Application of a Radiometric Model for Evaluation of Water Depths and Verification of Results with Airborne Scanner Data

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Abstract
An existing deterministic, algorithmic model was used to simulate the radiative transfer of light through various depths of water. A computational formula was developed for a given set of experimental conditions and used to predict radiances for water in 0.6-m increments to a depth of 3.7 m. In a verification test, the capability of the model was evaluated by comparing the model simulation results with data collected using an airborne multispectral scanner. Validation results indicated that the model-simulated data and scanner-measured data were correlated. Spearman-Rho comparisons of calculated model water radiances and scanner brightness values indicated very good to excellent agreement at a high level of significance.

Introduction
It is difficult to measure the resource characteristics of shallow water over large areas using traditional methods. Remote sensing technologies and models of light interactions in water can potentially supply additional data for determinations of water resource characteristics such as depths, bottom types, and concentration of water colorants. Previous studies have demonstrated the value of measurements of water variables in combined remote sensor and boat sampling experiments (Klemas *et al.*, 1974; Bukata *et al.*, 1976; Lathrop and Lillesand, 1986; Ackleson and Klemas, 1986; Lyon *et al.*, 1988). However, these projects generally restrict their analyses to one date or one time only, and use empirical models. The empirical approach is suitable for one-time analysis, but it provides limited assistance in analyses of frequently changing water characteristics. This makes the results of empirical experiments difficult to apply at other study sites, or to other kinds of remote sensing coverage. A deterministic approach potentially addresses these requirements and does so with a minimum of new field information to be supplied in the new application. Deterministic models may potentially help supply water characteristics over large areas at lower costs as compared with the requirements of traditional technologies.

The application of remote sensor data for measurement and modeling of water requires deterministic models based on physical and chemical processes (Weidmark *et al.*, 1981; Bukata *et al.*, 1983; Suits, 1984; Hollinger *et al.*, 1985; Lyon *et al.*, 1988). It remained to evaluate such an approach by optimizing an existing deterministic model for a given experimental application, develop a computational formula, derive results, and determine the relative accuracy of products with an independent remote sensor data set.

A computer simulation program was developed to model the flow of electromagnetic energy from the atmosphere through a water body and back to the surface where it could be measured by a remote sensor. The computational program (Hutchinson, 1989) was based on a radiation transfer (RT) model developed by Suits (1984), and it addressed the influence of water, water surface effects, and atmospheric influences on solar radiation and its propagation over space and time.

The model and program were composed of parameters that addressed physical and chemical processes in a deterministic fashion. Inputs to the program included the solar constant, extinction coefficients, and sand bottom type reflectance. These inputs were estimated from study area measurements and from the literature. To test the potential of the deterministic approach, a combined remote sensing and on-site sampling experiment was conducted. This experiment provided a detailed data set for verification of model results, and it was deemed valuable to conduct analyses of water resources using these data. The model simulated the radiance of a water column for a given bottom type and water depth, as measured by a sensor at the surface of the water body. The conditions of the model simulation were made to approximate those of the St. Marys River aircraft multispectral scanner overflight, allowing for verification of radiometric model results with airborne scanner data. This overflight was in support of environmental studies (USACE, 1988; Lyon *et al.*, 1992; Lyon *et al.*, 1994). The radiance results and depths from the computer model were compared to brightness values of multispectral scanner data in the verification of the effort.

Approach
The data analysis was performed in two steps. The first step involved calibration and model simulations using the RT model developed by Suits (1984) and computational model (Hutchinson, 1989), for determination of connecting channel water depths. The model accounted for each source of light and its contribution to the total signal measured by the re-

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mote sensor. The approach employs an RT model of water depth and water quality, and incorporates elements from several approaches to modeling (Suits, 1972; Bukata *et al.*, 1978; Jain *et al.*, 1981; Bukata *et al.*, 1983; Suits, 1984; Hollinger *et al.*, 1985).

The second step was to correct and enhance the remote sensor data set. This involved geometric and radiometric corrections, feature selection of data enhancement and reduction, and pattern recognition or computer categorization of water colorants classes (Lyon *et al.*, 1992). The second step yielded a database of water colorants classes which could be compared to the RT model calculations.

Background of the Radiometric Model

The available light to be measured by the sensor or scanner can be estimated by the radiance L. In this case, the water is a transparent material of varying thickness or depth. We can calculate (or measure) the radiance from various depths of water and predict the response if conditions are known.

To address the characteristics of water, we need to evaluate several physical and chemical processes of light, including the upwelling and downwelling quantities of diffuse irradiances, and the spectral irradiance components.

Observing the diffuse assumptions appropriate for fore- and back-scattering of light, Duntley's equations were rendered in the Suits (1972) notation. The differential equations that govern the flow of irradiance through a diffusing material were as follows:

$$dE(+d,x)/dx = -(\alpha+\beta)E(+d,x)+\beta E(-d,x)+\beta' E(s,x) \quad (1)$$

$$dE(-d,x)/dx = (\alpha+\beta)E(-d,x)-\beta E(+d,x)+(f' E(s,x) \quad (2)$$

$$dE(s,x)/dx = (\alpha'+\beta'+f')E(s,x) \quad (3)$$

where E was the irradiance in a narrow spectral band upon a plane oriented perpendicular to the flow of the light energy. $E(+d,x)$, $E(-d,x)$, and $E(s,x)$ denote diffuse irradiance in the upward and downward directions, and the specular (collimated) irradiance, respectively. These variables are shown as (c), (d), and (e) on Figure 1. The specular irradiance $E(s,x)$ had only a downward component (Figure 1), due to the fact that the water surface was directly illuminated from only one side of the sun. The symbols α, β, and f were the absorption, backscattering, and forescattering coefficients of water, respectively. The primed values of Equation 1, 2, and 3 refer to the specular irradiance and the unprimed values referred to the diffuse irradiance.

The differential equations allowed for the calculation of the change in irradiance moving through a diffusing medium, water, as well as for the calculation of changes due to scattering and absorption. For example, the change in irradiance moving in the positive x direction, $dE(+d,x)$, would be less than the amount of irradiance moving in the same direction that was absorbed and backscattered. Consequently, a calculation of irradiance at a given point would also gain the irradiance that was backscattered from both the diffuse and specular irradiance sources that were moving in the opposite direction.

Suits (1984) rewrote Equations 1, 2, and 3 and combined the absorption, backscattering, and forescattering components using five extinction coefficients: i.e.,

$$dE(+d,x)/dx = -aE(+d,x)+bE(-d,x)+cE(s,x) \quad (4)$$

$$dE(-d,x)/dx = aE(-d,x)-bE(+d,x)-c'E(s,x) \quad (5)$$

$$dE(s,x)/dx = kE(s,x) \quad (6)$$

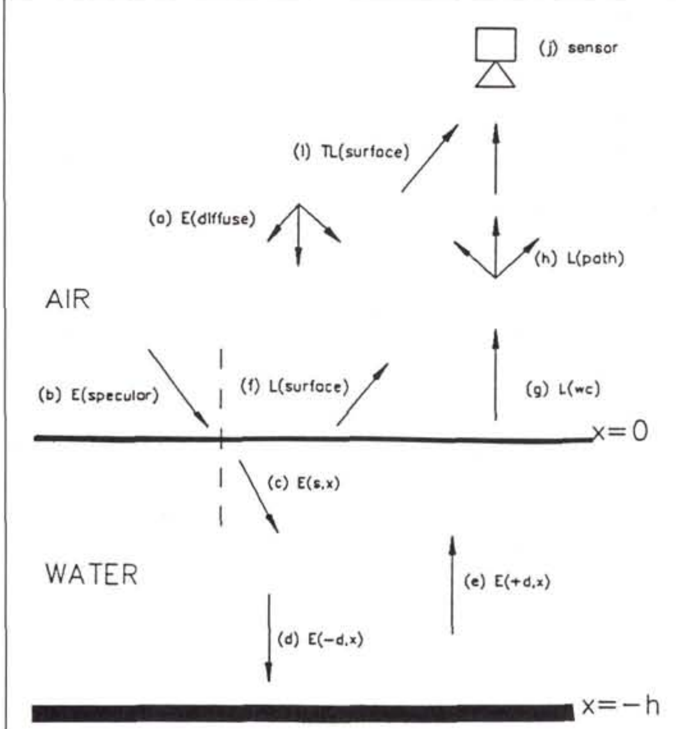

Figure 1. Diagram of light components of water bodies.

where a was the extinction coefficient (incorporating both absorption and scattering) for diffuse irradiance, b was the backscattering coefficient for diffuse irradiance, c was the backscattering coefficient for specular irradiance, c' was the forward scattering coefficient for specular irradiance, and k was the extinction coefficient for specular irradiance. By solving differential Equations 4, 5, and 6, one could then calculate the irradiance at any specific depth in the water column.

Suits also developed this model to take into account materials carried in the water. Unlike plastics that exist as one material, natural water bodies have added particulate composed of a variety of materials. The capability to address a variety of suspended and dissolved materials is a strong point of this modeling approach. These capabilities were not used here, due to the clear, clean water characteristics measured in the study area (<0.3 mg/l chlorophyll a and <1.0 mg/l suspended sediment).

Suits also developed mathematical expressions for these parameters by quantifying the interaction of light with water. These steps are defined in Suits' papers cited here, and in a master's thesis (Hutchinson, 1989).

The radiance from the water column was the sum of radiance from the water volume and radiance reflected from the bottom. From Suits (1984) the radiance above the surface of the water column was equal to

$$L(wc) = T_r(1/n_w^2)\,[\int_{-h}^{0} dL(\text{water volume})/(dx)^* e^{kh} dx + L(\text{bottom})e^{-kh}] \quad (7)$$

where T_r was the transmittance through the air-water interface, n_w was the index of refraction of water, k was the beam extinction coefficient in the angle of view of the sensor, and

h was the depth of the water column. The term kh was the probability of having a direct line of sight from the layer into the direction of view of the sensor.

In the case of given depth x, the radiance of an infinitesimally thin layer of water can be calculated from

$$dL(\text{water volume})/dx = (1/\pi)[uE(+d,x)+vE(-d,x)+wE(s,x)] \quad (8)$$

where u, v, and w were the factors expressing relative proportions of upwelling diffuse, downwelling diffuse, and specular irradiance that combine to make up the total irradiance on the horizontal plane or given depth of water.

For any depth x, these irradiances can be calculated from

$$E(+d,x) = A(1-f)e^{gx} + B(1+f)e^{-gx} + Ce^{kx} \quad (9)$$

$$E(-d,x) = A(1+f)e^{gx} + B(1-f)e^{-gx} + De^{kx} \quad (10)$$

$$E(s,x) = E(s,o)e^{kx} \quad (11)$$

A and B are constants that depend on the boundary conditions defined below. The values of f, g, C, and D were directly calculated from the extinction coefficients a, b, c, c', and k (Suits, 1984) and were dependent on the optical properties of the water: i.e.,

$$f = [(a-b)/(a+b)]^{1/2} \quad (12)$$

$$g = (a^2-b^2)^{1/2} \quad (13)$$

$$C = \frac{c(k-a)-c'b}{k^2-g^2}E(s,o) \quad (14)$$

$$D = -\frac{c'(k+a)+cb}{k^2-g^2}E(s,o) \quad (15)$$

The extinction coefficients input to the model a, b, c, c', and k were all optical properties of the water body. These values were wavelength dependent. These extinction coefficients were defined as

a = extinction coefficient for diffuse irradiance,
b = the upper hemisphere or backscattering coefficient for diffuse irradiance,
c = he upper hemisphere or backscattering coefficient for specular irradiance,
c' = the lower hemisphere or forward scattering coefficient for specular irradiance, and
k = the beam extinction coefficient.

The equation describing the boundary conditions at the air/water interface was

$$E(-d,o) = [r/n_w + (1-1/n_w^2)]E(+d,o) + (1-r)E(\text{diffuse}) \quad (16)$$

where E(diffuse) was the incident diffuse irradiance in a given spectral band from the sky, shown as a on Figure 1. This equation states that the downward diffuse irradiance was equal to any upwelling diffuse irradiance that was internally reflected, plus any incident diffuse radiation from the sky.

The bottom sediment material was assumed to be Lambertian and to exhibit diffuse hemispherical characteristics (Suits, 1984). These phenomena were expressed by

$$E(+d,-h) = r_b[E(-d,-h)+E(s,-h)] \quad (17)$$

The computer algorithm solved the boundary conditions for parameters A and B in Equations 9 and 10 and solved for the irradiance. In this application, however, there were a variety of depths at the study site. The dependence of A and B on water depth made it necessary to solve for a set of As and Bs which corresponded to the appropriate depth and bottom type.

For the conditions of this experiment, we simulated the radiance from homogeneous thicknesses of water in 0.6-m increments between the upper boundary (at $x = 0$) or the air/water interface, and the lower boundary (at $x = -h$) was the bottom of the river channel, with the positive x value in the upward direction as shown in Figure 1. Here, $-h$ was equal to 3 metres of water.

To complete this task, the radiance that could be recorded by a sensor was calculated by the model for water depths in 0.5-m increments above a given bottom sediment type. The depth-radiance data and model results were stored in a "look-up table" for further reference.

Methodological Approach

A computer program was developed to make calculations based on Suits (1984) radiometric model (the program is in Hutchinson (1989) or can be obtained from the authors). Results from this computer program were the radiance attributed solely to the water column. In Equation 7, $L(wc)$ was calculated because it approximated that quantity measured by the sensor. This calculated parameter was the same quantity as $L(wc)$ or g on Figure 1.

Estimation of Inputs to the Program

Inputs to the computer algorithm were selected as appropriate to the site conditions and assumptions used in the experiment. Sources for coefficients and parameters included actual measurements and literature references of conditions on the day of overflight of the site to be simulated using the radiometric model (USACE, 1988; Lyon *et al.*, 1992). These inputs included solar illumination conditions, water extinction coefficients, bottom type reflectances, beam angle of view extinction coefficient, and other parameters which describe the amount of irradiance on a horizontal plane such as a water surface (Valley, 1965; Jerlov, 1968; Wiedmark *et al.*, 1981; USACE, 1988).

The Suits model partitioned the solar illumination into two components, the diffuse and the specular. The diffuse irradiance was called the illumination contribution due to skylight (RCA, 1968; Suits, 1984). Diffuse solar illumination for skylight was estimated to be approximately equal to one-fifth the direct solar illumination (RCA, 1968; Wiedmark *et al.*, 1981).

Total solar illumination was obtained from the literature (Valley, 1965). Valley's document provides tabulations of incoming radiation from the sun in a variety of wavelength intervals or increments. These data were summed to estimate the amount of incoming radiation in a particular wavelength range. The percentage of direct solar illumination was dependent on the sun angle and wavelengths of the light. In this case, the sun elevation angle was approximately 40 degrees, as determined by the field notes of the flight crew and by solar elevation tables. These wavelength ranges were the same as those specified in the description of the scanner data, wavelength bands of 0.05 μm.

Estimates of the percentage of illumination that was composed by skylight were also found in the literature (Jerlov, 1968). Jerlov's approach and the values above were used in the calculation. The direct solar illumination was then calculated such that the results were equal to the total solar illumination minus the percent contribution that was attributed to skylight. These values were used as estimates of solar illumination conditions on the day of overflight.

There are five general bottom types found in the St. Marys River. These types included sand bottom, and bottoms that were combinations of sand/silt, silt/sand, silt/clay, and sand/rock-silt bottom types (USACE, 1988). Due to sample size limitations based on sediment type and depth class combinations, only the sand bottom sediment type was used in the analysis. The bottom type reflectance was estimated from reflectance spectra for equivalent wet soil. These soil types were selected by checking the geology of the area and using this knowledge to best estimate the mineral composition of the soil. Reflectance of a similar soil type was then selected from Leeman *et al.* (1971).

Extinction coefficients, such as k in Equation 7, were estimated from Secchi Disk measurements using the technique developed and tested by Scherz (1977). Secchi Disk depth on the day of overflight was 3 metres. In addition, Bukata *et al.* (1974; 1978; 1983; 1988) measured extinction coefficients and Secchi Disk depths of all the Great Lakes, and estimates of k taken from Lake Superior were similar to those of the St. Marys river during the overflight.

From Duntley (1942), Suits (1984), Weidmark *et al.* (1981), and Bukata *et al.* (1988), the following relationships were derived to address the extinction coefficients of optical characteristics of water and particles in the water:

$$
\begin{aligned}
k &= \alpha' + \beta' + f' \\
a &= \alpha + \beta \\
b &= \beta \\
c &= \beta' \\
c' &= f'
\end{aligned}
$$

where α was the absorption coefficient, β was the beam portion of irradiance backscattered, and f' was the portion of the beam irradiance scattered forward. The parameters which have prime notation referred to incident irradiance, and the other values referred to the diffuse irradiance. It was assumed that the few particles found in the river had a random orientation and position and, therefore, the scattering of diffuse irradiance was equal to the scattering of specular irradiance, of $\beta+f=\beta+f'$ (Duntley, 1942).

The values of the extinction coefficients a, b, c, c', and k (explained above) were estimated from the work of Weidmark *et al.* (1981), Duntley (1942), Liston *et al.* (1983), and Bukata *et al.* (1988).

The Weidmark article provided values for the irradiance attenuation coefficient, the scattering coefficient, and the backscatter probability. These values were particularly appropriate for evaluation of water resources using remote sensor data with the same wavelength band as used in this study. In addition, the Bruce Peninsula area of Lake Huron exhibits water quality conditions similar to those of the St. Marys River, and the extinction coefficients were similar to those measured on the St. Marys River (Liston *et al.*, 1983).

Verification of Model Results

Scanner Data

In order to test the computational model, it was necessary to correlate the results of model simulations with the independent data sources. These included airborne multispectral data provided by the U.S. Environmental Protection Agency (EPA), and water depths from the U.S. Army Corps of Engineers, Detroit District (USACE, 1988) and from available NOAA bathymetry maps.

On 19 October 1985 the USEPA flew an aircraft with a Daedalus 1260 multispectral scanner over the St. Marys River. The sensor had 11 spectral bands ranging in wavelengths from 0.38 to 14.0 µm. The sensor recorded radiance in digital numbers from 0 to 255. The exact details of the scanner data and the analysis were described in USACE (1988) and in Lyon *et al.* (1992).

At the time of overlight, the Detroit District of the U.S. Army Corps of Engineers set up 36 stations in the St. Marys River area to sample water characteristics. These sample variables included Secchi Disk, water depth, chlorophyll a, suspended sediments, water temperature, and bottom sediment type (USACE, 1988).

The ERDAS clustering algorithm was run on the scanner data in order to reduce the volume of data for this analysis (Lyon *et al.*, 1988). The clustering algorithm established training sets of pixels that were spectrally similar groupings, and were reduced to 50 classes in this case (Lyon *et al.*, 1992).

Atmospheric corrections were not performed on the data set because of (1) the uniform and clear weather conditions on the day of overflight; (2) visual inspections of concurrent aerial photos showed no appearance of atmospheric distortions; (3) the small size of the study area limited the possibility of distortion; (4) the relatively low altitude of the sensor reduced the possibility of path radiance; and (5) the use of categorized scanner data accounts for small, residual quantities of atmospheric turbidity as a natural component of the variability of the data (Lyon *et al.*, 1988).

The mean brightness values of the clustered groups were used for comparative analyses with the calculated radiance from the radiometric model.

Verification

To verify any model, model simulations and independently measured values need to be compared. In this study, the value of interest was the water depth estimated by each method. However, it was very difficult to invert the equations of this radiometric model (due to multiple variables) and solve directly for the depth from the radiance (Lara, 1978). To solve this inversion problem, a "look-up" table procedure was developed to relate the model results to the sensor data.

In this procedure, a series of water depths were input to the radiometric-model computer program developed in the study. The program output was a radiance value for each depth for a given bottom type (e.g., sand sediments). These values were produced for each of four wavelength bands, 3(0.45 to 0.50 µm), 4(0.50 to 0.55 µm), 6(0.55 to 0.60 µm), and 5(0.60 to 0.65 µm).

Two non-parametric tests were chosen to compare the results of the radiometric model and the scanner data in the verification experiment. The first test was Spearman's correlation coefficient. The second was Kendall's tau. Both statistics are non-parametric assessments of association between two variables. This approach was chosen for several reasons, including (1) the clustered scanner data had non-normal distributions, (2) the sample sizes in different depth classes were not the same, and (3) data were assumed to be monotomically increasing (Lyon *et al.*, 1988).

Results

Verifications were made from the model radiances and were correlated with the mean brightness values of the scanner data. Spearman's rho rank correlation demonstrated the relationship between model radiance values and brightness values from the categorized remote sensor data. Strong relationships were found in bands 4, 5, and 6 (e.g., the blue-green, green, and red wavelength ranges). Spearman's rho values ranged from $r_s = 0.77$ to 0.94, with probability $p>0.050$-0.005 in these three bands. The sample size was 25 for each band.

A poor relationship was found in the blue region of the visible spectrum band 3, with a rho value of $r_s = 0.14$. The poor correlation was probably due to scattering effects of the water that affects blue light to a greater extent, and results in poor return of light to the sensor (Bukata *et al.*, 1974).

For the model results as compared with the scanner data depth results, the Spearman's rho demonstrated strong association in the green (0.55 to 0.60μm) and red (0.50 to 0.65μm) wavelength regions, respectively. Spearman's rho values were $r_s = 0.82$ and $p>0.010$ for both bands 5 and 6. A poor association was again found in bands 3 and 4, with rho values ranging between $r_s = -0.06$ to 0.38.

The Kendall's tau test also indicated a strong relationship between calculated radiance from the Suits model and clustered brightness values from remote sensor data for equivalent depths and sand bottom types in the blue-green, green, and red channels, bands 4, 5, and 6. This was found for the data utilizing the NOAA map to derive depths. A poorer correlation was again found in band 3. Kendall's tau for bands 4, 5, and 6 ranged from $\tau = 0.60$ to 0.70. The level of significance used in both these tests was $p>0.10$. Both statistical tests on the two data sets indicated a strong association between variables in band 5 (green) and band 6 (red), and poorer association in band 3 (blue) and band 4 (blue-green).

Conclusions

Two nonparametric association tests confirmed the similar depth measurement results from similar experimental conditions simulated by the Suits' radiometric model and measured by the scanner. The strongest correlations were found in bands 5 and 6, the "green" and "red" bands. Spearman's rho values ranged from $r_s = 0.82$ to 0.94 and Kendall's tau values ranged from $\tau = 0.66$ to 0.87. These tests were significant at levels ranging from $p>0.010$ to 0.005 for these two bands of data.

The results for band 3 (blue channel) indicated no relationship (or a weak one) between the variables in both verifications. Band 4 had strong correlation in one of the verifications and random association in the other. Spearman's rho value in the verification had a strong correlation of $r_s = 0.77$ and $p>0.05$.

The results for bands 5 and 6 were quite good, considering that some of the inputs to the computer model were estimated from the literature and available field sampling data.

A previous verification of the Suits model was done under ideal and very controlled conditions in a laboratory environment (Lara, 1978). The results of this study show that the Suits model can be applied in at least one example of a more general, field-measurement situation using a few key input parameters to the model.

Discussion

There is great potential in remote sensing technologies to supply data and augment traditional hydrographic measures. Traditional measurement technologies can be qualitative, time consuming, and people intensive. A combined modeling and remote sensing approach can provide additional data and potentially increase the accuracy of water resource determinations or reduce the costs associated with on-site sampling (Lyon *et al.*, 1988; Lyon, 1993).

This modeling approach can hopefully provide opportunities to use models in more than one location or at different times. RT models can simulate various water resource and atmospheric conditions, and the general approach may offer the potential to fulfill the need for a model that can be used at different times and places upon appropriate calibration.

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Nomenclature

a extinction coefficient for diffuse irradiance
B constant determined by boundary conditions
b backscattering coefficient for diffuse irradiance
c backscattering coefficient for specular irradiance
c' forward scattering coefficient for specular irradiance
d diffuse irradiance
$dE(+d,x)/dx$ change in upwelling diffuse irradiance with x
$dE(-d,x)/dx$ change in downwelling diffuse irradiance with x
$dE(s,x)/dx$ change in specular irradiance with x
E_o total incident irradiance in a narrow spectral band
$E(+d,x)$ diffuse irradiance in a narrow spectral band at depth x moving in upward direction
$E(-d,x)$ diffuse irradiance in narrow spectral band at depth x moving in downward direction
$E(s,x)$ specular irradiance in narrow spectral band at depth x
E(specular) direct solar illumination in a narrow spectral band volume fraction occupied by water
F_w volume fraction occupied by water
h thickness of total depth of water layer
k extinction coefficient for specular flux
L_o incident radiance in a narrow spectral band
L_x radiance in a narrow spectral band at point x
$L(a)$ path radiance due to aerosol scattering
L(bottom) radiance in a narrow spectral band reflected off the bottom of water body
L(path) path radiance
L(path at i) path radiance of pixel i
L(path, clearwater) path radiance of a pixel that views clear water
$L(R)$ path radiance due to Rayleigh scattering
L(sensor) radiance in a narrow spectral band at the sensor
L(surface) radiance in a narrow spectral band reflected of the target surface
$L(wc)$ radiance in a narrow spectral band from the water column. Includes radiance from water column and from the bottom of the water body, but not the water surface effects
L(water volume) radiance in a narrow spectral band from the water volume. Does not include radiance from the bottom of the water body
n_w index of refraction of water
r hemispherical reflectance from air to water
r_b hemispherical reflectance of bottom
s specular
T_r transmittance through water-air interface
u fraction of upwelling diffuse irradiance upon a horizontal plane
v fraction of downwelling diffuse irradiance upon a horizontal plane
w fraction of specular irradiance upon a horizontal plane
x depth in feet or metres
α absorption coefficient for diffuse irradiance
α' absorption coefficient for specular irradiance
β portion of diffuse irradiance backscatter
β' portion of specular irradiance backscatter
f portion of diffuse irradiance scattered forward
f' portion of specular irradiance scattered forward
τ Kendall's tau correlation coefficient