The Use of Reference Surfaces to Determine Repeat-Orbit Variability in Satellite Altimetry

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Abstract
Two alternative techniques for estimating the variability of the radial orbit error for collinear tracks are investigated using Geosat altimeter data. The first uses sinusoidal fitting to ocean height differences around an orbit, and the second uses relatively flat areas of land (in the Simpson Desert, Australia, and the Antarctic Plateau). Using a non-ocean surface requires knowledge of the local surface slope, and we obtain this through the fitting of a plane to the set of repeat height measurements. The difference in the relative-orbit-error estimates from the two techniques is 12 cm root-mean-square (RMS), from which we conclude that relative orbit error can be reduced to less than 9 cm using ocean fitting, and to between 9 and 12 cm using land fitting. The Antarctic plateau could not be used as a reference as the orbit error appeared correlated with the cross-track displacement of repeat tracks, preventing the determination of the local surface slope. The land analysis was also limited by lack of waveform data and Geosat off-pointing; current altimeter missions (e.g., ERS-1 and Topex/Poseidon) should be able to achieve higher accuracies.

Introduction
Radial orbit error is the single largest source of error in determining surface heights from satellite radar altimetry. However, for many applications of altimeter data requiring repeat observations, e.g., determining changes in large-scale ocean topography or measuring the variation in water level in lakes or wetlands, it is sufficient to know the variation in the satellite orbit rather than its absolute value. This can be achieved through the use of height reference surfaces that are assumed to be fixed or varying in a known way. In this paper, we investigate two alternative techniques for estimating the variability of the radial orbit error for collinear tracks using Geosat data.

The first technique uses the mean ocean surface, corrected for tides, etc., as a reference. The effect of the dynamic topography of the ocean surface is reduced by taking advantage of the fact that difference in height measurements for repeat orbits (the relative orbit error) has a once per cycle sinusoidal form (Cartwright and Ray, 1990). A number of researchers have used collinear repeat tracks to estimate orbit error in this way (e.g., Cheney *et al.*, 1991; Van Geysen *et al.*, 1992).

The second technique uses land surfaces for which the topography and radar backscatter are constant. Previous research into altimeter measurements over land has shown that there are several large, uniform desert regions in the world that give "good-quality" radar echoes (Rapley *et al.*, 1987; Guzkowska *et al.*, 1990). For example, areas of the Libya Sand Sea, Taklimakan Shamo, and the Kalahari and Simpson Deserts are uniform over hundreds of kilometres and are very flat (i.e., have large-scale surface slopes less than 0.1°). In principle, these deserts, together with the flatter regions in Greenland and on the Antarctic Plateau, could be used to determine the relative orbit error at a number of points around an orbit arc. The relative orbit error at other locations around the orbit could then be determined through sinusoidal interpolation, as in the ocean technique. Many orbits will not cross suitable reference surfaces, and this may be a drawback compared to using the ocean for certain applications. The effect of the distribution of suitable reference surfaces is not addressed here as it is application dependent.

The two techniques have relatively independent sources of error, and so their merits and accuracy are assessed by comparing the relative orbit error derived using the ocean with that derived at two land reference sites for the same set of orbits. The Simpson Desert in central Australia was chosen as one of the reference sites because it was already known to provide "well-behaved" altimeter returns (Chua *et al.*, 1991); and Australia has extensive ocean on three sides, enabling good interpolation of the ocean results. A region on the Antarctic Plateau around 56°E, 72°S was chosen as the other reference area because it was crossed by two of the orbits that passed over the Simpson. In principle, much of the Antarctic Plateau could be used as a reference surface due to the uniformity of its terrain and its low surface slopes. Geosat Geophysical Data Records (GDRs) for four orbits with repeat data in 1987 and 1988 were used in the analysis.

The Geosat Data
The Geosat altimeter satellite, launched by the U.S. Navy in 1985, had the primary objective of mapping the marine geoid. Initially, it performed a classified Geodetic Mission between 1 April 1985 and 30 September 1986. During this period, the satellite was in a very long repeat orbit, with a ~5 km cross-track spacing at the equator, in order to perform fine-resolution geoid mapping. The second phase of the mission began on 8 November 1986 and was known as the exact repeat mission (ERM). During this period the satellite was placed in a 17-day repeat orbit with a ~160 km cross-track

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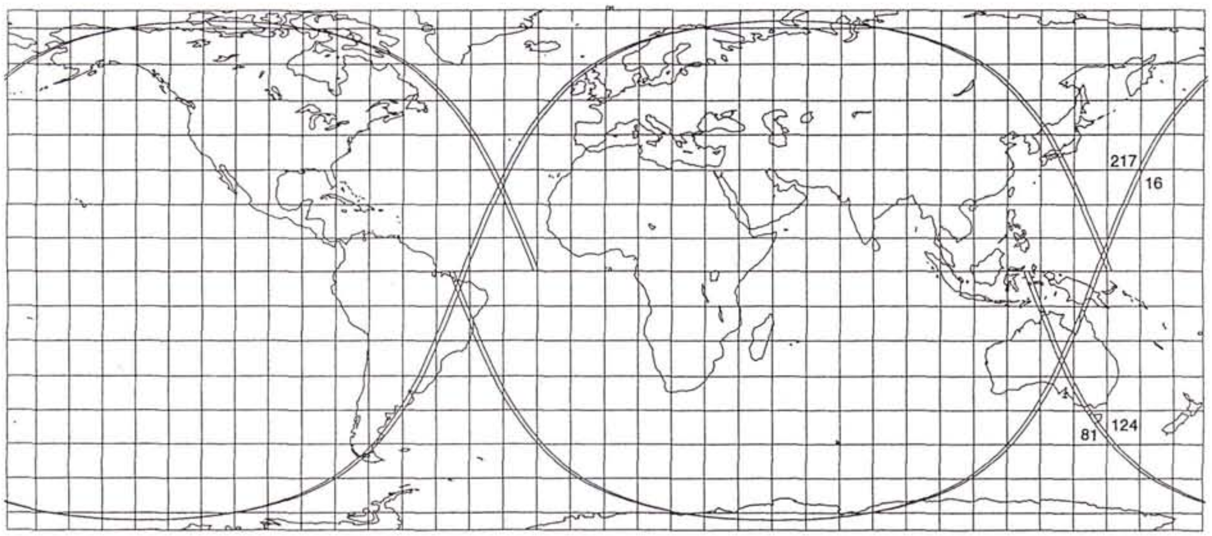

Figure 1. Global ground tracks for the four Geosat orbits that cross the Simpson Desert in Australia. The tracks are labeled with the track number assigned sequentially to the 243 orbits within each 17-day repeat period.

spacing at the equator, resulting in the orbit ground track repeating every 243 orbits. Sixty-two repeat cycles (usually referred to as ERMs) were completed before the satellite finally failed in January 1990, almost five years after launch. The orbit of the satellite was controlled by rocket burns, with updates being required every few weeks, to ensure that the ground tracks repeated to within ±1 km. The data from this part of the mission are not classified, and 1 Hz averaged data (Geophysical Data Records) are available from the NOAA National Oceanographic Data Center in the U.S.A. (Cheney *et al.*, 1991).

There are four Geosat ground tracks that cross the Simpson Desert, two ascending (tracks 81 and 124) and two descending (tracks 16 and 217). The track numbers used here refer to the orbit number within each 17-day repeat period. Figure 1 shows the location of the ground tracks on a map of the world. The results from the analysis of track 16 are presented in this paper. The Antarctic Plateau is crossed only by the two descending tracks due to the asymmetric distribution of land around the South Pole, and the latitudinal limit of 72° for Geosat. The ERS-1 satellite has a larger latitudinal limit (82°) which will result in over half of its orbits crossing the Antarctic Plateau.

Geosat completed 62 ERMs in all. Improved orbit ephemeris data, based on the GEM-T2 geopotential model (Haines *et al.*, 1990), were available to us for only the first 43 ERMs (covering 1987 and most of 1988). Data error problems meant that ERMs 1, 5, and 20 could not be included in the analysis, resulting in 40 ERMs of data for processing. Excluding the remaining ERMs from the analysis is not a serious drawback because towards the end of 1988 the stability of the spacecraft began to be seriously affected by the increased solar activity associated with the maximum of the sunspot cycle which occurred in 1989. The pendulum motion of the satellite increased, resulting in more frequent "off-pointing," which caused a loss of data quality and the instrument to loose lock of the surface more often.

Satellite off-pointing, which results in the boresight of the antenna not pointing to the nadir, causes a change in the pattern of radar illumination of the surface, and this distorts the shape of the return echo. This has an effect on the onboard height tracker that is dependent on the surface topography. Over the ocean, the topography is quantified using the significant wave height (SWH) parameter. The SWH itself can also alter the tracking characteristics, and so the combined effect of off-pointing and SWH is theoretically calculated before the mission and entered into look-up tables. Off-pointing and SWH estimates derived from the return-echo waveform shape are used to extract the appropriate correction from the tables, and the value is inserted into the GDR data.

In this analysis, when calculating the surface heights, the ionospheric corrections and the Fleet Navy Operational Center (FNOC) atmospheric corrections given in the GDR data were used. Data over the ocean were also corrected for tides and the SWH/off-pointing correction. For data over the land, only the Earth tide correction was added. The SWH/off-pointing correction was not applied because the off-pointing estimates are known to be incorrect over land. This is a result of the return-echo waveform shape being non ocean-like, and the long averaging time of ~2 minutes required for the off-pointing calculation. The land data are therefore uncorrected for Geosat off-pointing. This can introduce errors of up to ±17 cm in individual surface height measurements for SWH values of up to 5 m (the maximum value allowed in the analysis).

Altimeter Corrections

Many corrections have to be applied to convert the time delay measured by a radar altimeter into first a range measurement and then a surface height measurement. The corrections required and their accuracies have been discussed extensively elsewhere (Seasat Special Issue I, 1982; Geosat Special Issue I, 1990; Guzkowska *et al.*, 1990; Cheney *et al.*, 1991), and a brief but comprehensive overview of the operating principles and terminology of space borne radar altimeters can be found in Rapley (1990).

The surface height, h, relative to the Earth's reference ellipsoid is given by

$$h = (h_{sat} + \Delta h_{orb}) - (r_m + \Delta r_{inst} + \Delta r_{atmos} + \Delta r_{est} + \Delta h_{tide} + \Delta h_{dyn} + \Delta h_{slope}) \quad (1)$$

where

h_{sat} is the altitude of the satellite above the Reference Ellipsoid at the instant of measurement;
Δh_{orb} is the radial orbit error (including the effect of time-tag bias error);
r_m is the range estimate derived directly from the telemetered time-delay measurement;
Δr_{inst} is the net instrument correction, which includes the center-of-gravity correction, antenna corrections, electronic delays, etc.;
Δr_{atmos} is the net atmospheric correction and includes contributions from the dry, wet, and liquid water tropospheric corrections, together with the ionospheric correction; and
Δr_{est} is the net correction derived from estimating the location within the range window of the point on the leading edge of the return echo corresponding to the mean surface; it includes the on-board tracker error (or retrack error if the data are retracked (Martin *et al.*, 1983)), the SWH/off-pointing correction, and a surface bias correction (referred to as the sea-state bias correction, over the ocean).

The remaining corrections are concerned with the conversion of range to surface height:

Δh_{tide} is the net tidal correction (including Ocean, Ocean Loading, and Earth tides) required to convert the height measurement at the time of observation to the mean height (over time);
Δh_{dyn} (only applicable over water) accounts for the height changes produced by such things as eddies and currents, and the depressing effect of high pressure systems (the barometric correction); and
Δh_{slope} (only significant over land) accounts for the fact that the altimeter ranges to the nearest point on the surface rather than the nadir point. The correction is usually referred to as slope-induced error.

The bias component of many of these corrections will be canceled out when height differences are calculated. We can write the relationship between height difference, δh, and relative orbit error, δh_{orb}, as follows:

$$\delta h = \delta h_{orb} + L\sin\alpha + \delta r_{atmos} + \delta r_{est} + \delta h_{dyn} + \delta h_{tide} \quad (2)$$

where

L is the horizontal distance between the measurement points, and $L\sin\alpha$ is the vertical displacement between the measurement points assuming the local topography is uniform and can be characterized by a mean surface gradient, α;
δr_{atmos} is the error in the atmospheric corrections;
δr_{est} is the error in the tracking and surface-bias corrections;
δh_{dyn} only applies to ocean data and is the height difference resulting from changes in dynamic effects (eddies, currents, atmospheric pressure, etc.). The features extend typically just a few hundred kilometres and constitute the "noise" signal observed in height difference plots over the ocean; and
δh_{tide} is the error in the tidal corrections. It can be several metres in coastal regions. It is insignificant over land.

Typical large-scale surface gradients over the ocean are ~2 cm/km; therefore, the ±1-km cross-track displacement of repeat orbits introduces a height variation of <2 cm, which is small enough to be ignored. Observations in the vicinity of ocean trenches and seamounts, where larger surface gradients exist, are excluded from this analysis. Generally, surface gradients over land are much larger, and the uncertainty in this gradient constitutes the largest potential source of error in the use of land as a height reference.

Errors derived from Δr_{inst} and Δh_{slope} do not appear in Equation 2 because they constitute a constant bias that cancels for repeat observations. The magnitudes and effects of the error terms in Equation 2 are discussed in detail in later sections.

Ocean Reference Surfaces

Theoretical calculations (Cartwright and Ray (1990) and references therein) have shown that radial orbit error has a dominant frequency of 1 cycle/revolution, and height differences for collinear tracks can be expressed as

$$h(t) = A(t)\cos\Omega t + B(t)\sin\Omega t + C(t) \quad (3)$$

where $\Omega = 2\pi$/orbital period, and $A(t)$, $B(t)$, $C(t)$ are coefficients which vary with time but on a time scale long compared with Ωt. For single-orbit height differences, $A(t)$, $B(t)$, and $C(t)$ can be regarded as constants. Note that Equation 3 is functionally identical to $h(t) = D(t)\sin(\Omega t + \phi) + C(t)$, where $D(t)$ is the amplitude and ϕ the phase of a single sinusoid.

For repeats of ground track 16, we used ERM 22 as the reference track as it is near the median of the cross-track spread of orbits, and it is towards the middle of the period of the data being analyzed. The height differences were generated using the time into the orbit after aligning the orbits at the ascending node equator crossing. The repeat data were linearly interpolated from adjacent 1-Hz values to the reference times, and subsequently averaged over 17 data points (~112.2 km) in order to reduce the data volume for further analysis. Averaging over 17 samples gives a point spacing of about 1° and about 200 points per orbit for fitting.

Visual inspection of the sinusoidal fits to the height differences appear satisfactory even when over half the ocean data were lost due to excessively large satellite off-pointing. This occurred mainly towards the end of 1988 when the solar activity was increasing and was affecting the stability of the spacecraft. Figure 2 illustrates two fits to ocean height differences.

There are a number of factors which can affect the quality of the fit, as indicated by the components of Equation 2.

Collocation Error ($L\sin\alpha$)

As mentioned earlier, the surface gradients over the ocean are sufficiently small that the effect of the ±1 km cross-track

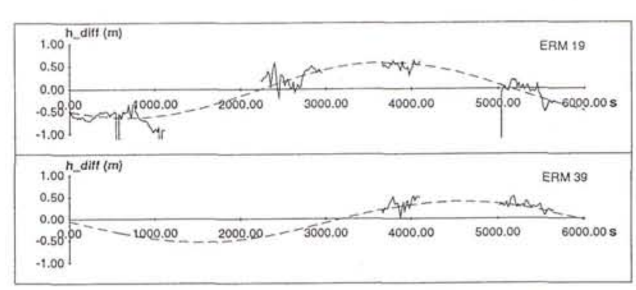

Figure 2. Typical sinusoidal fits to ocean height differences (h_diff) for track 16. The *x*-axis is the time in seconds from the start of the orbit (ascending equator crossing). Four segments of ocean data are usually seen (e.g., ERM 19) although for some orbits less data are available (e.g., ERM 39) due to the altimeter losing lock of the surface through satellite off-pointing.

displacement of repeat orbits can be ignored. However, because we use the time into the orbit to match repeat data, rather than the location itself, there is a small along-track error introduced by changes in the orbit timing. Analysis of the effect shows that, although the orbit period is constant to about 0.01 seconds, the difference in the time spent in the northern and southern hemispheres varies seasonally with peak to peak amplitude of about 0.3 seconds. However, given the satellite velocity of ~6.6 km/sec and typical basin-scale gradients of 2 cm/km, the resulting error is at most a few centimetres and so can be ignored.

Atmospheric Correction Error (δr_{atmos})

The main components of the atmospheric residual errors will be due to the wet tropospheric correction and the ionospheric correction. The ionospheric correction in the geosat GDRs is discussed in detail in Cheney *et al.* (1991). They conclude that the Geosat ionospheric model underestimated the global ionosphere as the solar maximum approached through 1988 and 1989. Errors of a few centimetres are likely. Emery *et al.* (1990) discuss the FNOC wet tropospheric correction and conclude that the FNOC model consistently underestimates water vapor in the tropics by as much as 10 cm, with the error varying with time. The maximum error in the atmospheric correction difference is therefore likely to be on the order of 10 cm.

Range Estimation Correction Error (δr_{est})

The contribution to this term from the performance of the on-board tracker is insignificant over the ocean because the data are averaged along track. It is not insignificant over land because only limited along-track averaging is possible.

The error in the SWH/off-pointing correction is also included in this term, although, over the ocean, the error in this correction is assumed to be small. However, Hayne and Handcock (1990) have shown that further corrections could be applied to the "standard" SWH/off-pointing corrections given in the GDR data. These additional corrections, although typically <10 cm, can be as large as 20 to 30 cm for combined extremes of off-pointing and significant wave height. However, there will be a reduction in the associated error when a whole orbit is analyzed because the typical wavelength of the off-pointing variation is less than an orbit.

Similarly, the sea-state bias correction is also included in this term, but a correction is not applied in this analysis because it is of sufficiently short wavelength to be averaged out around the orbit.

Tidal Correction Error (δh_{tide})

The Schwiderski Tide Model given in the GDRs is likely to contain errors of 10 cm or so in the open ocean, increasing to a metre or more in coastal regions.

Ocean Dynamic Feature Error (δh_{dyn})

The ocean surface has dynamic features such as eddies and currents which can have relatively large amplitudes of ~50 cm, over scales of 100's of kilometres, and changing on a time scale of days.

One advantage of the ocean fitting is that all these errors have typical wavelengths that are short compared to the length of an orbit and so the effect of the errors is reduced when data from a whole orbit are fitted. It is difficult to quantify this reduction because the behavior of the errors around the orbit depends on many factors. Therefore, the best measure of the uncertainty in the relative orbit error estimates is obtained through a comparison with values derived independently over land reference surfaces.

For the ERMs of track 16, the standard deviation of the residual from the fit is typically 15 cm, reflecting mainly the variability in the surface due to dynamic features, etc. The amplitudes of the fitted functions (the maximum relative orbit error) have a mean of 45 cm, and a maximum amplitude of 1.06 m. This is consistent with the published precision for the GEM-T2 orbit ephemeris of 10 to 25 cm root-mean-square (RMS) initially, rising to 40 to 60 cm through 1988 (Cheney *et al.*, 1991).

Having derived the functional form of the relative orbit error for each ERM, the values over the Simpson Desert were calculated and are displayed in Figure 3. The variation has an RMS of ~30 cm (1.6 m peak to peak), indicating the extremes of the orbit error tend not to occur over the Simpson Desert.

Land Reference Surfaces

Determination of relative orbit error using land reference surfaces has the advantage that the surface topography is static, i.e., has no significant tidal or dynamic signal as there is over the ocean. A complication, however, is that over land the effect of the cross-track surface slope cannot be ignored. With a cross-track displacement of repeat orbits of up to ±1 km, to achieve a target orbit error measurement accuracy of 5 cm, the cross-track surface slope must be known to an accuracy of about 5 cm/km (0.003°). In this analysis we determine the overall slope at a point by fitting a plane surface to the set of repeat height measurements accumulated about that point. The residuals from the plane fit are then taken to be the relative orbit error associated with each repeat observation. Successive estimates of the orbit errors can be obtained along track and the results averaged to reduce the effect of random noise.

For a set of repeat tracks, the height measurements can be represented by $h(\mathbf{x}_{ij})$, $i = 1$ to N, $j = 1$ to M, where $\mathbf{x}_{ij}$ is the latitude and longitude position vector of the *j*th sample on the *i*th track. For a typical reference surface, the repeat tracks are effectively parallel and have equal sample spacing (~6.6 km for 1-Hz data), although the actual along-track sample location varies from track to track. Using a set of reference points, $\mathbf{r}_j$, parallel to the along-track direction and with a spacing equal to the sample spacing of the altimeter measurements, each repeat height measurement can be assigned to the nearest reference point. Figure 4 illustrates the geometry of the situation. If we now assume that the local surface around each reference point can be represented by a plane over the 6.6- by 2-km area of the repeat observations, we have

$$h(\mathbf{x}_{ij}) = h_o(\mathbf{r}_j) + \mathbf{b}(\mathbf{r}_j)\cdot(\mathbf{r}_j - \mathbf{x}_{ij}) + \Delta h_{ij} \quad (4)$$

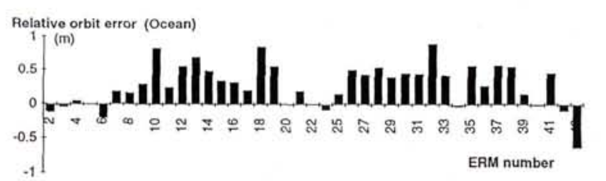

Figure 3. The relative orbit error at the Simpson Desert derived from the sinusoidal fits to repeat-track height differences over the ocean. ERM 22 was used as the reference orbit.

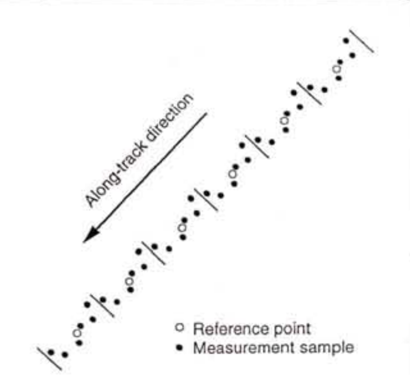

Figure 4. A schematic drawing showing the relationship between the samples of repeat-track measurements and a set of reference points. The bars delineate the areas over which a plane is fitted to the repeated height measurements.

where $h_o(\mathbf{r}_j)$ is the height at the reference point, $\mathbf{b}(\mathbf{r}_j)$ is the local gradient vector of the surface, and Δh_{ij} is the residual of the measurement. We assume that Δh_{ij} can be regarded as having a constant and a variable component

$$\Delta h_{ij} = \delta h'_j + \delta h_{ij}. \quad (5)$$

$\delta h'_j$ is the constant element with components from both the altimeter measurement corrections and the radial orbit error. δh_{ij} represents the variable components of the corrections and radial orbit error and is assumed to be a zero-mean random variable.

Substituting Equation 5 into Equation 4 and rearranging gives

$$h(\mathbf{x}_{ij}) = [h_o(\mathbf{r}_j)+\delta h'_j] + \mathbf{b}(\mathbf{r}_j)\cdot(\mathbf{r}_j - \mathbf{x}_{ij}) + \delta h_{ij}. \quad (6)$$

Fitting a plane surface to the set of repeat height measurements around each reference point allows the height, $[h_o(\mathbf{r}_j) + \delta h'_j]$, the gradient, $\mathbf{b}(\mathbf{r}_j)$, and the residuals, δh_{ij}, to be determined. The residuals, δh_{ij}, have three main components (see Equation 2):

$$\delta h_{ij} = \delta h_{\text{orb}_i} + \delta r_{\text{est}_{ij}} + \delta r_{\text{atmos}_i}. \quad (7)$$

δh_{orb_i} is the variable component of the orbit error for which we use the term relative orbit error. Given the size of a typical reference surface, the relative orbit error can be regarded as being constant along track (hence no *j* subscript) as its variation around the orbit is relatively slow. $\delta r_{\text{est}_{ij}}$ is the error in the range-estimation corrections and is composed mainly of the tracker noise and the off-pointing error. $\delta r_{\text{atmos}_i}$ is the error arising from the atmospheric and ionospheric corrections. It can also be regarded as being constant along track for a typical reference surface.

Assuming the components of Equation 7 are independent, then the variance of the required parameter, δh_{orb_i}, is equal to the sum of the variances of the other parameters. Providing there are a sufficient number of repeat observations to give a reasonable estimate of the surface gradient, and the variances of $\delta r_{\text{est}_{ij}}$ and $\delta r_{\text{atmos}_i}$ are small, then the residual, δh_{ij}, will be a good estimate of the relative orbit error, δh_{orb_i}. Note that the uncertainty in the relative orbit error estimates can be reduced to some extent by averaging the residuals along track.

Variations over time in the radar backscatter coefficient of the surface may also introduce changes in altimeter-measured heights through a change in the relationship between the mean radar surface and the mean geometric surface. The variability of the backscatter coefficient in deserts and the high Antarctica plateau is difficult to determine using Geosat data because of the severe effect of satellite off-pointing on backscatter measurements. Given the arid and unchanging nature of the areas under consideration, the effect is not expected to be significant at the levels of accuracy being considered here. Further investigation of backscatter stability and its effect on height measurement should be carried out with data from ERS-1 and Topex/Poseidon.

Simpson Desert Analysis

The sandy area of the Simpson Desert extends approximately between latitudes 23.5°S and 27°S and between longitudes 136°E and 138°E. Figure 5 shows the location of the four different Geosat ground tracks that pass over this area. Figure 6 gives the parameter profiles for height, backscatter coefficient, and significant wave height (SWH) for track 16 of ERM22 as an example of the typical behavior of the observations. The height profile reveals that the surface gradient is uniform and relatively smooth with a mean of ~27 cm/km (<0.02°). However, there are a few undulations towards the center of the profile where the SWH value is particularly high (~20 m). The return-echo waveforms in the Simpson area are known to be ocean-like (Guzkowska *et al.*, 1990; Chua *et al.*, 1991); therefore, the SWH values can be interpreted as representing the typical dune heights in the region.

The main steps in the analysis are

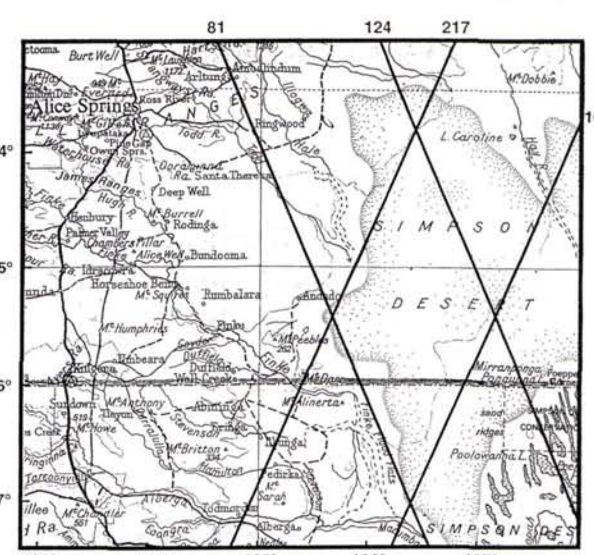

Figure 5. Location of the four Geosat ground tracks that pass over the Simpson Desert superimposed on an extract from *The Times Atlas* (Bartholomew, 1986). The region with the dotted outline shows the main area of sand dunes.

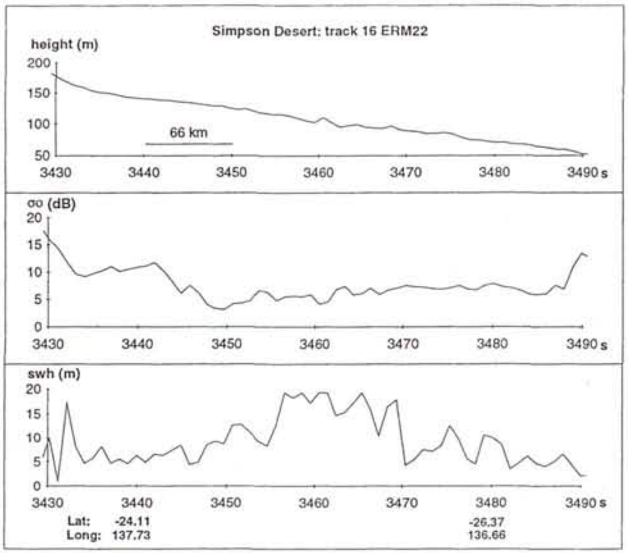

Figure 6. Typical parameter profiles for track 16 over the Simpson Desert. The *x*-axis is the time in seconds from the start of the orbit (ascending equator crossing). The height and radar backscatter coefficient (σ^o) profiles are relatively uniform, whereas the significant waveheight (SWH) shows a marked increase towards the center of the desert.

- Establish a set of ground-track reference points for the reference area,
- Extract a set of repeat height measurements for each reference point and fit plane surfaces, and
- Identify reference points with "acceptable" fit and average the residual along track for each ERM.

Sixty-two reference points for the track-16 repeats were established using the locations of the samples for ERM 22. ERM 22 was chosen because it was known from the ocean analysis to lie at the median of the spread of possible cross-track positions. The repeat height measurements about each of these reference points were extracted, and fitted with a plane using a standard least-squares method. Figure 7 shows the distribution of the samples about each reference point. The samples spread over a rectangle of dimensions 6.6 km by 2 km as a result of the along-track 1-Hz sample spacing and the ±1-km cross-track variation.

The goodness-of-fit statistic for each reference point showed that the assumption of the surface being a plane over the area of the samples is a reasonable one for most of the points. However, the RMS of the residuals at each reference point varied widely from 0.28 m (Point 58) to more than 2 m (Point 32) (see Figure 8). Because the relative orbit error is effectively constant for each reference point over the length of the Simpson, then the RMS of the residuals should also be constant. Clearly, the additional sources of error (from the range-estimation correction and the atmospheric correction) identified in Equation 7 are not insignificant. The error in the range-estimation correction has two main parts, namely, the tracker noise and the off-pointing error. These two components, together with the atmospheric correction error, are now discussed.

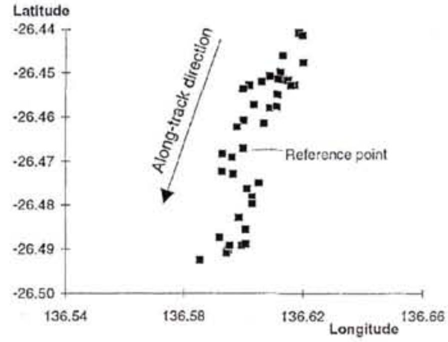

Figure 7. Typical distribution of repeat-track samples about each reference point for track 16 over the Simpson Desert.

Tracker Noise

The *precision* of Geosat height measurements is dominated by "tracker noise" and is 5 to 10 cm over the ocean for 1-Hz samples (a mean value every 6.6 km) (Cheney *et al.*, 1987). The precision is worse over the Simpson Desert because the dune height is larger than typical ocean waves and the terrain is more variable. An examination of very close repeat tracks (i.e., whose samples were within 50 m) showed the height tracker noise ranges from 30 to 50 cm RMS for dune areas with SWH typically <5 m. Differences of several metres were observed in areas of larger dunes.

In order to further investigate the effect of tracker noise and the averaging of data in the GDR data set, a number of tracks of waveform data for track 81 were processed. Analysis of a close repeat of threshold-retracked waveform data resulted in a height noise in the range 14 to 30 cm for dune areas with SWH <5 m, i.e., about half the value recorded for the non-retracked data in the GDRs. Clearly, the orbit-error analysis would be greatly improved if carried out with retracked waveform data. However, such data are not readily available for the Geosat mission. The few repeat tracks of waveform data for track 81 available to us were provided by special arrangement with the Applied Physics Laboratory, John Hopkins University.

Tracker noise is obviously a significant contributor to the observed RMS of the residuals. To limit the effect of tracker noise, the sections of track with the lowest SWH have to be

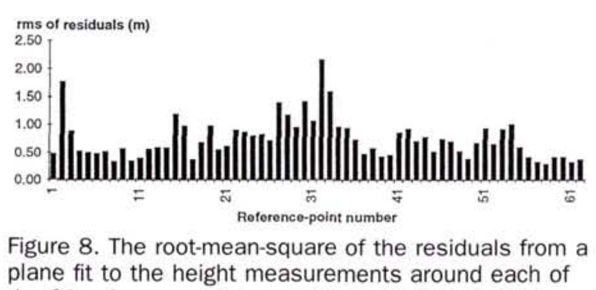

Figure 8. The root-mean-square of the residuals from a plane fit to the height measurements around each of the 62 reference points established for track 16 over the Simpson Desert.

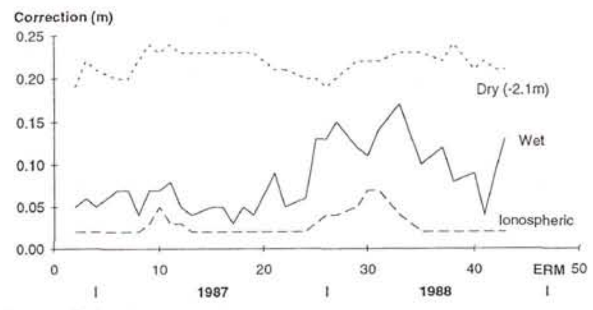

Figure 9. Variation of the FNOC Wet and Dry tropospheric corrections and the Ionospheric correction over the period of the analysis. The Dry correction has been offset by −2.1 m to place it on the same scale as the other corrections.

used and its effect further reduced by averaging the results along track.

Satellite Off-Pointing

The effect of Geosat off-pointing contributes to the residuals, and this will be exaggerated by the relatively large SWH in the Simpson Desert, particularly towards the center of the region. It was mentioned earlier that an off-pointing correction was applied over the ocean, but not over land, because it is difficult to estimate correctly. The off-pointing is calculated over the ocean from the slope of the trailing-edge of the return echo. However, over land the trailing-edge slope is dominated by terrain effects and the standard off-pointing calculation does not give the correct result. Fortunately, examination of the off-pointing from the ocean either side of Australia indicates that it rarely exceeded 0.5° over the Simpson, so for dune areas with SWH <5 m, the off-pointing error was nearly always in the range 10 to 17 cm. In addition, there did not appear to be a significant correlation between the off-pointing angle and the difference between the ocean and land orbit-error estimates derived later. We conclude therefore that the effect of off-pointing is less than that due to tracker noise for this area.

Atmospheric Correction Error

As discussed for ocean data, the wet tropospheric correction is likely to be the main source of error in the atmospheric correction. However, one of the advantages of using an arid region as a reference surface is that the water-vapor content of the atmosphere is likely to be low, so the error in the correction will be correspondingly small. Figure 9 shows the variability of the wet, dry, and ionospheric corrections versus ERM (i.e., every 17 days) for the period of the analysis. The wet tropospheric correction reveals that during 1988 there was significantly more water vapor in the atmosphere compared to 1987. Nevertheless, the correction rarely exceeds 15 cm, and even assigning a 50 percent error to the FNOC values gives a maximum error of 8 cm. The contribution to the RMS of the relative orbit error is just a few centimetres. This is acceptable given the other sources of error.

Distribution of Orbit Error

One of the assumptions required for the plane-fitting technique is that the relative orbit error is a zero-mean random variable, that is, its value does not depend on location over the area of the fit. To test this assumption, the interpolated relative orbit errors from ocean fitting were assumed correct and assigned to the relevant samples about an arbitrary reference point. If the relative orbit error is a zero-mean random variable, then the fitted plane would show no significant slope. In addition, the difference between the residuals and the input values gives an indication of the effect of the error in the slope estimate. The results of the fitting were

No. of samples in fit	= 40
Lat. gradient (m_1)	= 5.4 ±5 cm/km
Long. gradient (m_2)	= −13.4 ±13 cm/km
St. dev. of input/residual differences	= 6.0 cm

The observed latitude and longitude gradients are not significant given the error associated with the fit, showing the zero-mean random variable assumption to be valid. The differences between the input values and the measured residuals have a standard deviation of 6 cm. This implies that, when fitting a plane to the real observations, the variation of the orbit error over the plane will contribute ~6 cm RMS towards the overall error budget. This is acceptable given the size of other contributing errors. It must be pointed out that this error analysis applies for 40 repeats with the given ground pattern for track 16 (Figure 7), and the result may be different for a different number of repeats with a different ground pattern (see Antarctic analysis in next section).

Relative Orbit Error Calculation

Despite the large variation in the residuals seen in Figure 8, there are a number of points, mostly lying towards the end of the track, where the RMS of the height residuals is less than 40 cm. This is close to the expected value of about 30 cm for the GEM-T2 orbit, and, as the SWH is less than 5 m here, the residual is almost certainly dominated by orbit error rather than tracker noise and off-pointing. The four points with the lowest residuals were selected and the mean residual for each ERM was calculated (Figure 10). We interpret these mean residuals as the relative orbit error as derived over land, and in a later section we compare them with the ocean-derived values. Averaging the residuals from these four points helps to reduce the effect of tracker noise, but the off-pointing and atmospheric correction errors act as a bias over short track lengths and will not be reduced. This is one of the drawbacks of using land reference surfaces as opposed to the ocean surface around an orbit.

Antarctic Analysis

Data from the section of track 16 that passes over the Antarctic Plateau appear to be relatively smooth and uniform. Data

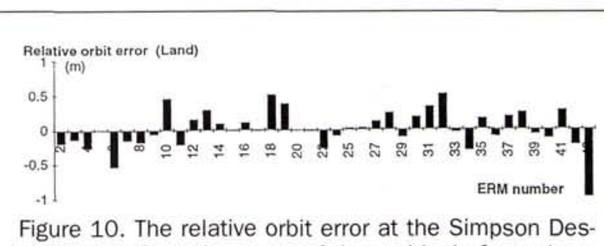

Figure 10. The relative orbit error at the Simpson Desert derived from the mean of the residuals from plane fits to the height measurements around four reference points.

spanning the longitude range of 48°E to 55°E near the latitudinal limit of −72° for Geosat were extracted and processed. Figure 11 illustrates typical profiles for the parameters height, radar backscatter coefficient (σ^o), and SWH taken from ERM 22. It can be seen that the σ^o is relatively constant and that the SWH is lower than that over the Simpson, suggesting that the region should be good for surface fitting.

Unfortunately, as a result of increased Geosat off-pointing in polar regions, about half the passes in the Antarctic data set fail to maintain lock of the surface. This loss of data, while obviously reducing the number of orbits that can be analyzed, also increases the error associated with estimating the surface slope.

To determine the effect of the reduced number of orbits and to test again if the relative orbit error behaves as a zero-mean random variable, the interpolated relative orbit error from the ocean was assigned to the repeat observations around an arbitrary reference point as described for the Simpson analysis. For the full set of repeat observations, the results of the plane fitting were

No. of samples in fit	= 40
Lat. gradient (m_1)	= −45.3 ±14 cm/km
Long. gradient (m_2)	= 0.5 ±2 cm/km
St. dev. of input/residual differences	= 11 cm

In contrast to the Simpson result, there appears to be a marked correlation between the latitude (cross-track location) of the point and the orbit error. This results in a significant slope to the fitted plane, and the standard deviation of the difference between the input orbit error and output residuals is 11 cm, compared to 6 cm for the Simpson.

The situation is made worse by the fact that half the data are lost through loss of lock. If we just use the 20 points for which we actually have Antarctic observations, then the discrepancies are larger, the standard deviation of the difference being an unacceptable 24 cm, as shown below:

No. of samples in fit	= 20
Lat. gradient (m_1)	= −61.7 ±16 cm/km
Long. gradient (m_2)	= 1.3 ±3 cm/km
St. dev. of input/residual differences	= 24 cm

The relative orbit error shows a significant gradient over the area of the fit, so the residuals from a plane fit to the height measurements cannot be interpreted as orbit error. The relationship between orbit error and cross-track location is surprising, given the small size of the cross-track displacement. The effect may be related to the fact that the data are near the latitudinal limit of the orbit and that errors in the geoid models used in calculating the orbit ephemeris are larger near the poles. This problem may not be as severe for the ERS-1 altimeter because the latitudinal limit of −82° will allow observations over flatter regions of the plateau away from the limit. Geoid models with improved representation of the Antarctic will also be used in the orbit calculations.

We have to conclude that these Geosat data over the plateau cannot be used to estimate relative orbit error because of our inability to determine the cross-track surface slope. Further work will be required to investigate whether alternative techniques could be used to determine the slope. Possible alternatives include using adjacent orbits, or regions of the Antarctic with particularly low surface slopes, such as ice shelves or areas with sub-glacial lakes.

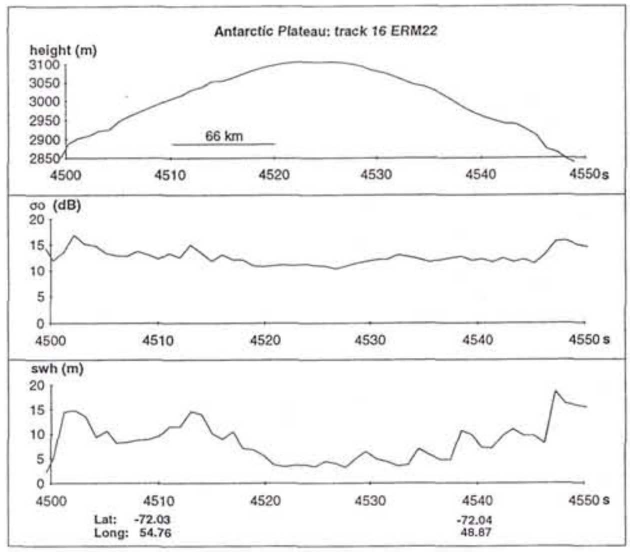

Figure 11. Typical parameter profiles for track 16 (ERM 22) over the section of the Antarctic Plateau considered as a reference surface. The *x*-axis is the time in seconds from the start of the orbit (ascending equator crossing).

Our inability to use the Antarctic Plateau as a reference surface is unfortunate because the plateau has the potential to act as a reference surface for many orbits and is almost essential if interpolated relative orbit errors are to be derived from land reference surfaces.

Comparison of Ocean and Land Results

Figure 12 shows the ocean-derived value of relative orbit error for each ERM (as shown in Figure 3) plotted against the corresponding land-derived value (as shown in Figure 10). A consistent relationship is revealed, with a mean offset of 30 cm due to the fact that the ocean values were derived relative to ERM 22 whereas the land values were derived relative to a mean orbit error. For the land observations, ERM 22 is 25 cm from the mean, which is consistent with the 30-cm offset, given the error in the individual altimeter measurements. The presence of such an offset is not relevant for relative orbit error analysis.

Figure 13 shows the difference between the ocean and land derivations (after the subtraction of the 30-cm offset). The RMS of the difference is 12 cm (51 cm peak to peak), and this represents the worst-case uncertainty in using either technique. The differences should be the RMS sum of the errors in the individual techniques, because the main sources of error are independent. Assigning the error equally suggests an accuracy of about 9 cm RMS in the use of either technique. However, there are a number of reasons to suggest that this combined error is dominated by errors in the land measurement:

- The tracker noise is significant because only limited along-track averaging of the data has been possible;
- The errors associated with off-pointing have not been accounted for over land; and
- In some of the cases that have a large discrepancy between the land and ocean estimate, the ocean fit looks particularly good.

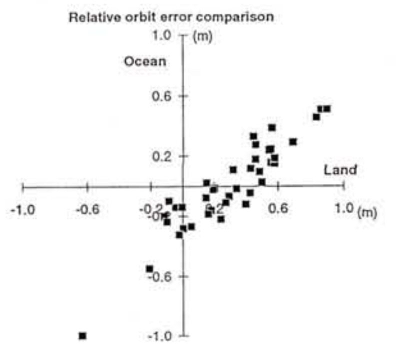

Figure 12. Comparison of the ocean-derived relative orbit error estimates with those obtained using the Simpson Desert height measurements. The non-zero intercept is not relevant for relative orbit error analysis.

The use of ocean data around the orbit as a reference surface is likely therefore to give better results than using desert data, at least for Geosat observations over regions containing dunes a few metres or more in height. The quality of relative orbit error estimate from the ocean data is therefore likely to be better than the 9-cm equal division of error might suggest.

There are a number of ways in which the analysis of the data could be improved. Over the ocean, the residual after the fit could be analyzed to exclude regions of high variability, improved wet tropospheric corrections (available in the CD-ROM edition of the GDR data (Cheney *et al.*, 1991)) and the barometric correction could be applied, together with the revised SWH corrections of Hayne and Handcock (1990). Over the land, the distribution of points around the reference point could be improved by repositioning the 1-Hz height values by recalculating the means from the 10-Hz height data available in the GDR product. In addition, the surface slope estimate could be improved by using a more recent Geosat orbit ephemeris, e.g., the University of Texas Geosat ERM Ephemeris which is available for the first 44 ERMs and has a quoted radial accuracy of 17 cm RMS (C. Shum, private communication). However, none of these improvements are worth pursuing while the land analysis is dominated by the tracker noise inherent in the GDR data. The most significant

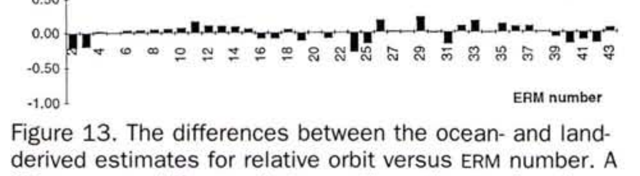

Figure 13. The differences between the ocean- and land-derived estimates for relative orbit versus ERM number. A 30-cm mean difference between the estimates has been removed.

improvement would come from the use of retracked waveform data.

Discussion

This analysis has demonstrated that the mean ocean surface around an orbit provides a better reference surface for the altimeter than does a desert region. However, many of the problems associated with the land may be particular to the use of Geosat data. Current altimeter missions (e.g., ERS-1 and Topex/Poseidon) should have improved attitude control and better access to waveform data (for retracking). Eventually, the limiting factor in the use of land surfaces as height references will be the ability to determine the cross-track surface slope.

For some applications, it may be possible to select land surfaces known to have zero surface slope, such as salars or clay pans. However, this would severely limit the number of orbits that could be analyzed. Lakes with monitored water levels might also be used as a height reference, but the effect of wind set-up and denivellations, which could introduce errors of several decimetres, might be significant. Further research is required in this area.

The current advantage of Geosat data over data from current altimetric satellites is the large quantity of repeat data available. Although later altimeter missions are likely to produce better land observations than Geosat, it will be several years before the number of repeat orbits available is sufficient for self cross-track slope determination. In the meantime, it should be possible to use Geosat data to determine the local slope at, for example, Geosat/ERS-1 orbit crossovers. This will allow the determination of relative orbit error for later satellites with very little repeat data.

In the longer term, Topex/Poseidon has the best prospect for accurate local slope estimation because it is intended that this satellite will accumulate 5 years of altimetric data in a 10-day repeat cycle. Unfortunately, the orbit separation is rather large (~300 km), which may limit some of the applications of the data. The ERS-1 satellite, in its 35-day repeat phase, will give better ground coverage with 80-km orbit spacing, but will acquire less than 20 repeats. ERS-1 will obtain about 30 repeats in each of the two 3-month ice-phases of the mission, but again the large orbit separation may limit the applicability of the data.

Summary and Conclusions

The analysis demonstrates that both ocean and land reference surfaces can be used to improve on the relative orbit error of the Geosat orbit ephemerides currently available. Assigning the error from the comparison equally gives 9 cm RMS error compared with the ~20 to ~50 cm RMS radial orbit error over the Simpson for the GEM-T2 orbit ephemeris used in the analysis. The ocean technique appears slightly superior to that of land because some of the contributing errors are reduced through the averaging effect of analyzing data from a whole orbit. Also, the Geosat land data used here suffered from tracker noise and off-pointing.

The Antarctic Plateau cannot be recommended as a reference surface because our analysis revealed that the Geosat orbit error was correlated with the cross-track location of the orbit and so did not behave as a zero-mean random variable over the 2-km spread of ground tracks. This prevented the determination of the cross-track surface slope, an essential prerequisite for the use of land reference surfaces. It has yet to be determined whether all Geosat orbits over the Antarctic are affected in this way.

Using the ocean also has the advantage that it is easy to interpolate the result to any location. For land, unless the reference surface is close to the area under analysis, then two or more suitable land reference sites along the orbit are required for interpolation, and this may not be easy to achieve given the problems with the Antarctic data and the effect of large dunes in deserts.

Data from later altimetric missions such as ERS-1 and Topex/Poseidon will potentially yield better results over land than those presented here because these altimeters have better satellite attitude control and there should be better access to waveform data for retracking. The land technique is important for some ocean-related applications because the relative orbit error is determined completely independently of the ocean surface. In the longer term, it may be possible to use land surfaces as a reference datum for sea-level change measurements.

The accuracy of the relative orbit error determination suggests that desert areas could also be used as absolute height references for orbit determination if the surface height was known in the same reference frame as the satellite orbit (e.g., using GPS) and the relevant altimeter biases were also known.

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References

Bartholomew & Son Ltd., 1985. *The Times Atlas of the World*, Times Books Ltd, London.

Cartwright, D.E., and R.D. Ray, 1990. Ocean Tides from Geosat Altimetry, *J. Geophys. Res.*, 95(C3):3069–3090.

Chua, P.K., A.H.W. Kearsley, J.K. Ridley, W. Cudlip, and C.G. Rapley, 1991. The Determination and Use of Orthometric Heights Derived from the Seasat Radar Altimeter over Land, *Photogrammetric Engineering & Remote Sensing*, 57(4):437–445.

Cheney, R.E., B.C. Douglas, R.W. Agreen, L. Miller, D.L. Porter, and N.S. Doyle, 1987. *Geosat Altimeter Geophysical Data Record (GDR) User Handbook*, NOAA Technical Memorandum NOS NGS-46, Natl. Ocean Serv., N/CG112, NOAA, Rockville, Maryland, 32 p.

Cheney, R.E., N.S. Doyle, B.C. Douglas, R.W. Agreen, L. Miller, E.L. Timmerman, and D.C. McAdoo, 1991. *The Complete Geosat Altimeter GDR Handbook*, NOAA Technical Memorandum NOS NGS-7, Natl. Ocean Serv., N/CG112, NOAA, Rockville, Maryland, 80 p.

Geosat Special Issue I, 1990. *J. Geophys. Res.*, 95(C3):1.

Guzkowska, M.A.J., C.G. Rapley, J.K. Ridley, W. Cudlip, C.M. Birkett, and R.F. Scott, 1990. *Developments in Inland Water and Land Altimetry*, ESA CR-7839/88/F/FL, ESA Scientific and Technical Publ. Branch, ESTEC, Noordwijk, Holland.

Emery, W.J., G.H. Born, D.G. Baldwin, and C.L. Norris, 1990. Satellite-Derived Water Vapor Corrections for Geosat Altimetry, *J. Geophys. Res.*, 95(C3):2953–2964.

Haines, B.J., G.H. Born, Rosborough, J.G. Marsh, and R.G. Williamson, 1990. Precise Orbit Computation for the Geosat Exact Repeat Mission, *J. Geophys. Res.*, 95(C3):2871–2886.

Hayne, G.S., and D.W. Hancock III, 1990. Corrections for the Effects of Significant Wave Height and Attitude on Geosat Radar Altimeter Measurements, *J. Geophys. Res.*, 95(C3):2837–2842.

Martin, T.V., A.C. Brenner, H.J. Zwally, and R.A. Bindschadler, 1983. Analysis and Retracking of Continental Ice Sheet Radar Altimeter Waveforms, *J. Geophys. Res.*, 88(C3):1608–1616.

Rapley, C.G., 1990. *Microwave Remote Sensing for Oceanographic and Marine Weather-Forecast Models* (R.A. Vaughn, editor), Kluwer Academic Publishers, pp. 45–63.

Rapley, C.G., M.A.J. Guzkowska, W. Cudlip, and I.M. Mason, 1987. *An Exploratory Study of Inland Water and Land Altimetry Using Seasat Data*, ESA Contract Report 6483/85/NL/BI, ESA Scientific and Technical Publ. Branch, ESTEC, Noordwijk, Holland.

Seasat Special Issue I, Geophysical Evaluation, 1982. *J. Geophys. Res.*, 87(C5):1.

Tapley, B.D., G.H. Born, and M.E. Parke, 1982. The Seasat Altimeter Data and its Accuracy Assessment, *J. Geophys. Res.*, 87(C5): 3179-3188.

Van Gysen, H., R. Coleman, R. Morrow, B. Hirsh, and C. Rizos, 1992. Analysis of Collinear Passes of Satellite Altimeter Data, *J. Geophys. Res.*, 97(C2):2265–2277.

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