels to 1153 pixels with a mean of 248.85 pixels and a standard deviation of 198.95 pixels. As shown in Figure 2, those pixels that are on ridge lines and that are peaks tend to have larger numbers of visible pixels. Alternatively, those pixels in pits and ravines tend to have smaller numbers of visible pixels.

To observe the relationship between pixel elevations and the sizes of their visibility regions, a crosstable of frequency counts is constructed to show the distribution of the numbers of pixels that are in a given elevation range and in a given range of the numbers of visible pixels. Along the horizontal axis, Table 1 divides DEM pixels into ten groups of equal intervals of elevations: E1, E2, ..., E10, representing intervals of (264.12 to 269.97 metres), (269.97 to 275.80 metres), ..., (316.82 to 322.67 metres). Along the vertical axis, Table 1 divides DEM pixels into ten groups of equal intervals according to the number of visible pixels: N1, N2, ..., N10, representing intervals of (6 to 114 pixels), (115 to 229 pixels), ..., (1039 to 1153 pixels).

Higher frequencies are shown in the upper-right triangular part where lower frequencies are shown in the lower-left triangular part. This trend implies that pixels of higher elevations tend to have larger visibility regions than do the pixels of lower elevations. A regression analysis using the elevations of DEM pixels (*Elev*) as the independent variable and the numbers of visible pixels (*V_count*) as the dependent variable reveals the following relation: $V\_count = -559.78 + 2.73\ Elev$.

The trend between elevations and the numbers of visible pixels is significant as the t-test statistic on the slope of the regression line is statistically significant ($t = 8.75$ with degrees of freedom of 2498). This trend agrees with what was observed from the frequencies in Table 1. However, the correlation between the two variables is 0.17 which represents a weak positive association of the two variables. This low correlation, which may be due to the large degrees of freedom,

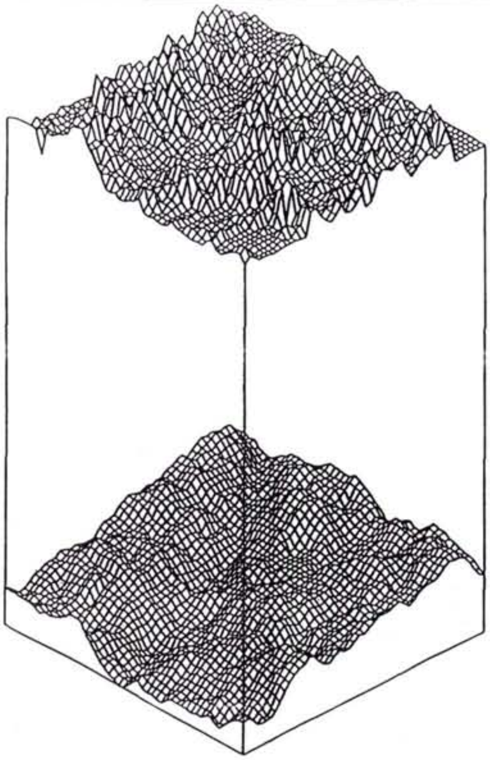

Figure 2. Numbers of visible pixels (upper diagram) and the sample DEM (lower diagram).

suggests that other variables should be investigated to fully explain the variation. The investigations in the following sections are to compliment this first result.

Elevations and Visibility Dominance

For every pixel in the sample DEM, we computed two forms of visibility dominance: (1) the number of other pixels that a given pixel dominates in visibility (dominating visibility) and (2) the number of times that this given pixel is dominated by any other pixels in visibility (dominated visibility). Computing the dominating visibility will allow us to identify pixels in the DEM that may be better candidates for visibility sites because visibility sites are presumably those locations that "see" more areas than other locations. Moreover, computing the dominated visibility will allow us to detect pixels that are less likely to be good candidates for visibility sites. The purpose is that we can quickly locate potential visibility sites by identifying and processing only those pixels that dominate others in visibility because they tend to be better candidates.

The results of the computed visibility dominance are organized in Table 2 which shows means, standard deviations (SDs), and medians of pixel elevations in the DEM, grouped by different topographic features and visibility dominance. After testing the differences between the means, SDs, and medians of the two dominance forms and the overall means, NSDs, and medians in Table 2, no significant relationships were found. In addition, elevations of pixels of various topographic features do not suggest much more than a simple fact that mean elevations are higher for peaks and ridges than for pits and ravines.

Topographic Features and Visibility Dominance

Unlike elevations, topographic features of DEM pixels are more significantly related to visibility dominance among DEM pixels. Table 3 and Table 4 show the statistics of frequency of the dominated visibility and the dominating visibility, respectively. Also from Table 3, pit pixels are the most dominated in visibility because the maximum frequency that a pixel is dominated by other pixels in visibility is accounted by ravine pixels. Similarly, pits are frequently dominated by other pixels in visibility. On the other hand, peaks and ridge pixels are rarely dominated by other pixels in visibility as shown by the mean frequencies.

In Table 4, ridge pixels account for the maximum frequency in dominating other pixels in visibility. We can also see in Table 4 that peak and ridge pixels generally have higher frequencies in dominating other pixels in visibility than those of pits and ravines. Similarly, pits and ravine pixels very seldom dominate other pixels in visibility.

Finally, with a visual inspection of Figures 3 and 4 which further endorse the observations from Tables 3 and 4, pixels along ridge lines generally show higher frequencies of dominating visibility (Figure 3) and ravine pixels along channel lines generally show higher frequencies of dominated visibility (Figure 4).

In summary, the sizes of the visibility regions associated with DEM pixels are found to be related to the elevations of the pixels, and visibility dominance is found to be common among raster DEM pixels. No significant relationship between

TABLE 1. NUMBERS OF VISIBLE PIXELS AND ELEVATIONS IN 10 BY 10 INTERVALS. DEM PIXELS ARE CLASSIFIED INTO TEN EQUAL INTERVALS ACCORDING TO THEIR ELEVATIONS. THE NUMBERS OF VISIBLE PIXELS ASSOCIATED WITH DEM PIXELS ARE ALSO CLASSIFIED INTO TEN EQUAL INTERVALS ACCORDING TO THEIR VALUES. FROM N1, N2, TO N10, THE DIRECTION REPRESENTS INCREASING NUMBERS OF VISIBLE PIXELS. FROM E1, E2, TO E10, THE DIRECTION REPRESENTS INCREASING ELEVATIONS OF THE DEM PIXELS.

	E1	E2	E3	E4	E5	E6	E7	E8	E9	E10	Total
N1	19	50	87	96	116	169	109	103	77	12	838
N2	30	28	51	64	59	153	79	57	47	7	575
N3	6	27	49	49	47	87	70	46	43	12	436
N4	0	9	21	28	31	59	48	43	34	8	281
N5	0	0	9	20	21	30	31	30	23	10	174
N6	0	0	1	18	12	21	11	19	12	11	105
N7	0	0	0	6	4	15	15	10	6	5	61
N8	0	0	0	1	1	4	6	1	5	1	19
N9	0	0	0	0	0	0	5	0	2	0	7
N10	0	0	0	0	3	0	0	0	0	1	4
Total	55	114	218	282	294	538	374	309	249	67	2500

TABLE 2. ELEVATIONS DISTRIBUTION BY VISIBILITY DOMINANCE AND FOUR TYPES OF TOPOGRAPHIC FEATURES. PIXELS OF DOMINATED VISIBILITY INCLUDE ONLY THOSE PIXELS THAT ARE DOMINATED IN VISIBILITY BY AT LEAST ONE OTHER PIXEL OR MORE. PIXELS OF DOMINATING VISIBILITY INCLUDE ONLY THOSE PIXELS THAT DOMINATE AT LEAST ONE OTHER PIXEL OR MORE IN VISIBILITY.

	Pixel Elevations		
	Mean	St. Dev.	Median
All Pixels	295.81	12.47	296.94
Pixels of dominated visibility	293.18	12.74	294.64
Pixels of dominating visibility	295.98	12.52	297.00
Pixels of peaks	301.52	11.69	300.65
Pixels of pits	289.52	12.28	290.96
Pixels on ravines	294.24	12.50	295.82
Pixels of ridges	298.19	12.31	298.34

TABLE 3. FREQUENCY ANALYSIS OF THE DOMINATED VISIBILITY AMONG PIXELS OF DIFFERENT TOPOGRAPHIC FEATURES. NOTE THAT PITS AND RAVINES ARE MOST OFTEN DOMINATED BY OTHER PIXELS IN VISIBILITY. THE PIXELS WHICH ARE THE MOST FREQUENTLY DOMINATED BY OTHER PIXELS IN VISIBILITY ARE OF THE TYPE OF RAVINE PIXELS.

Dominated Visibility: Frequency by Topographic Features of Pixels					
	All Pixels	Peaks Pixels	Pits Pixels	Ravine Pixels	Ridge Pixels
Mean Frequency	2.698	0.104	17.139	5.087	0.703
Frequency S.D.	4.478	0.484	9.302	5.894	1.466
Maximum Frequency	38	4	37	38	18
Median Frequency	1	0	15	3	0
Minimum Frequency	0	0	4	0	0

TABLE 4. FREQUENCY ANALYSIS OF THE DOMINATING VISIBILITY AMONG PIXELS OF DIFFERENT TOPOGRAPHIC FEATURES. NOTE HERE THAT THE MOST DOMINATING PIXELS IN VISIBILITY ARE THE PEAKS AND RIDGE PIXELS. THE HIGHEST FREQUENCIES ARE ACCOUNTED BY THE RIDGE PIXELS.

Dominating Visibility: Frequency by Topographic Features of Pixels					
	All Pixels	Peaks Pixels	Pits Pixels	Ravine Pixels	Ridge Pixels
Mean Frequency	2.698	5.765	0.444	1.235	4.009
Frequency S.D.	3.795	5.366	0.735	1.733	4.704
Maximum Frequency	36	31	2	14	36
Median Frequency	2	4	0	1	2.5
Minimum Frequency	0	1	4	0	0

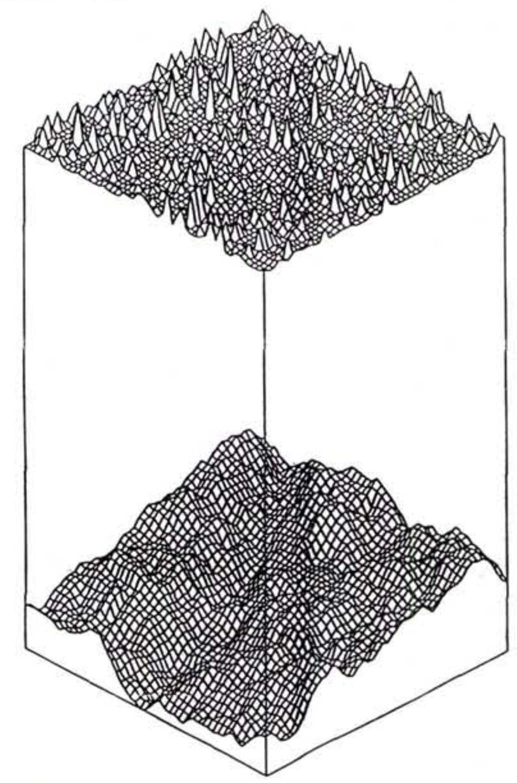

Figure 4. Frequencies of dominated visibility.

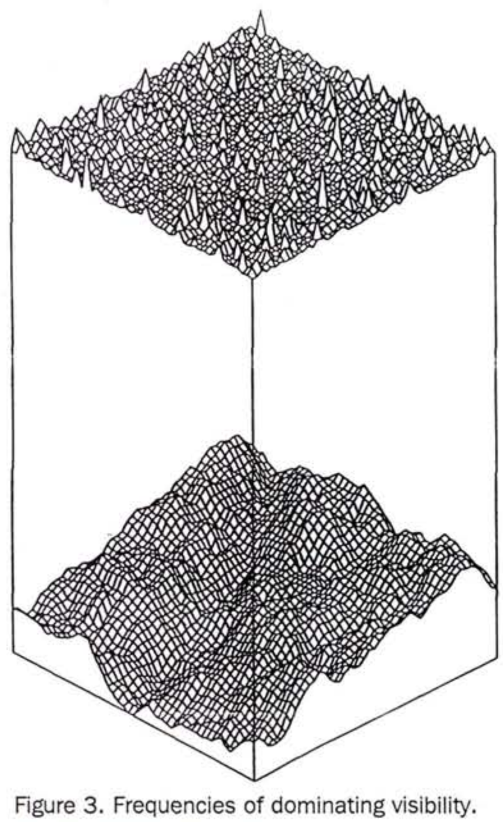

Figure 3. Frequencies of dominating visibility.

visibility dominance and pixel elevations is found. However, it is observed that there are significant differences in visibility dominance among pixels of various topographic features. More specifically, pixels of peaks and ridges tend to dominate other pixels more frequently than pixels of pits and ravines.

Concluding Remarks

With a sample DEM, we demonstrated in this paper that elevations are generally higher for peaks and ridge pixels than those of pits and ravines. The sizes of visibility regions of the DEM pixels are related to their elevations in that pixels of higher elevations tend to have larger visibility regions than those pixels of lower elevations. Between elevations and the visibility dominance of the DEM pixels, the relationships have been observed to be less significant.

The results of the frequency analyses between topographic features and visibility dominance of the DEM pixels suggest that peaks and ridges tend to dominate more pixels in visibility than pits and ravines. Moreover, peaks and ridges are less frequently dominated by other pixels than pits and ravines. Not only did the mean frequency of the visibility dominance support this observation, but also the maximum and minimum frequencies verified the same results.

In this paper, we provide a set of formal definitions for visibility dominance of DEM pixels for further applications. We document the relationships between visibility dominance and topographic features of DEM pixels. Although these relationships may seem to be trivial, the results of investigations in this paper are in fact very important as they will have significant impacts on the efficiency of many visibility analyses. For instance, DEM pixels can be filtered first with respect to their importance in visibility dominance for various visibility analyses. Consequently, analyses of visibility sites can be drastically speeded up by only evaluating those DEM pixels of greater visibility dominance.

Acknowledgment

This research has been partially funded by the Ohio Supercomputer Center. The author wishes to express his appreciation for the assistance from OSC and its personnel.

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(Received 10 August 1992; revised and accepted 2 February 1993)